Basics of RF superconductivity and Nb material

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Tutorial program Jun 22nd 2023 SRF2023 @ Michigan State University

Outline

- Introduction: why superconducting RF?
- Finite surface resistance of superconductors
  - Superconductors in equilibrium
  - BCS resistance
  - Residual resistance
- Field limitations
  - Fundamental limit
  - Practical limits
- Niobium as a cavity material
  - Required feature
  - Beyond niobium

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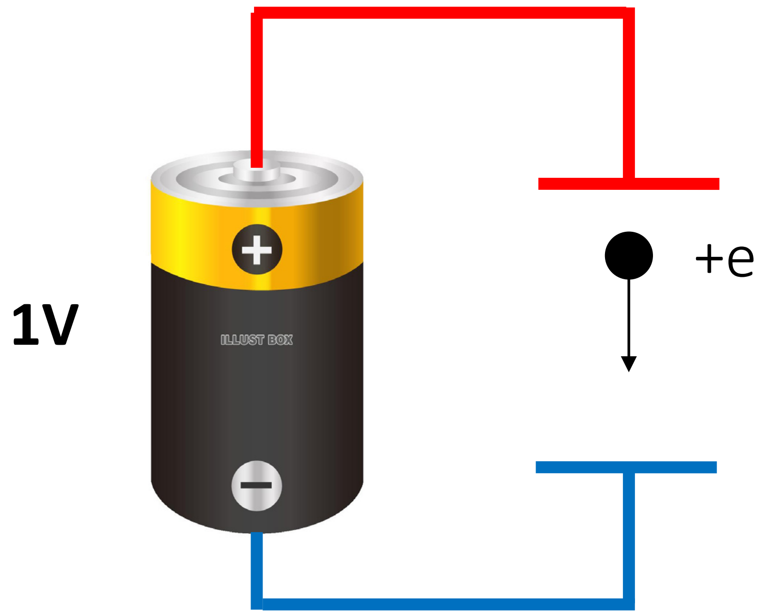
How to accelerate charged particles



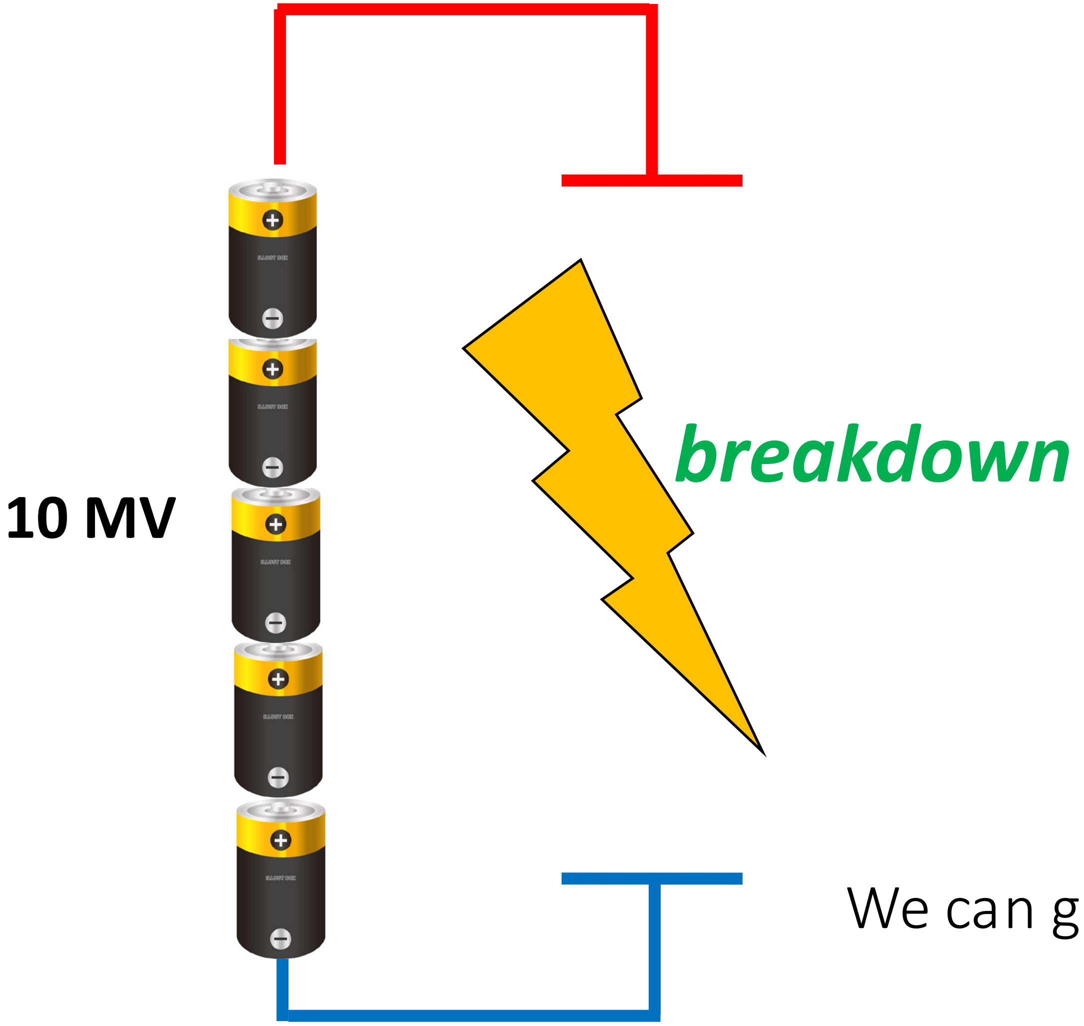
Electron's rest mass
in the natural unit
$m(c^2) = 511$ keV

Kinetic energy of a charge +e (1.6×10^{-19}C) accelerated by 1 V

$E = 1$ **eV**

Modern science ≫ MeV (Neutrons>**1GeV**, hard X-rays>**10GeV**, Higgs boson>**125+90 GeV**)

DC cannot provide high accelerating gradient ($\boldsymbol{E_{acc}}$)



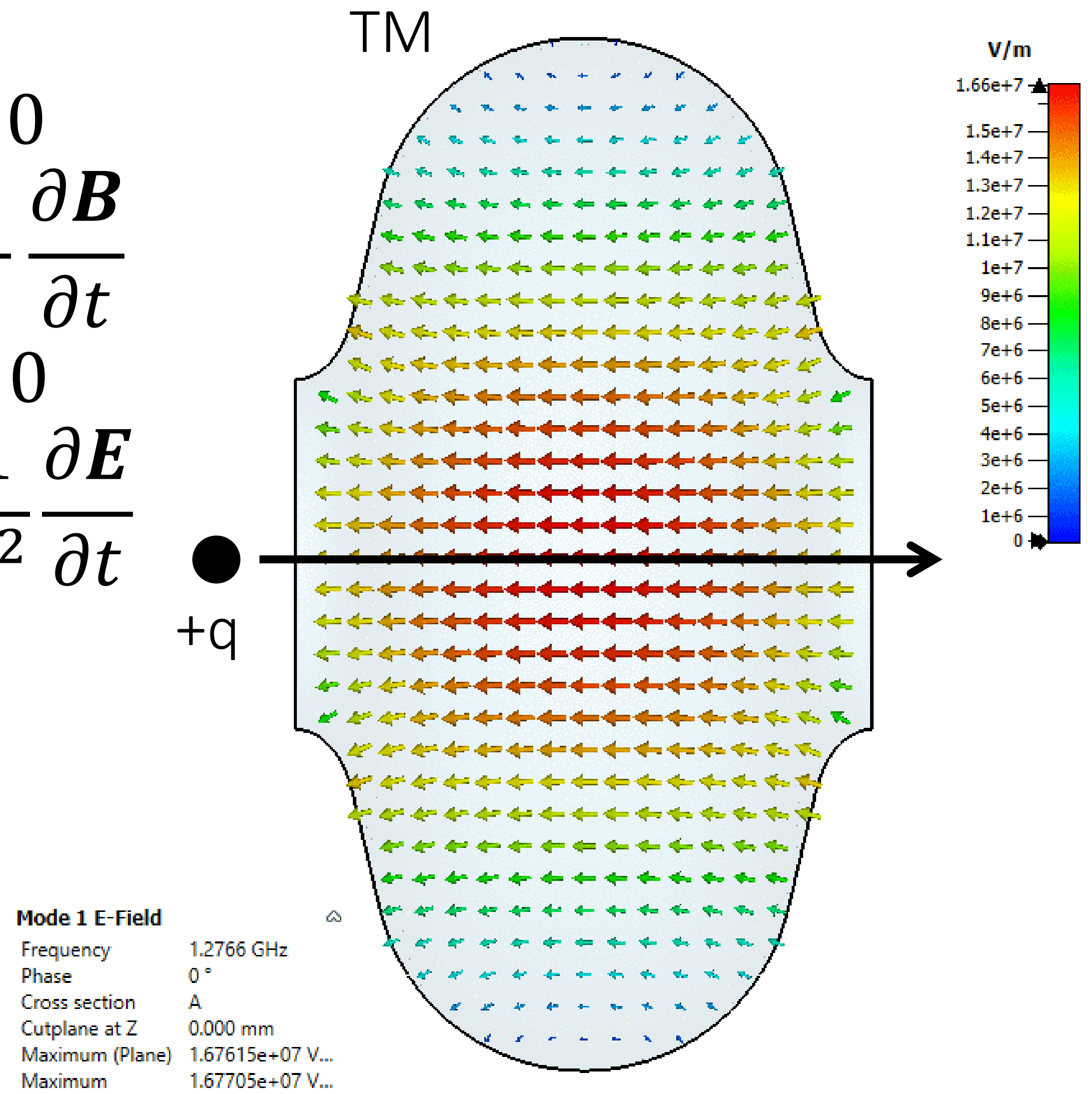
We can generate high DC voltage but is limited to O(10MV)

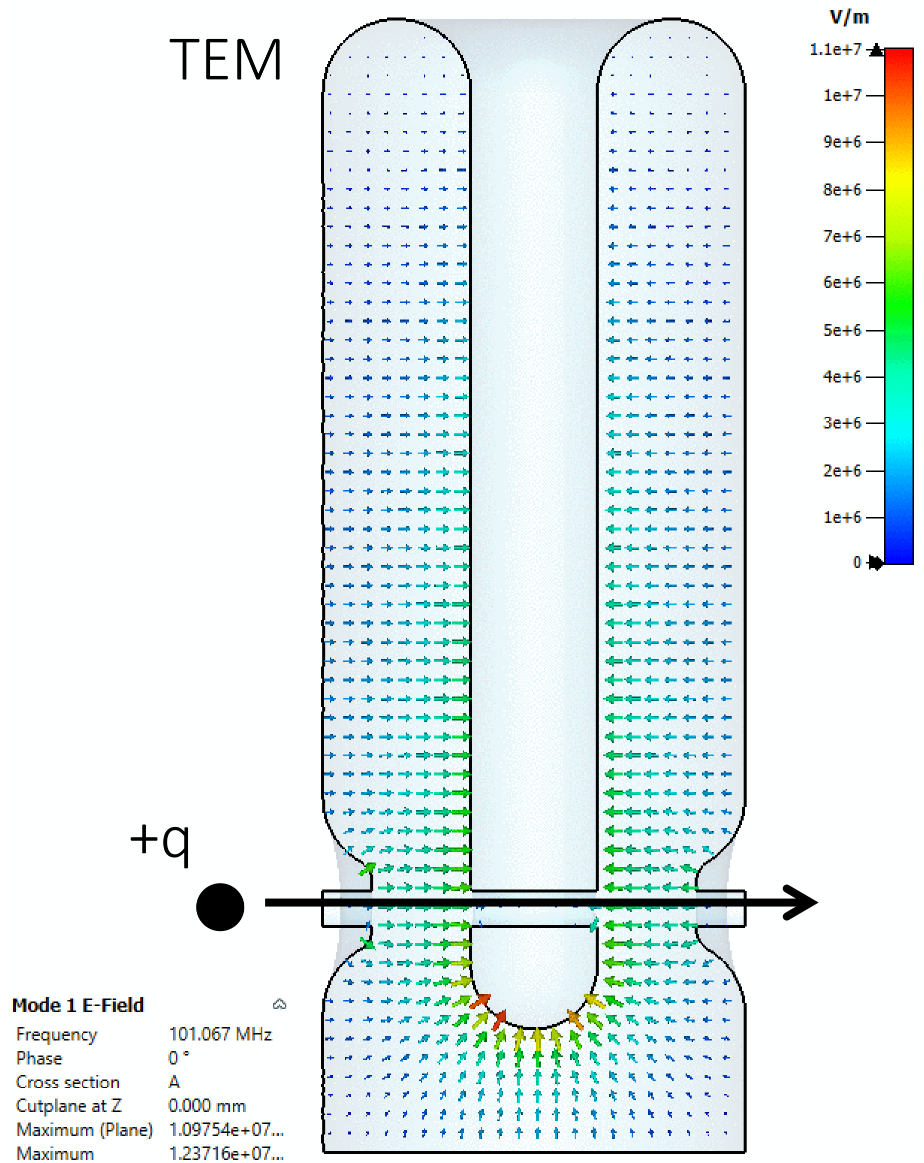
→ For GeV science **RadioFrequency (RF)** is one option

Confine electromagnetic waves inside RF resonant cavities

Charged particles synchronized with RF can be accelerated

$$
\begin{cases}
\nabla \cdot \boldsymbol{E} = 0 \\
\nabla \times \boldsymbol{E} = -\dfrac{\partial \boldsymbol{B}}{\partial t} \\
\nabla \cdot \boldsymbol{B} = 0 \\
\nabla \times \boldsymbol{B} = \dfrac{1}{c^2}\dfrac{\partial \boldsymbol{E}}{\partial t}
\end{cases}
$$

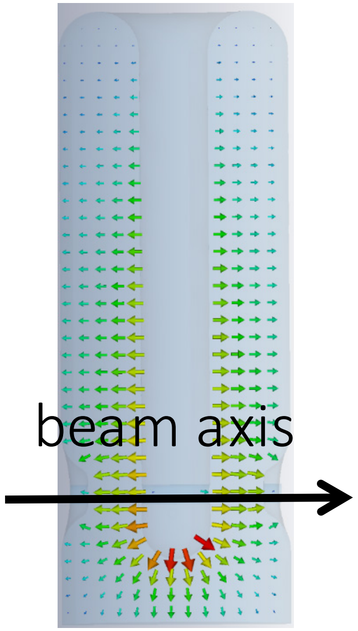




Lecture by
M. Nishiwaki
D. Valuch

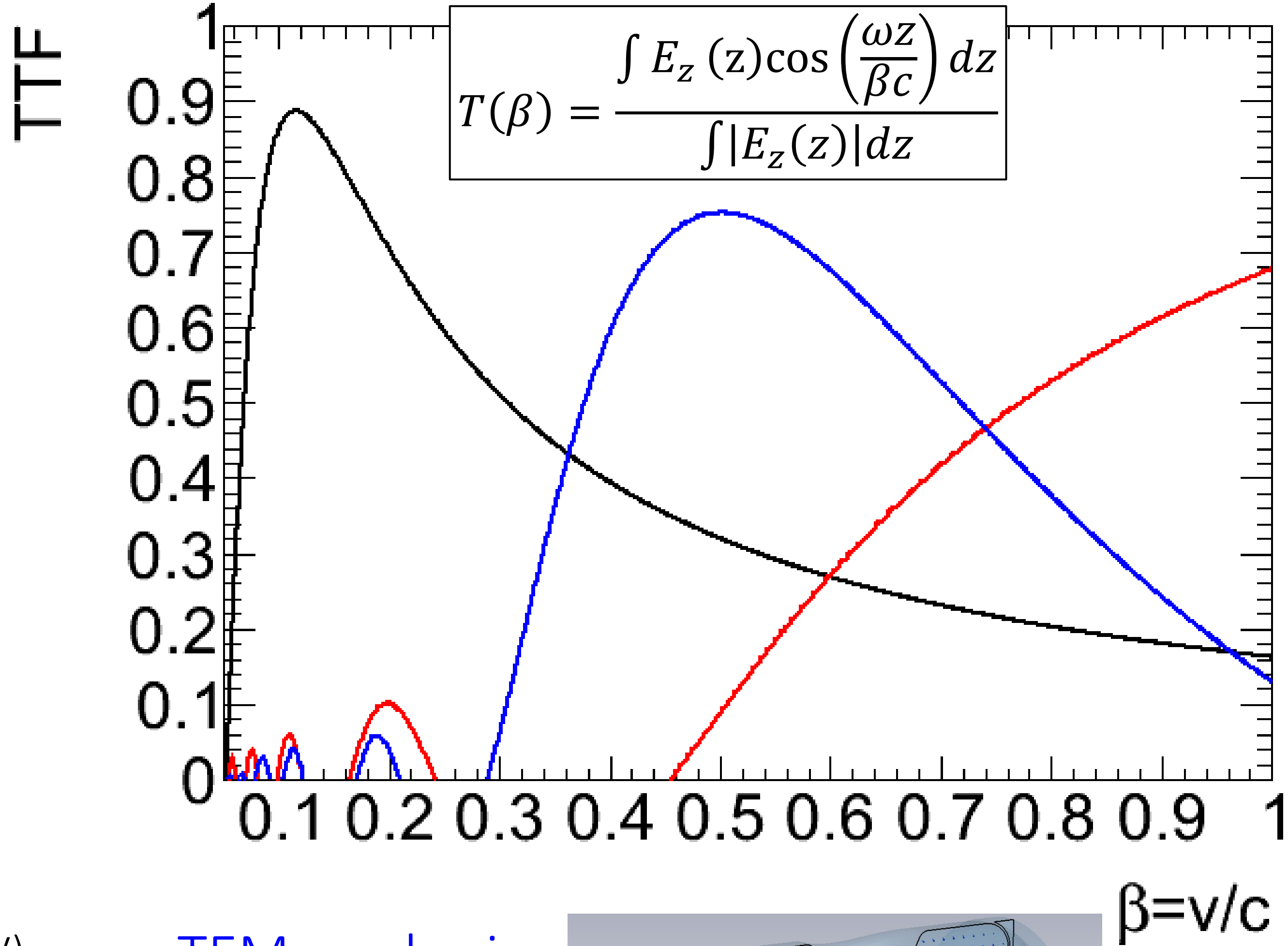
Geometrical consideration: low-β , middle-β, and high-β

TEM_{00} modes in a quarter-wave or half-wave cavity



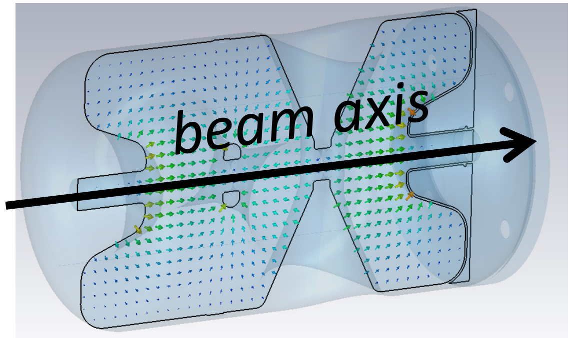
- p+ upstream (<1GeV)
- Heavy ion

Lecture by Subashini De Sliva

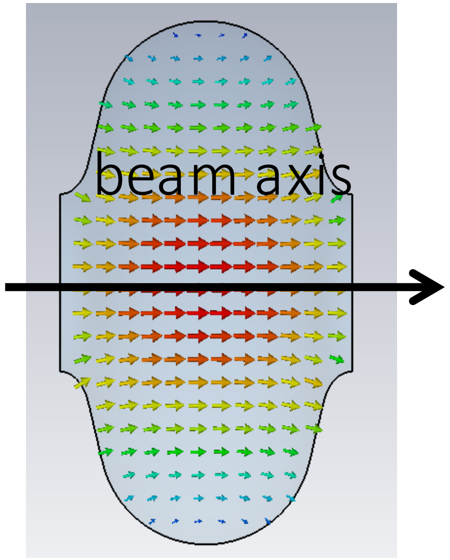


TEM modes in a spoke cavity

- p+ (<1GeV)



TM_{010} modes in an elliptical cavity



- p+ downstream (>1GeV)
- e-, e+ (>0.5MeV)

Lecture by Rongli Gen

Our interest: (unloaded) quality factor

Higher Q → higher field E_{acc} with smaller power dissipation P_c

Geome trical

$$Q_0 = \frac{\omega U}{P_c} = \frac{\kappa E_{acc}^2}{P_c}$$

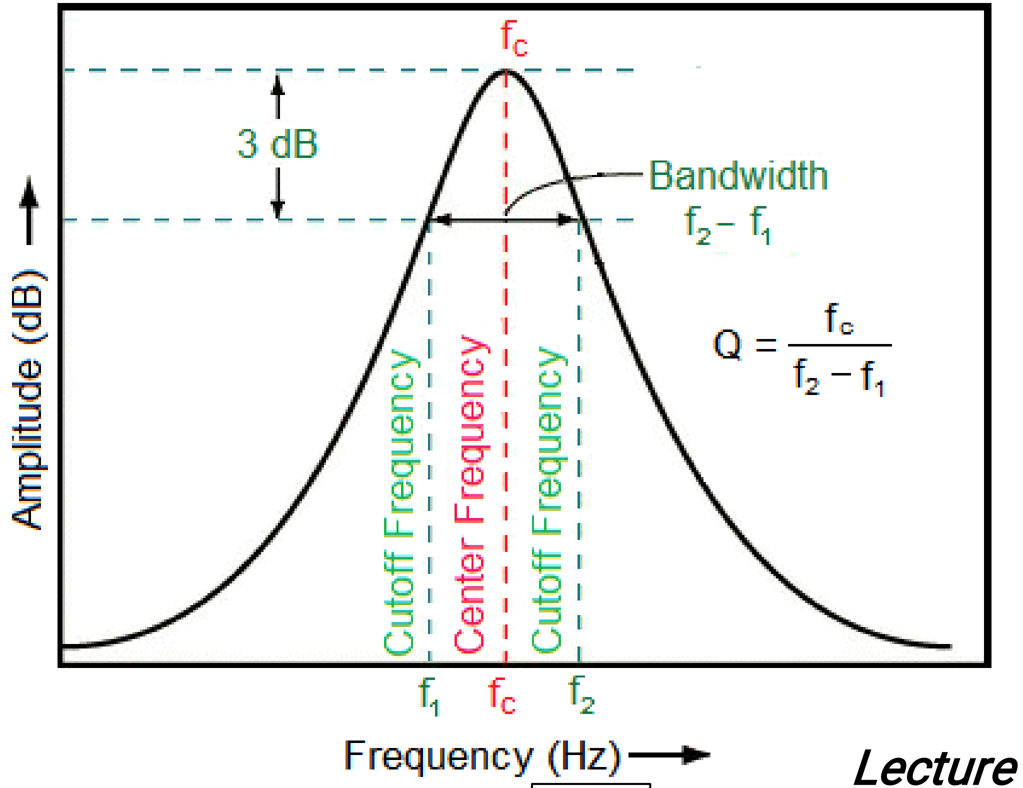
Smaller surface resistance R_s
→ high Q & low P_c

Geometrical

Experimental observable $$Q_0 = \frac{G}{R_s}$$ From material

Experimental observable $$P_c = \frac{\kappa R_s}{G} E_{acc}^2$$

http://lossenderosstudio.com/glossary.php?index=q



Lecture by W. Hartung

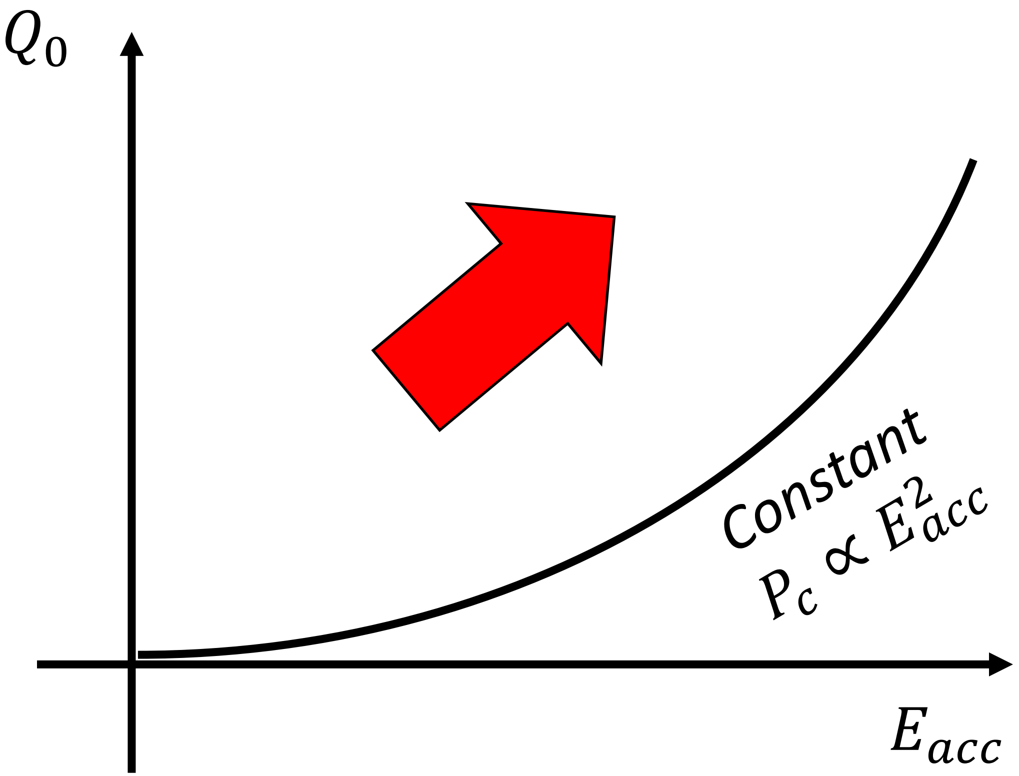
Q1

G is a geometrical factor

- Elliptical cavity $G \sim 250\ \Omega$
- Spoke cavity $G \sim 133\ \Omega$
- Quarter-wave resonator $G \sim 30\ \Omega$

SRF cavity has <1Hz bandwidth. How to measure? Hint: Fourier transform

High-Q (Q_0) and high-gradient (E_{acc}) is the keyword



One of our goals in SRF is to go

High-gradient: E_{acc}

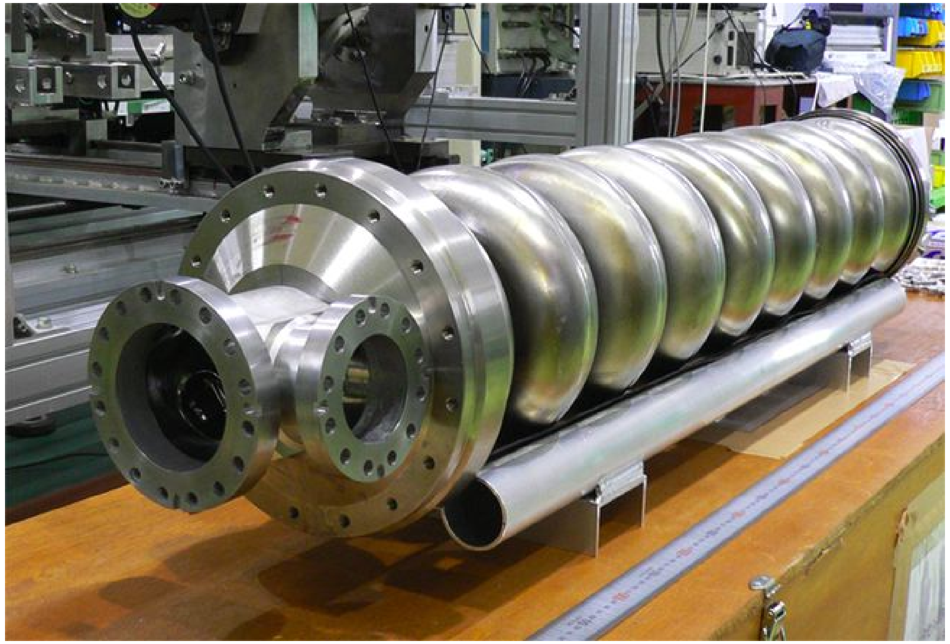
with lower power consumption P_c

High-Q: $Q_0 = \frac{G}{R_s}$

We first consider lower $\boldsymbol{R_s}$

Superconducting

cavity



cryogenics

Lecture by N. Hasan

https://www.123rf.com/stock-photo/old_radio.html?sti=mbqiuc5egnu4ynov6q|&mediapopup=41753240

Superconducting cavity for $R_s \to 0$?

Ohm's law

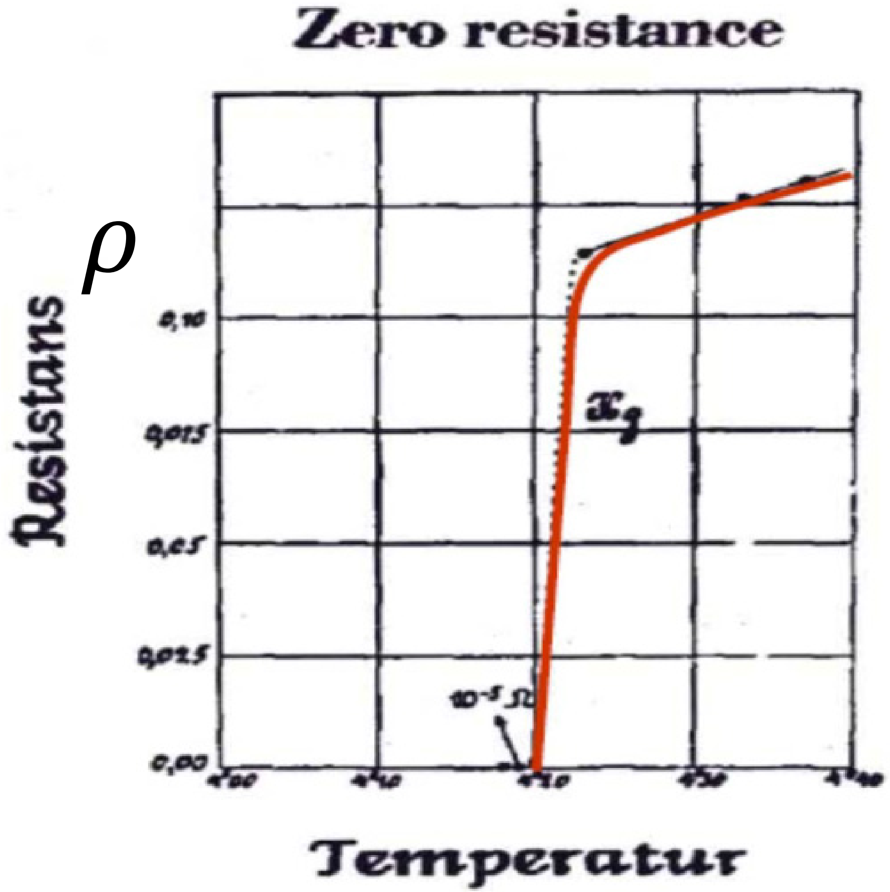
DC resistivity ρ

$$\rho \equiv \frac{E}{J}$$

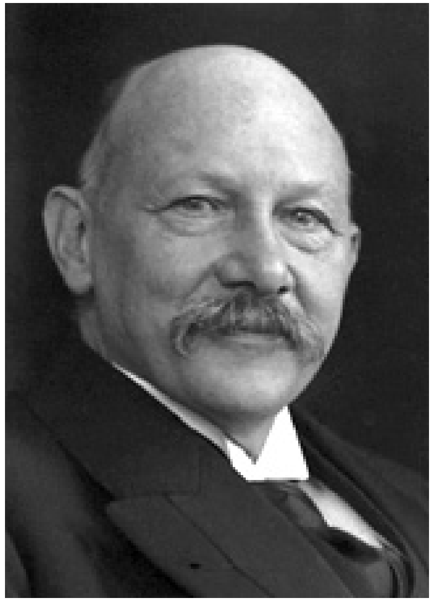
DC conductivity σ

$$\sigma = \frac{1}{\rho} \equiv \frac{J}{E}$$

Cool down the resistor…



$\boldsymbol{\rho = 0}$ below transition temperature T_c

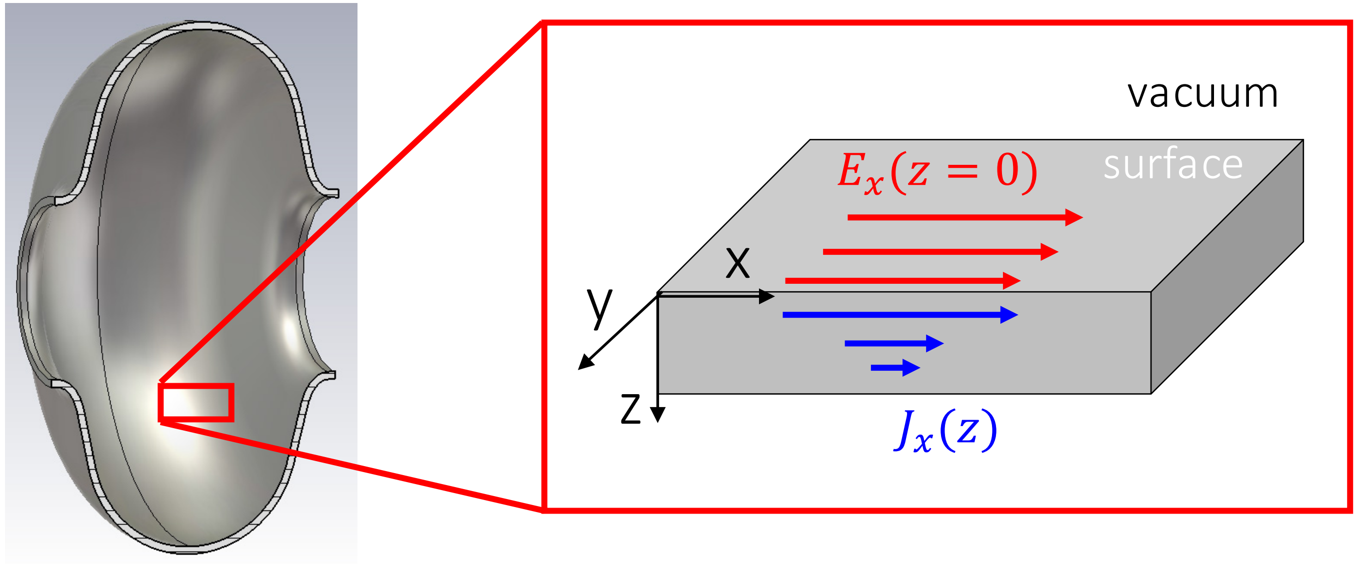


Heike Kamerlingh Onnes

Nobel prize in 1913

RF resistance R_s is non zero

Materials provide boundary conditions with finite power dissipation

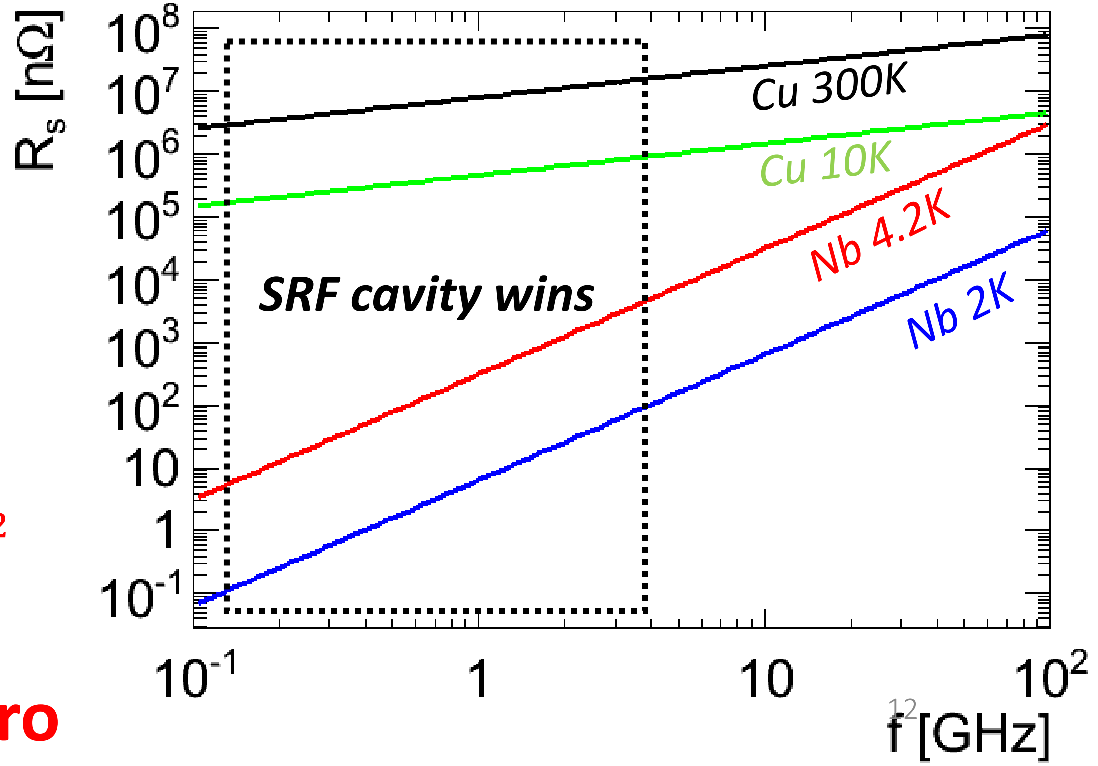


Local surface resistance

$$R_s \equiv \mathrm{Re}\left(\frac{E_x(z=0)}{\int_0^\infty J_x(z)dz}\right)$$

Normal conducting (Cu) $\quad R_s = \sqrt{\frac{\pi f \mu_0}{\sigma}} \propto f^{1/2}$

Super- conducting (Nb) $\quad R_s = \frac{A f^2}{T}\exp\left(-\frac{\Delta}{k_B T}\right) \propto f^2$



Superconducting R_s is small but non zero

After this lecture, you will be able to answer…

1. What are the intrinsic and extrinsic origins of finite R_s and RF loss in SRF cavities?
2. What are the fundamental and practical limitations of the field E_{acc} inside SRF cavities?
3. What is the requirement for materials and why niobium?

I also list up open questions on the research frontier

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Challengers for **microscopic** theory of superconductors

J. Schmalian, arxiv:1008.0447

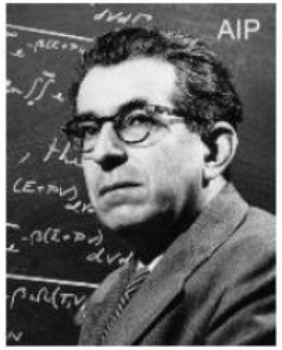
Albert Einstein (1879-1955)

Niels Bohr (1885-1962)

Ralph Kronig (1905-1995)

John Bardeen (1908-1991)

Werner Heisenberg (1901-1976)



Fritz London (1900-1954)

Lev D. Landau (1908-1968)

Felix Bloch (1905-1983)

Léon Brillouin (1889 -1969)

Max Born (1882-1970)

Herbert Fröhlich (1905-1991)

Richard Feynman (1918-1988)

A lot of models…all failed ☹

Development of quantum field theoy in many body problems was necessary…

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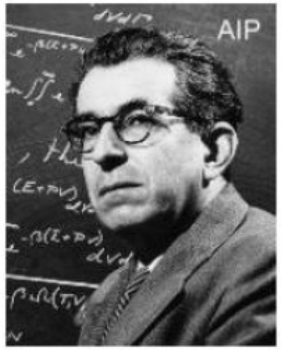

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Feynman tried to get superconductivity by **perturbation theory** including attraction forces between electrons caused by lattice vibration → **failed** ☹

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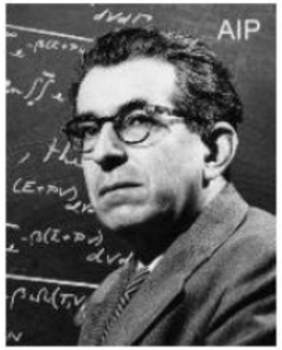

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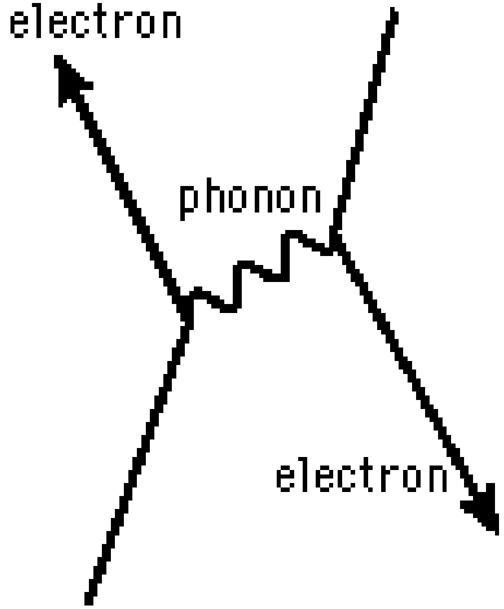
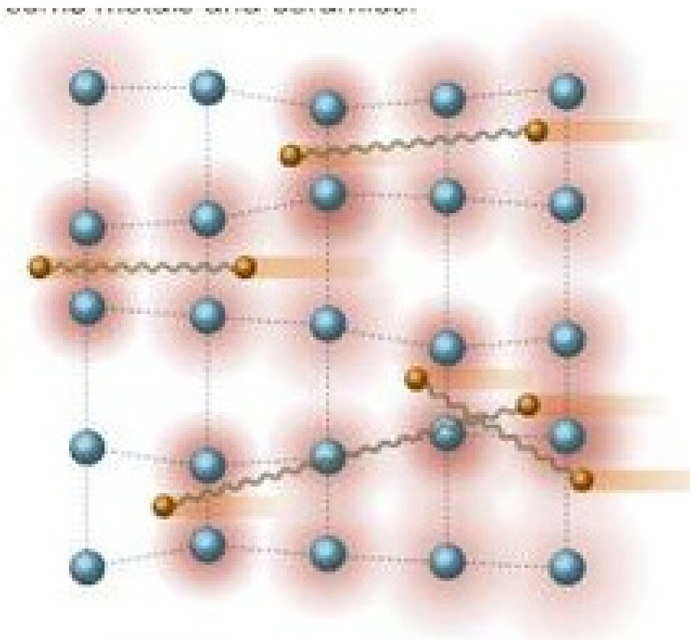
Bardeen and Fröhlich had a good idea but needed young talents

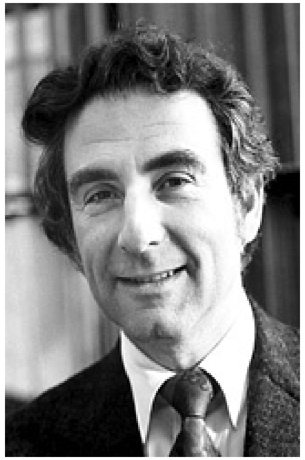
- Many body problem (Quantum field theory)
- Application of techniques developed in particle physics

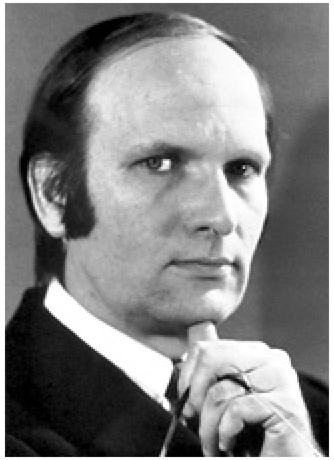
Theory of superconductor in equilibrium






John Bardeen


Leon Cooper


John Robert Schrieffer

<u>Cooper pair: Composite boson</u>

Two electrons are bounded by something (phonon) → effective Hamiltonian $\mathcal{H}_{BCS}$

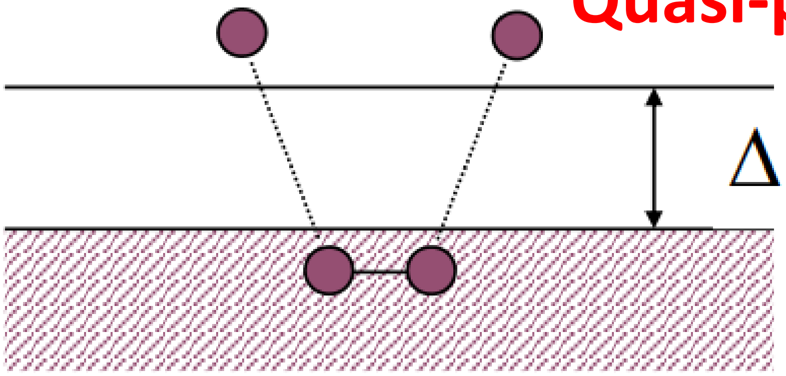
Mean field approximation + Variational method (+other approximations…)

$$\mathcal{H}_{BCS}|\Phi_0\rangle = E|\Phi_0\rangle$$

Non-perturbative!

Solution: superconducting gap

Quasi-particles



$$\Delta_o \propto \Psi_o$$

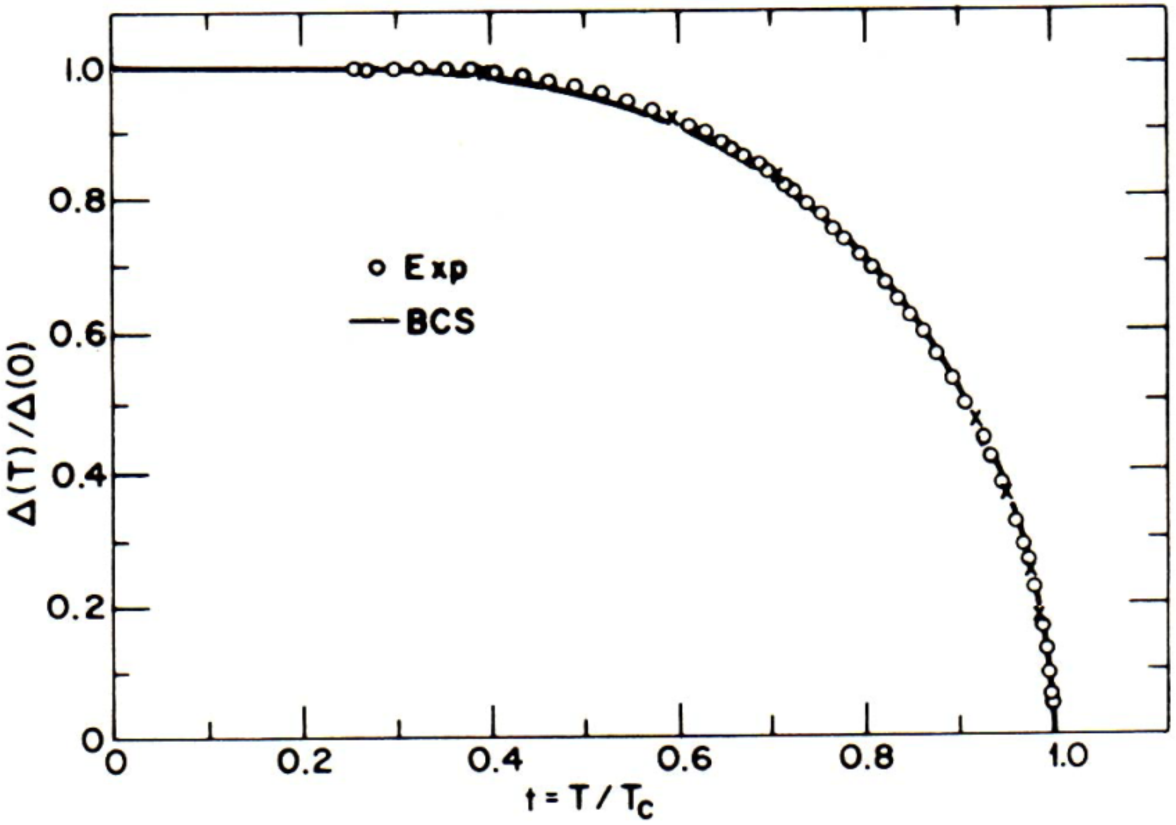
Scaler wavefunction

- The Cooper pair needs certain amount of energy to be broken
- The cause of Ohmic loss, stochastic scattering of one single electron by phonon or impurity **cannot break the pair**

  → No DC loss

Self-consistent gap equation

$$\Delta = N(E_F)V \int_{\Delta}^{\hbar\omega_D} \frac{\Delta}{\sqrt{\xi^2+\Delta^2}} \tanh\left(\frac{1}{2}\frac{\sqrt{\xi^2+\Delta^2}}{k_B T}\right) d\xi$$



The Equilibrium state of conventional superconductor was understood !

→ In this lecture, we try to obtain qualitative insight of the phenomenon

Electrons in a *perfect* metal are free (or independent)

Q3 Electron-electron scattering?
→Pauli's exclusion principle
Cf. Fermi-liquid theory by Landau

Perfectly periodic potential by ions does ***NOT*** scatter electrons (Bloch's theorem)

Q2

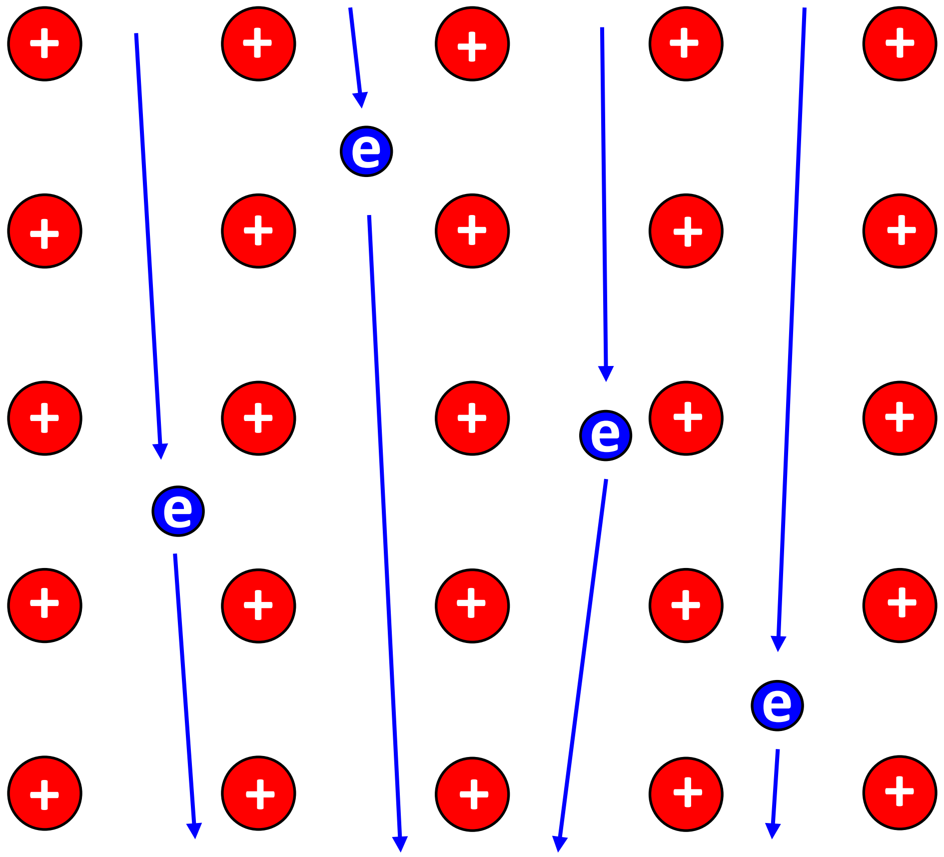
Check this

These electrons are ***NOT*** our favorite elementary particle of

$$m = 511\ \mathrm{keV}$$

These electrons are ***dressed*** by complicated electromagnetic property of metals to have an effective mass m^*given by a band structure

→ ***Quasi-particles***



In reality, imperfection causes quasi-particle scattering

Electrons in real metals show Ohmic loss

<u>Imperfections causes ***local*** scattering</u>

1. Impurity, defects (scattering time τ_{def})
2. Lattice vibration, phonon (τ_{ph})

Total scattering time

$$\frac{1}{\tau} = \frac{1}{\tau_{\text{def}}} + \frac{1}{\tau_{ph}}$$

<u>Macroscopic phenomenology (Drude model)</u>

An electron accelerated by an electric field

$$m^* \frac{dv}{dt} = -eE$$

is scattered by imperfections per τ, and its velocity relaxes to a mean velocity

$$\langle v \rangle = -\frac{e}{m^*} E\tau$$

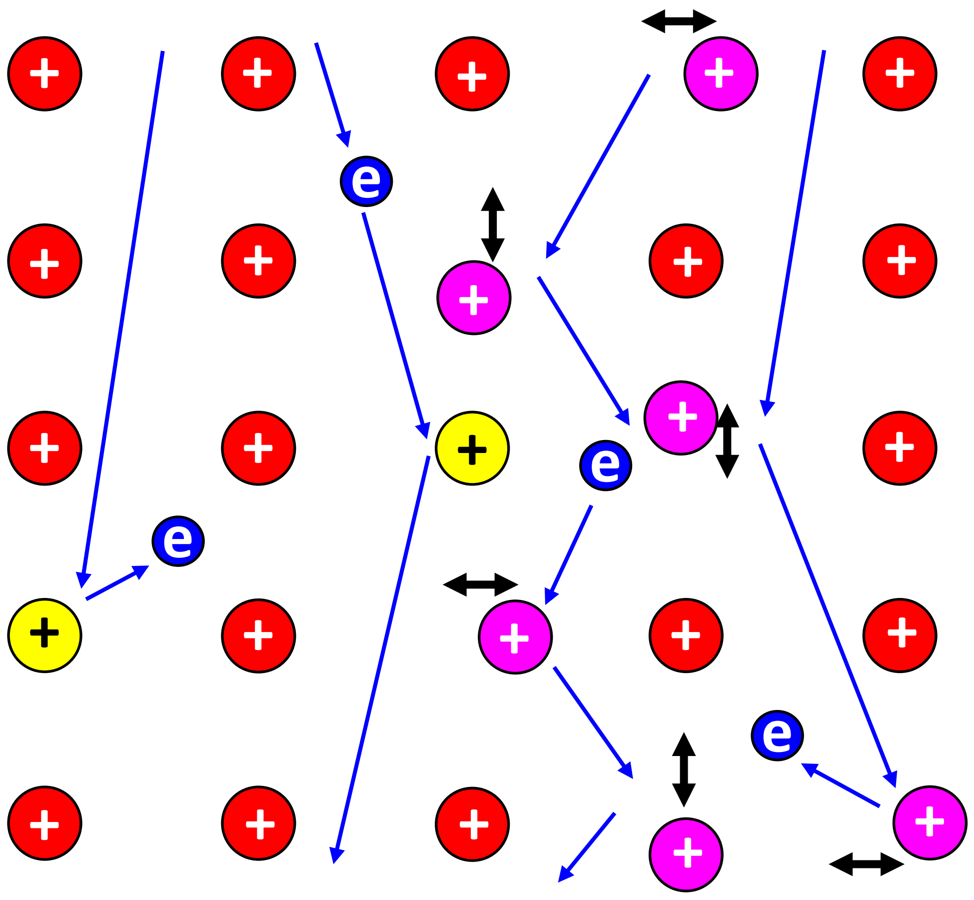
Electric current is a collective flow of n electrons

$$j = -en\langle v \rangle = \boxed{\frac{e^2 n\tau}{m^*}} E$$

Electrical conductivity σ

Ohm's law

$$\boldsymbol{j} = \sigma \boldsymbol{E}$$



Residual resistivity ratio

$$T \downarrow \rightarrow \tau_{ph} \uparrow \rightarrow RRR \equiv \frac{\sigma(< 10K)}{\sigma(300K)} \gg 1$$

Paired electrons can avoid Ohmic loss

If electrons ***in a distance*** (>39 nm) are bounded, ***local*** (< 0.5 nm) scattering can be avoided

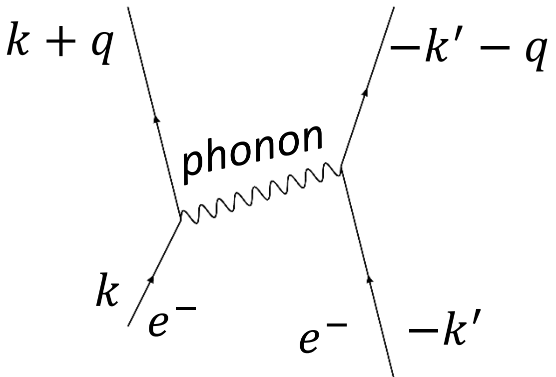
Any small attractive interaction V between electrons can lead to a ***Cooper pair*** coupled with an energy 2Δ, below critical temperature T_c

<u>BCS gap equation (1957)</u>

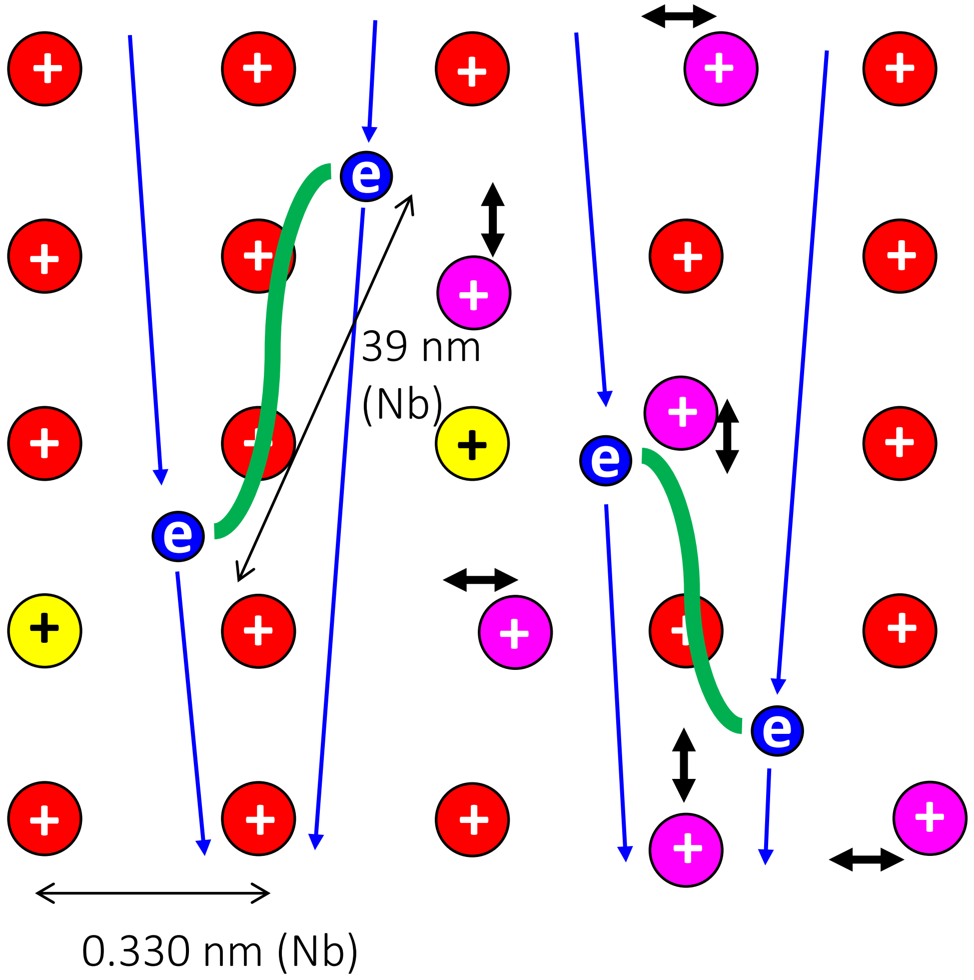
Non-perturbative!

$$\Delta = n(E_F)V \int_{\Delta}^{\hbar\omega_D} \frac{\Delta}{\sqrt{\xi^2+\Delta^2}} \tanh\left(\frac{1}{2}\frac{\sqrt{\xi^2+\Delta^2}}{k_B T}\right) d\xi$$

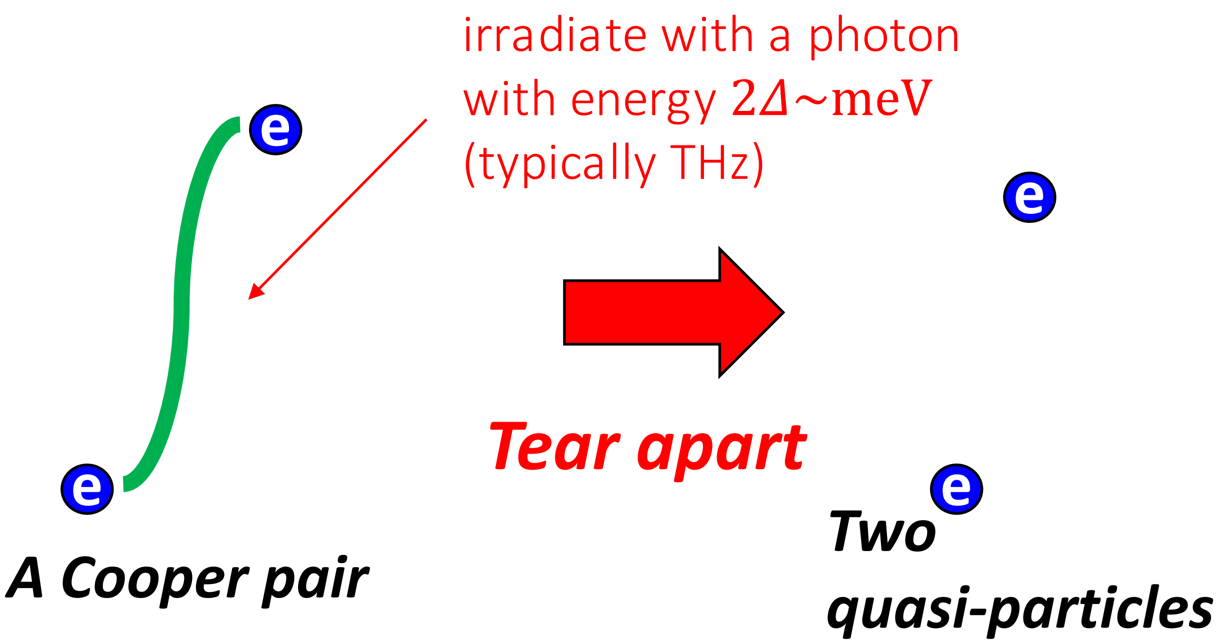
Classical superconductors' attractive potential is from ***longitudinal mode of lattice vibration***

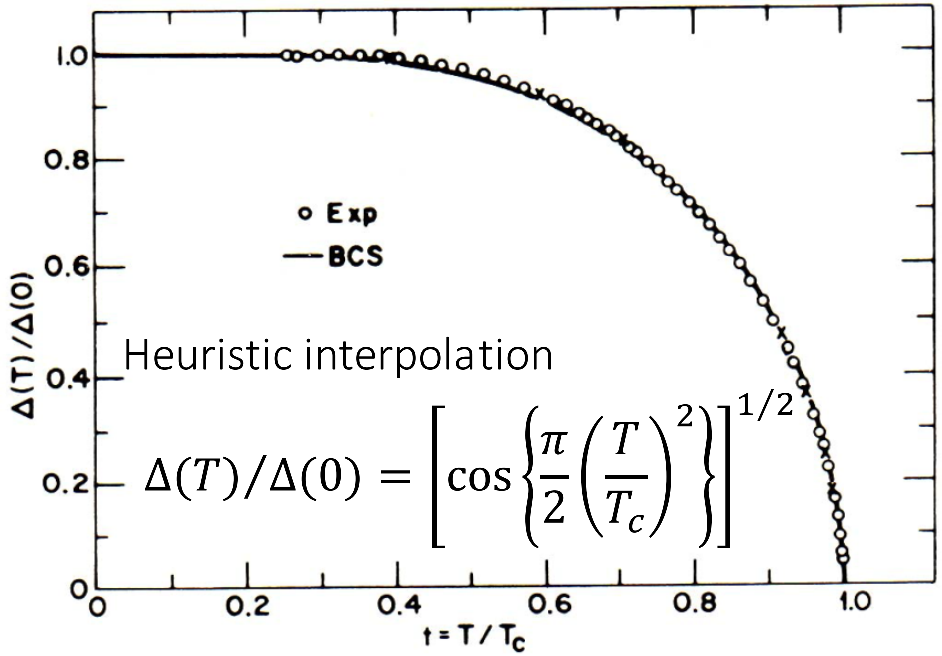


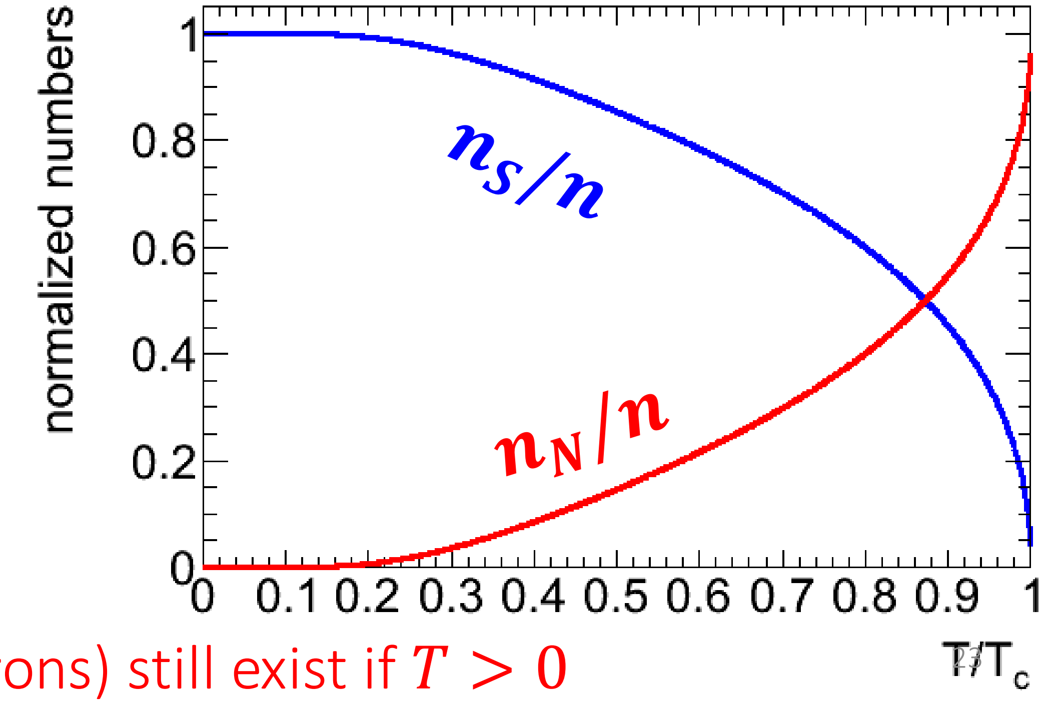
If energy transfer $|\epsilon_{k+q} - \epsilon_k|$ is smaller than phonon energy the interaction is attractive (Flöhlich)
→ Eliashberg's strong coupling superconductor (1960)



Superconducting gap







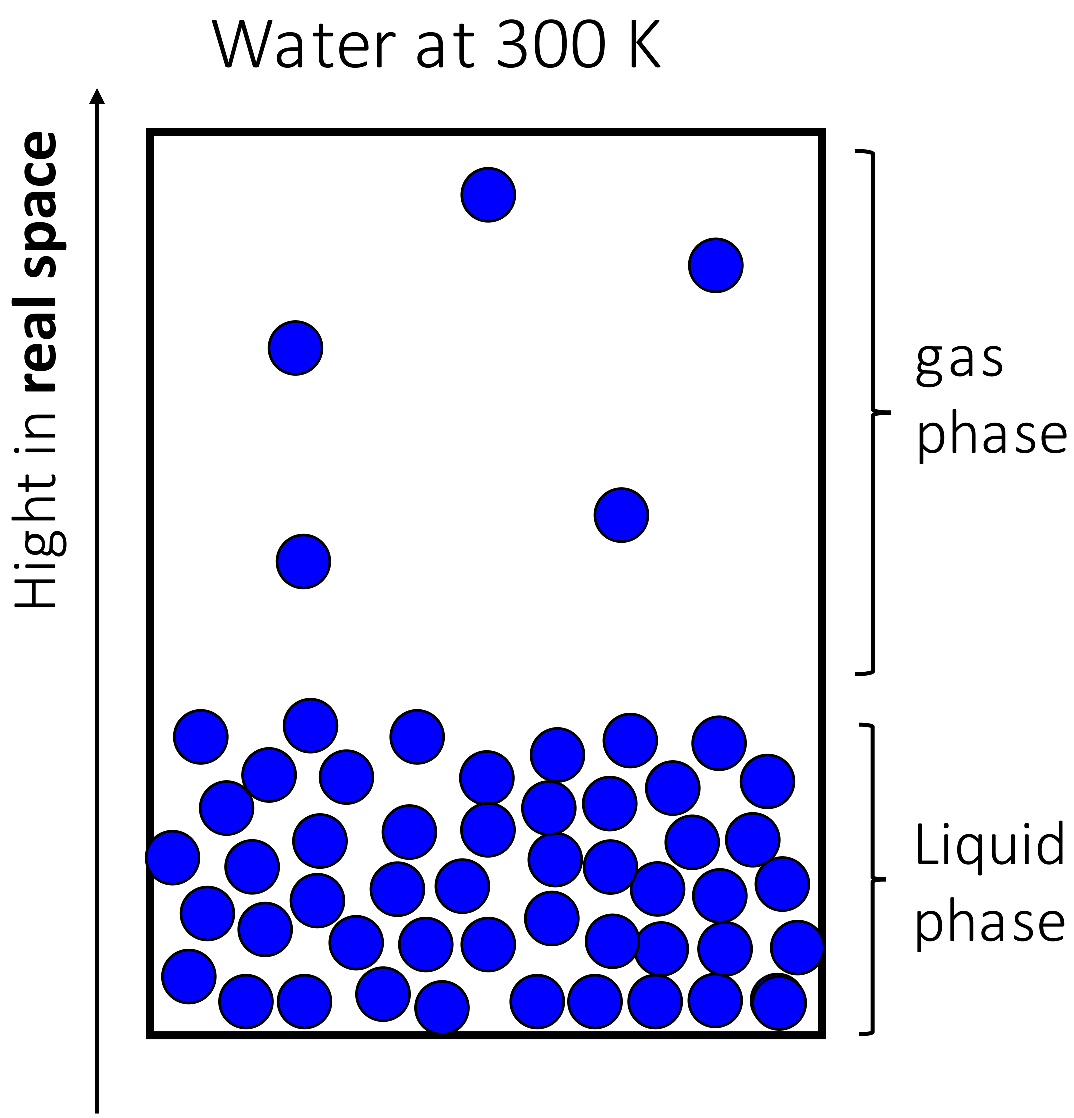
At finite temperature $0 < T < T_c$, these two states are ***in thermal equilibrium***

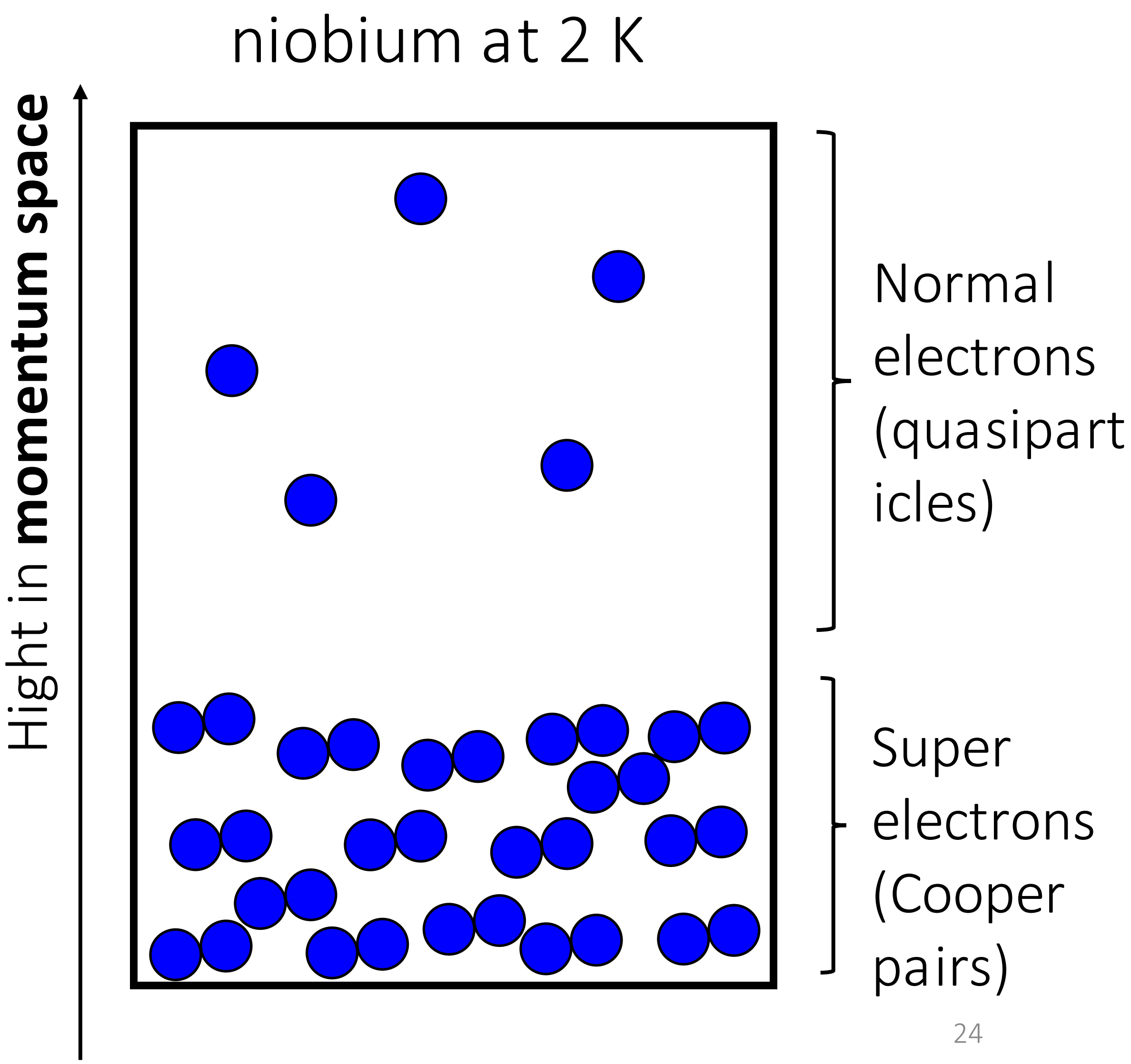
\# of quasiparticles: $n_N \sim \exp\left(-\frac{\Delta}{k_B T}\right)$

\# of electrons in Cooper pairs: $n_S \sim n - n_N$

Quasi-particles (~normal conducting electrons) still exist if $T > 0$

Why normal and super electrons at a time?





Implication of *no* scattering?

No scattering

$$m^* \frac{\partial \langle v \rangle}{\partial t} = -eE$$

Q4

Follow the math

generates super-current

$$j_s = -en_s \langle v \rangle$$

$$\rightarrow \frac{\partial j_s}{\partial t} - \frac{n_s e^2}{m^*} E = 0$$

Apply $\nabla \times$ from the left

$$\frac{\partial}{\partial t}(\nabla \times j_s) - \frac{n_s e^2}{m^*} \nabla \times E = 0$$

$\nabla \times j_s \sim \nabla \times \boldsymbol{B}/\mu_0$, $\quad \nabla \times E = -\frac{\partial \boldsymbol{B}}{\partial t}$

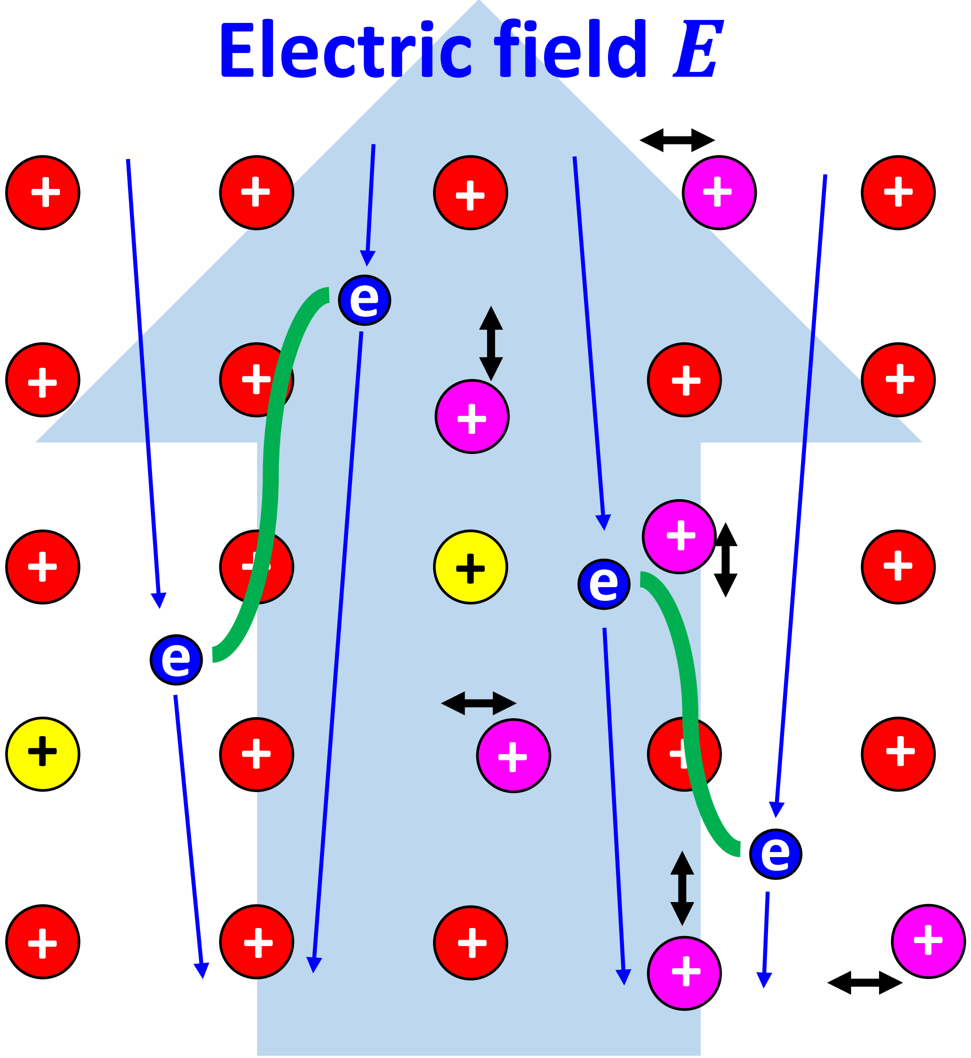
$$\lambda_L^2 \equiv \frac{m^*}{n_s e^2 \mu_0}$$

leads to

$$\frac{\partial}{\partial t}\left[\nabla^2 \boldsymbol{B} - \frac{1}{\lambda_L^2} \boldsymbol{B}\right] = 0$$

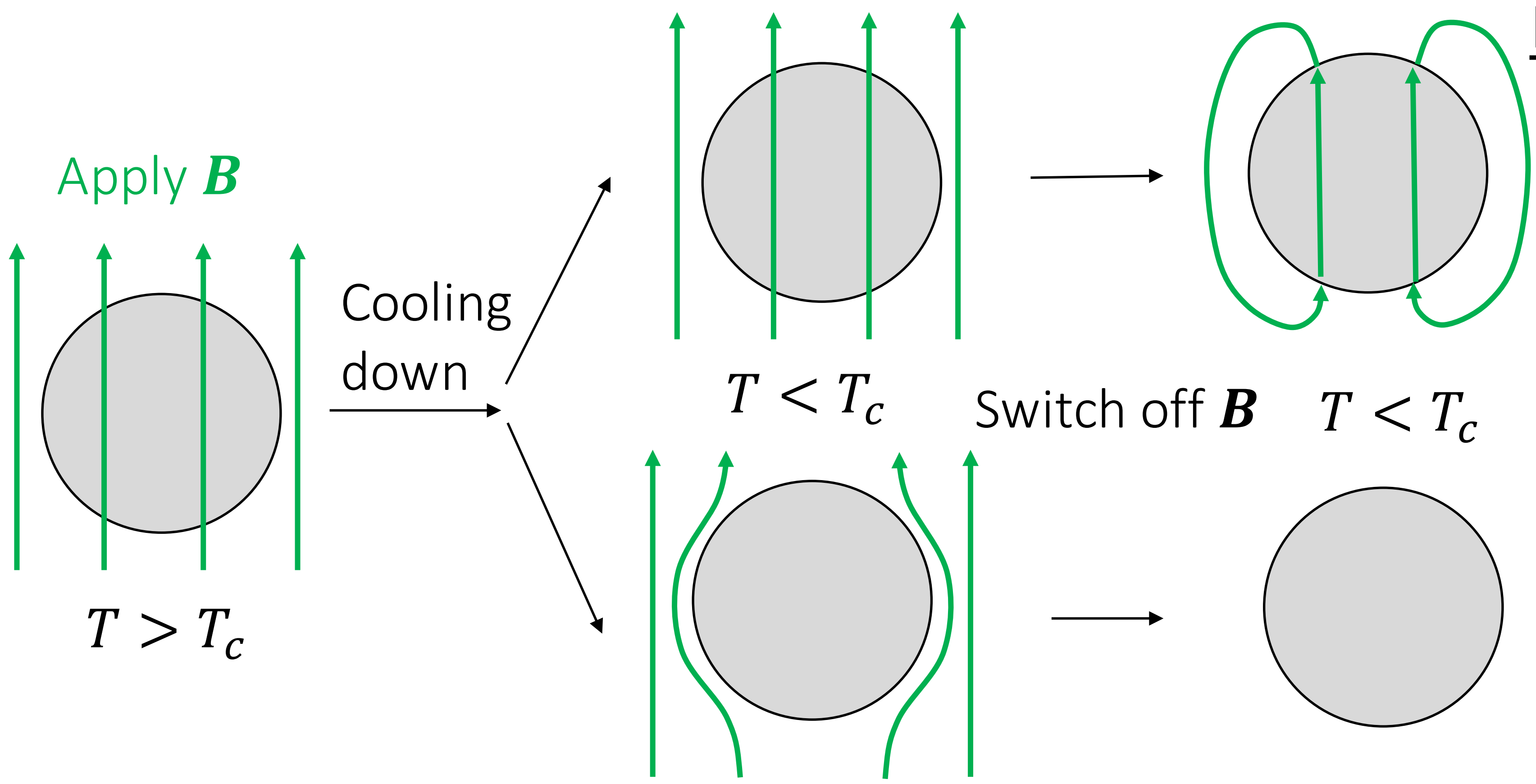
Constant of time

→ Initial condition before phase transition $T > T_c$ must be preserved



Superconductor ≠ Perfect electric conductor

***Meissner effect* differentiates them**



<u>Perfect Electric Conductor</u>

$$\nabla^2 \left(\frac{\partial \boldsymbol{B}}{\partial t}\right) - \frac{1}{\lambda_L^2}\left(\frac{\partial \boldsymbol{B}}{\partial t}\right) = 0$$

Preserve initial condition!

<u>Superconductor</u>

Zero field!

$$\nabla^2 \boldsymbol{B} - \frac{1}{\lambda_L^2}\boldsymbol{B} = 0$$

London equation

Additional constraint (broken Gauge symmetry)

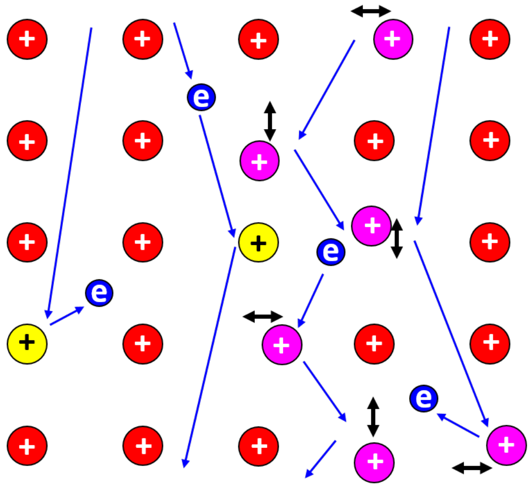
Superconductivity is a thermodynamical state which expels magnetic fields and cannot be explained by classical electrodynamics (input from quantum physics!)

Three characteristic lengths

Cf. Lattice constant of Nb is **0.330 nm**

Mean free path

$$l = \langle v \rangle \tau$$



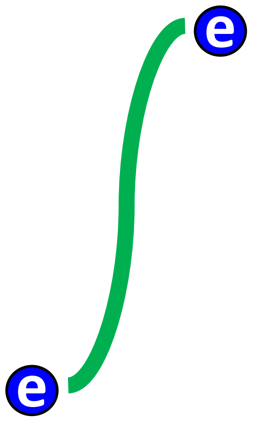
How often quasi-particles are scattered

l depends on RRR ($l \sim 2.7 \times RRR$)

RRR=300 → $l =$ **810 nm**

Coherent length

$$\xi_0 = \frac{\hbar v_F}{\pi \Delta}$$

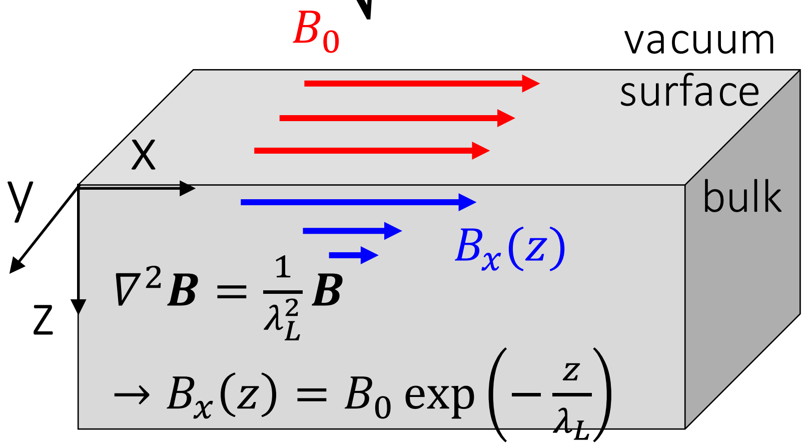


Characteristic size of Cooper pairs

$\xi_0 \sim$ **39 nm** for Nb

(London) Penetration depth

$$\lambda_L = \sqrt{\frac{m^*}{n_s e^2 \mu_0}}$$



How much magnetic fields can penetrate into a superconductor

$\lambda_L \sim$ **36 nm** for Nb

Penetration depth vs skin depth: similar but totally different origin

Superconductor

Quantum mechanics

$$\lambda_L = \sqrt{\frac{m^*}{n_s e^2 \mu_0}}$$

$\ll$

Normal conductor

$$\delta = \sqrt{\frac{1}{\pi f \mu_0 \sigma}}$$

From classical electrodynamics

From London equation (broken gauge symmetry)

$$\nabla^2 \boldsymbol{B} - \frac{1}{\lambda_L^2}\boldsymbol{B} = 0$$

Both **static** magnetic field and **RF** electromagnetic field and currents

From a RF screening effect of quasi-particles

$$\boldsymbol{j}_n = \sigma \boldsymbol{E}$$

$$\nabla \times \nabla \times \boldsymbol{E} = -\frac{\partial(\nabla \times B)}{\partial t} \sim \mu_0 \frac{\partial \boldsymbol{j}_n}{\partial t}$$

$$(= -\nabla^2 \boldsymbol{E})$$

$$E = E_0 exp(i2\pi f t)$$

$$\nabla^2 \boldsymbol{E} - \frac{1}{\delta^2}\boldsymbol{E} = 0$$

Math looks similar…

RF electromagnetic fields and currents

For niobium (<9.25K)

$\lambda_L \sim 36$ nm

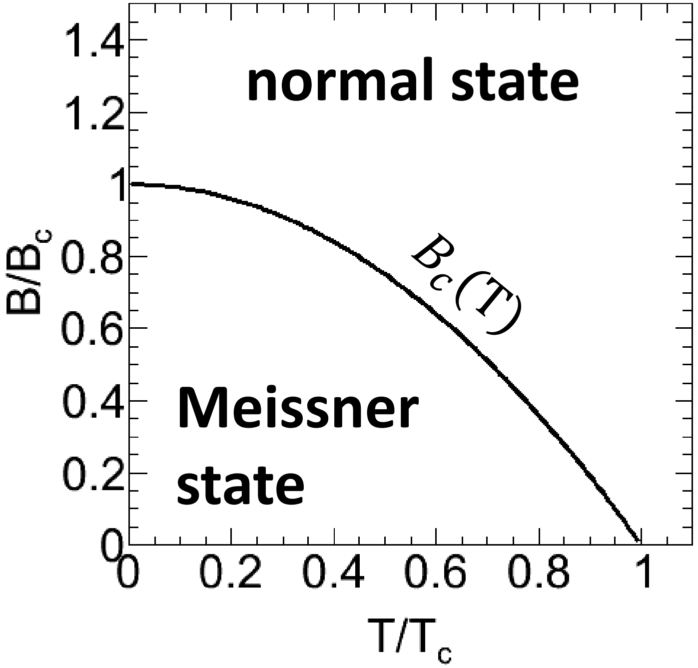
For 300K copper and $f = 0.1 - 1$ GHz

$\delta > 2$ μm

Under strong but *static* magnetic field: Type-I vs Type-II

Type-I

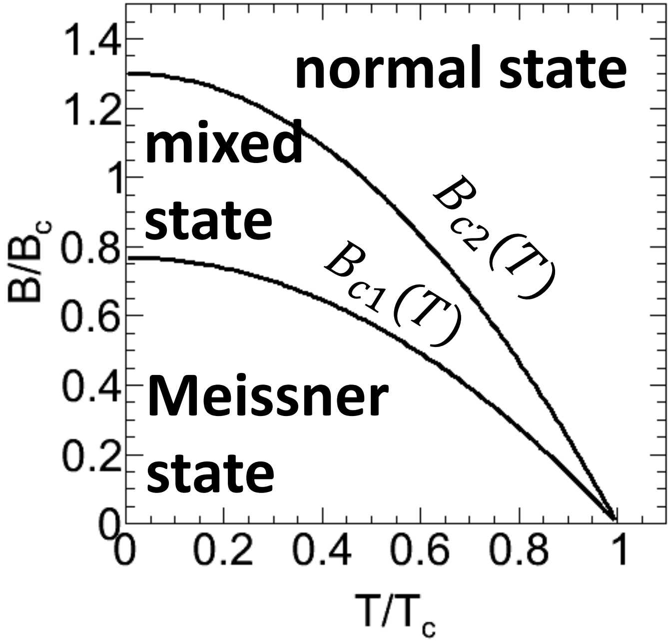
$$\kappa = \frac{\lambda}{\xi} < \frac{1}{\sqrt{2}} = 0.71$$



$$\kappa_{Pb} \sim \frac{28\ \text{nm}}{71\ \text{nm}} \sim 0.40$$

Type-2

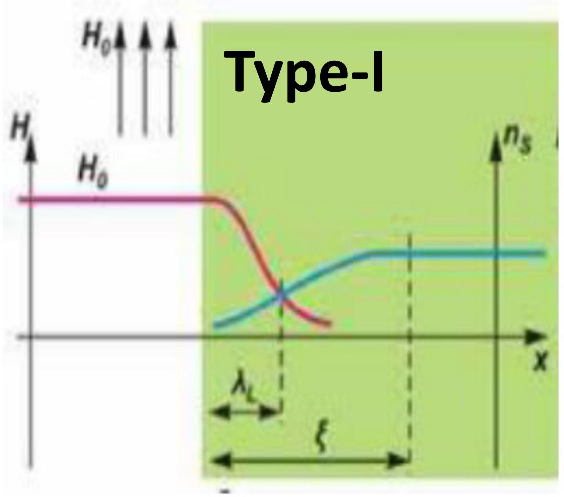
$$\kappa = \frac{\lambda}{\xi} > \frac{1}{\sqrt{2}} = 0.71$$

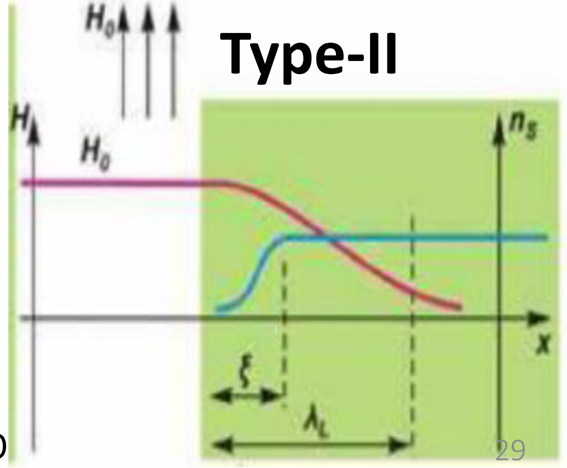


$$\kappa_{Nb} \sim \frac{36\ \text{nm}}{39\ \text{nm}} \sim 0.92$$

Stabilized by NC/SC boundary energy

$$\frac{1}{2\mu_0}\left(\xi_0 B_c^2 - \lambda_L B^2\right) < 0 \text{ for } B > B_{c1}$$





Type-II superconductors become energetically favorable to create normal conducting boundaries inside if $B > B_{c1}$

→ How to maximize interface area? → **Quantized flux** $\Phi_0 = \frac{h}{2e} = 2.07 \times 10^{-15}$ Wb

Three limits in the literature on (ξ, λ, l)

O. Klein et al PRB 50 6307 (1994)

Dirty limit (***dirty*** does not mean oil, dusts, finger prints, …)

$l \ll \xi, \lambda$ (both type-I and type-II)

Clean limits

The Pippard or anomalous limit: $\lambda \ll \xi, l$ (clean type-I)

The London limit: $\xi \ll \lambda, l$ (clean type-II)

Pure niobium is at the border of type-II

$\xi = 39$ nm
$\lambda = 36$ nm
} $\xi \sim \lambda$

$l = 2.7 \times RRR$ SRF cavity quality in a bulk → none of limits

RRR=300 → $l = 810$ nm $\gg \xi \sim \lambda$

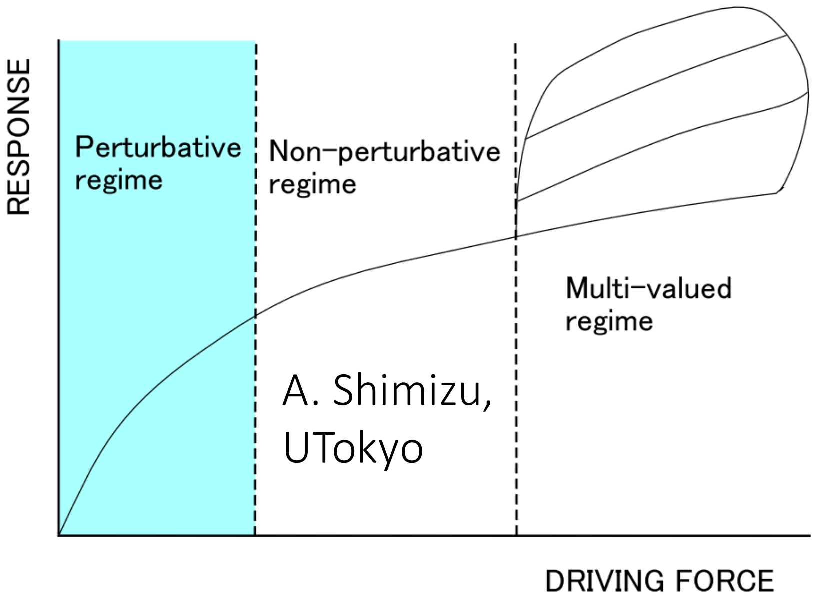
RRR=10→ $l = 27$ nm $< \xi \sim \lambda$

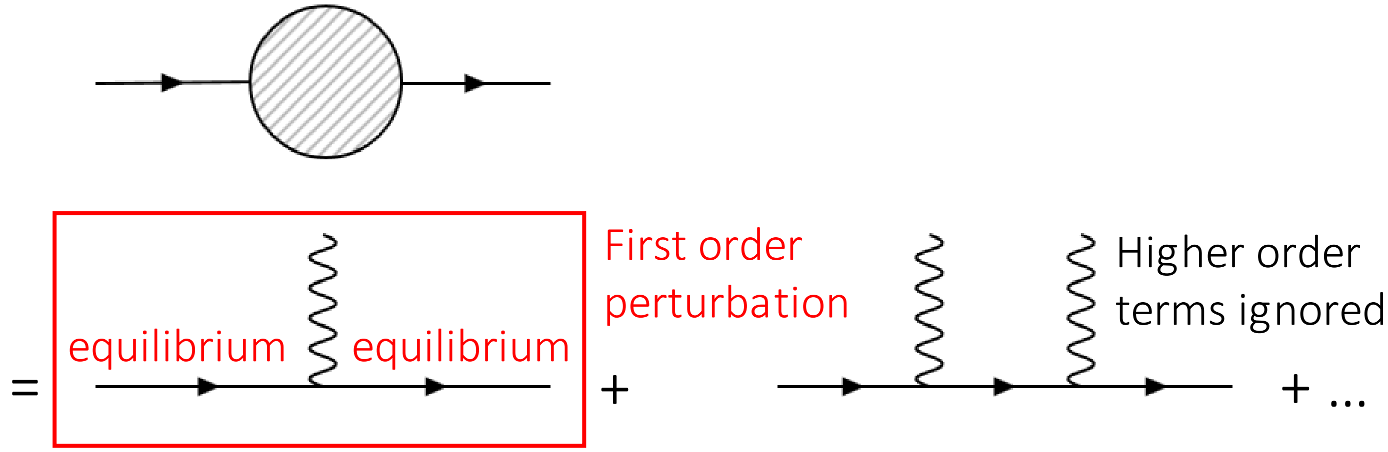
Optimized SRF cavity surface→ dirty limit

Linear response to RF→ BCS resistance R_{BCS}

Quamtum mechanical ***derivation*** of R_s requires quantum many body theory

$$\mathcal{H} = \mathcal{H}_{BCS} + \mathcal{H}_{RF}(t)$$ If the RF field is "small"





The responding current is given by the equilibrium state

$$J(q) = -\frac{1}{c}K(q)A(q)$$

Quantum ***derivation of*** Ohm's law is equally complicated...

$$\left.\begin{aligned} \sigma &= -\frac{1}{i\omega}[\Phi^R(\omega) - \Phi^R(0)] \\ \Phi^R &= \frac{i}{\hbar V}\theta(t)\langle\hat{J}(t)\hat{J}(0) - \hat{J}(0)\hat{J}(t)\rangle \end{aligned}\right\} \to \sigma = \frac{ne^2\tau_{\boldsymbol{k}}}{m}\frac{\widetilde{\rho_0}}{\rho_0}$$

Response to RF – classical *derivation* –

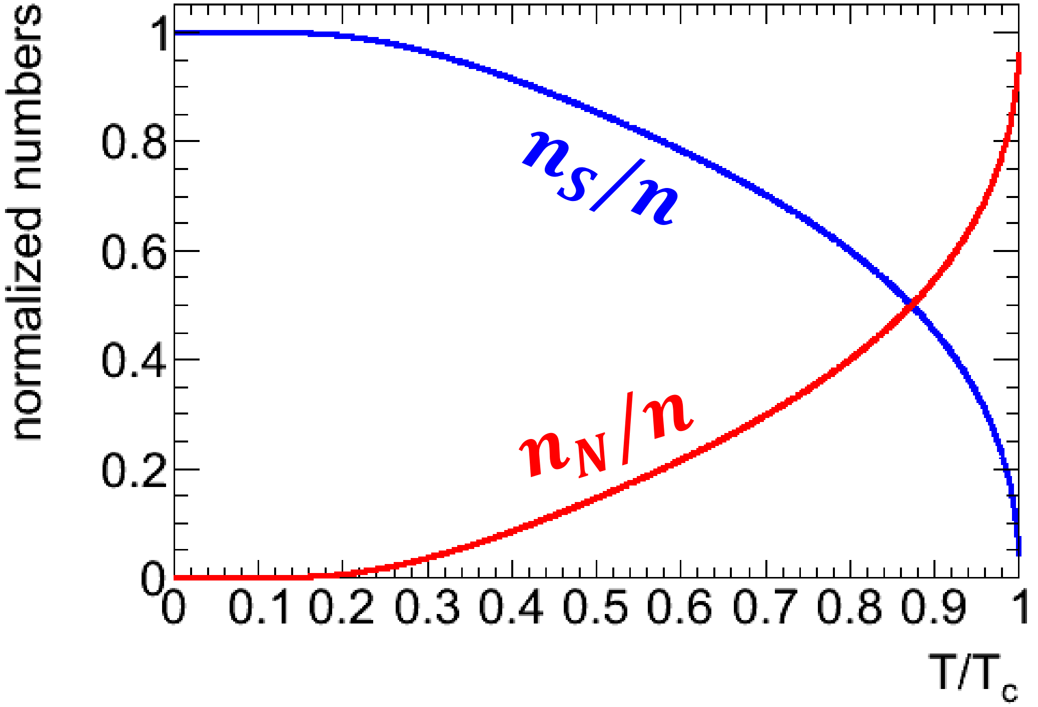
Supercurrent

$$\frac{\partial j_s}{\partial t} - \frac{n_S e^2}{m^*} E = 0$$

$$j_s = j_0 \exp(i\omega t)$$

$$j_s = -i \frac{n_S e^2}{m^* \omega} E$$

$$\equiv \sigma_S$$



Normal current

Ohm's law →

$$j_N = \frac{n_N e^2 \tau}{m^*} E$$

$$\equiv \sigma_N$$

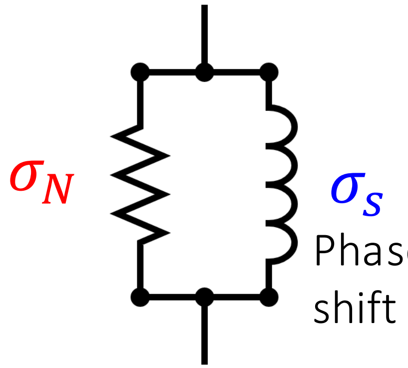
Total current induced by RF

$$j = j_S + j_N \quad \rightarrow \quad j = (\sigma_N - i\sigma_S)E$$

Dissipation by quasi-particles →resistive

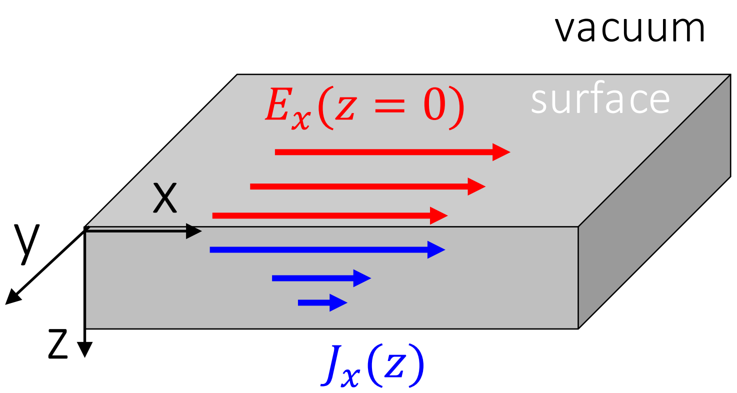
Inertia of Cooper pairs → inductive

Equivalent circuit



Two fluid model

Surface resistance of superconductor



$$\begin{cases} j_x = (\sigma_N - i\sigma_S)E_x \\ E_x(z) = E_0 \exp(-z/\lambda_L) \end{cases}$$

Q5 Follow the math

$$\rightarrow R_s \equiv \mathrm{Re}\left(\frac{E_x(z=0)}{\int_0^\infty J_x(z)dz}\right) \sim \frac{1}{2}\frac{\sigma_N}{\sigma_S}\sqrt{\frac{\omega\mu_0}{\sigma_S}} = \frac{\mu_0^2}{2}\lambda_L^3\sigma_N\omega^2 > 0$$

$$\sigma_N = \frac{e^2 n_N \tau}{m^*} \propto n_N \propto \exp\left(-\frac{\Delta}{k_B \boldsymbol{T}}\right)$$

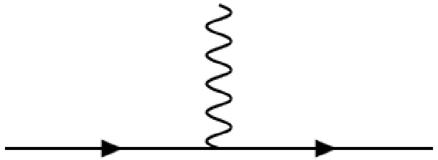
Lessons

- One origin of the finite R_s of superconductors is quasi-particles
- Quasi-particles are thermally activated from Cooper pairs at $0 < T < T_c$
- R_s exponentially decreases by lower T because quasi-particles are frozen out
- Higher RF frequency increases $R_s \sim \omega^2$

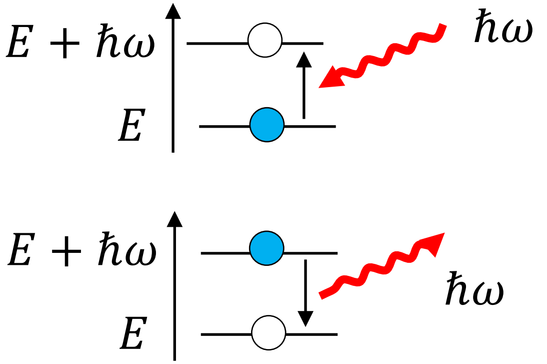
Classical understanding is sufficient in most of the SRF activities

Introduction to *quantum* mechanical derivation: *Integrate* contribution of all the quasi-particles

Fermi's Golden rule [Z. Physik 266 p.209 (1974)]

$R_s \propto P \cong \Sigma_{p,p',\hbar\vec{k}}$ $\left|\right.$  $\left.\right|^2$ $\propto$ (photon energy) x (net # of absorbed photons)

$$= \hbar\omega(n_+ - n_-) = \hbar\omega \int_{\Delta}^{\infty} dE\,[f(E) - f(E+\hbar\omega)] \times N(E)N(E+\hbar\omega)$$



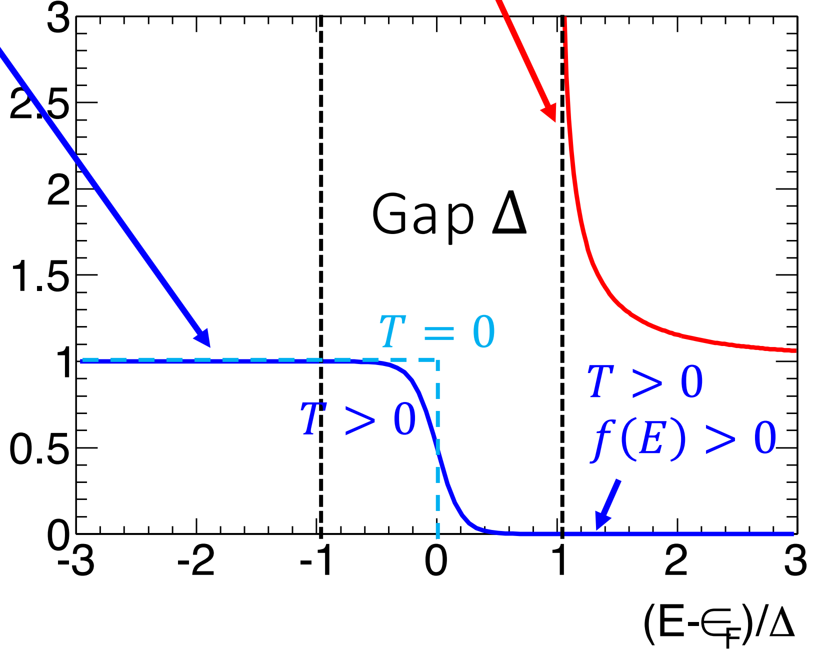
All quasiparticles $\Delta < E < \infty$

$N(E)$: density of states (how many quantum states at energy E: a kind of degeneracy)

$f(E)$: distribution function (how many electrons are in one state at energy E)

Fermi-Dirac function in equilibrium state

$$f(E) = \frac{1}{\exp(-\mathrm{E}/k_B T) + 1}$$



Q6	Follow the math

$$R_s \propto \frac{\omega^{1.5}}{T} \exp\left(-\frac{\Delta}{k_B T}\right)$$

$R_s(T = 0) = 0$

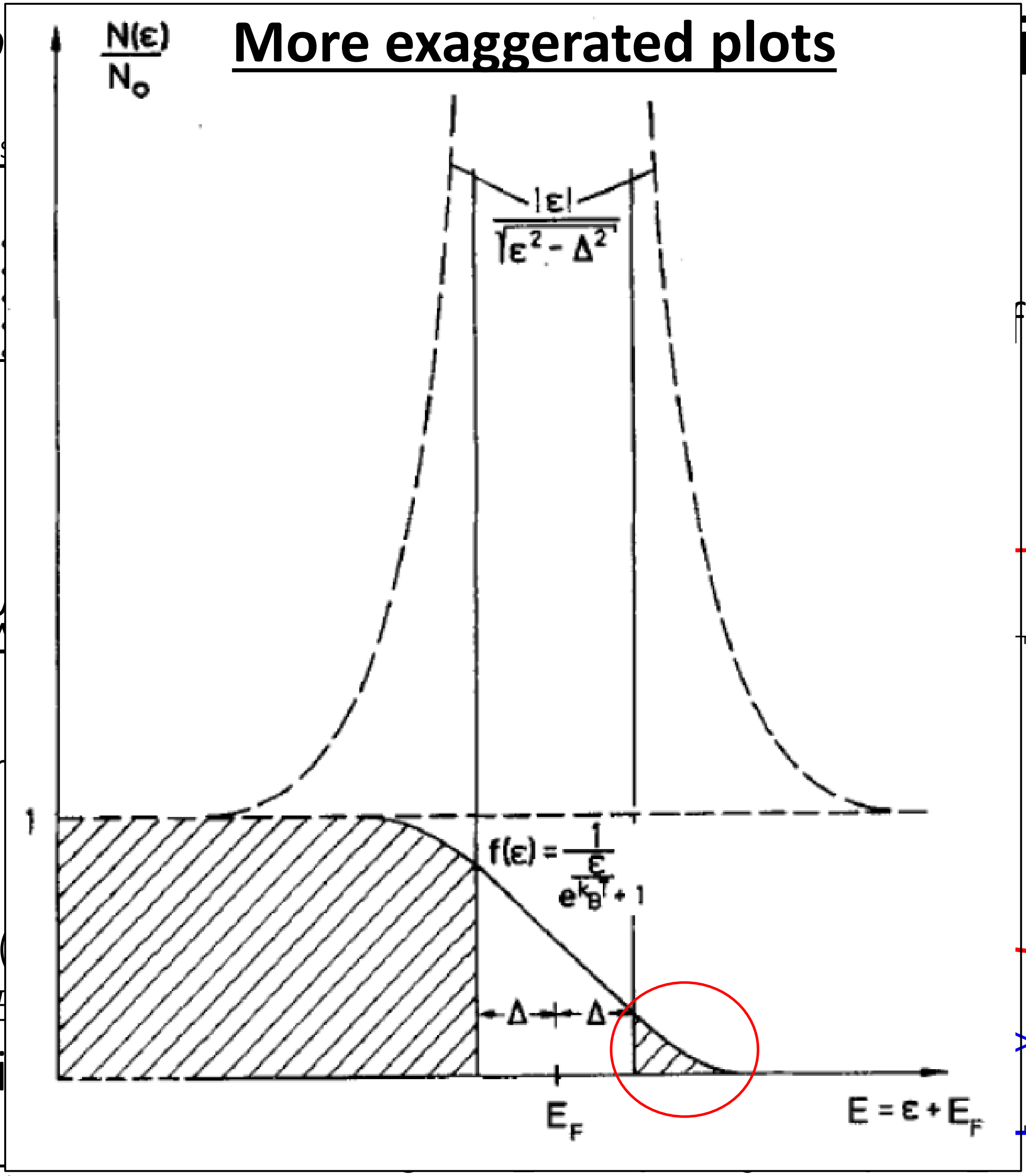
Introduction to *quantum* mechanical derivation:
Integrate ... i-particles

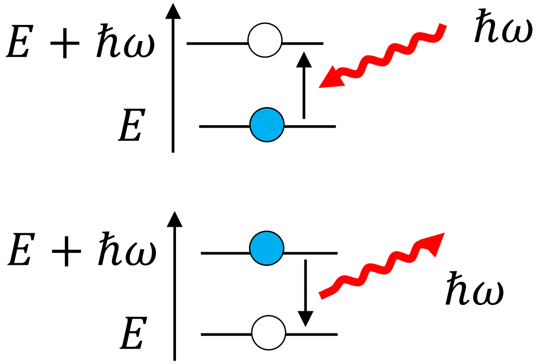
Fermi's Golden rule [Z. Phys...

$R_s \propto P \cong \Sigma_{p,p',h\vec{k}}$

... f absorbed photons)

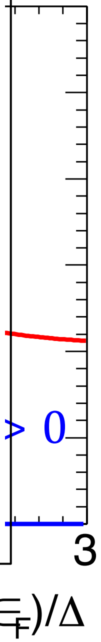
$= \hbar\omega(n_+ - n_-) = \hbar\omega$ ... $+ \hbar\omega)$





All quasiparticles

$\Delta < E < \infty$



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$f(E)$: distribution function (... electrons are in one state at energy E ...

Fermi-Dirac function in equi...

$$f(E) = \frac{1}{\exp(-E/k_BT) + 1}$$

Q6	Follow the math

$$R_s \propto \frac{\omega^{1.5}}{T} \exp\left(-\frac{\Delta}{k_BT}\right)$$

$R_s(T = 0) = 0$

Reality in the literature…

Mattis and Bardeen Phys Rev 111 2 1958

$$\mathbf{j}(\mathbf{r},t)=\sum_{\omega}\frac{e^2N(0)v_0}{2\pi^2\hbar c}\times\int\frac{\mathbf{R}[\mathbf{R}\cdot\mathbf{A}_\omega(r')]I(\omega,R,T)e^{-R/l}dr'}{R^4}$$

$$I(\omega,R,T)=-\pi i\int_{\epsilon_0-\hbar\omega}^{\epsilon_0}[1-2f(E+\hbar\omega)]\times[g(E)\cos(\alpha\epsilon_2)-i\sin(\alpha\epsilon_2)]e^{i\alpha\epsilon_1}dE$$
$$-\pi i\int_{\epsilon_0}^{\infty}\{[1-2f(E+\hbar\omega)]\times[g(E)\cos(\alpha\epsilon_2)-i\sin(\alpha\epsilon_2)]e^{i\alpha\epsilon_1}-[1-2f(E)]\times[g(E)\cos(\alpha\epsilon_1)+i\sin(\alpha\epsilon_1)]e^{-i\alpha\epsilon_2}\}dE,$$

$$\frac{\sigma_1}{\sigma_N}=\frac{2}{\hbar\omega}\int_{\epsilon_0}^{\infty}[f(E)-f(E+\hbar\omega)]g(E)dE+\frac{1}{\hbar\omega}\int_{\epsilon_0-\hbar\omega}^{-\epsilon_0}[1-2f(E+\hbar\omega)]g(E)dE,$$

$$\frac{\sigma_2}{\sigma_N}=\frac{1}{\hbar\omega}\int_{\epsilon_0-\hbar\omega,\,-\epsilon_0}^{\epsilon_0}\frac{[1-2f(E+\hbar\omega)](E^2+\epsilon_0^2+\hbar\omega E)}{(\epsilon_0^2-E^2)^{\frac{1}{2}}[(E+\hbar\omega)^2-\epsilon_0^2]^{\frac{1}{2}}}$$

Abrikosov et at JTEP 35 182 1959

$$\mathbf{j}(\mathbf{k},\omega)=\frac{3e^2N\mathbf{A}(\mathbf{k},\omega)}{32mc}\int_{-1}^{1}d\cos\theta\sin^2\theta\int_{-\xi_0}^{\xi_0}d\xi\left[\left(1-\frac{\xi_1\xi_2+\Delta^2}{\varepsilon_1\varepsilon_2}\right)\left(\tanh\frac{\varepsilon_1}{2T}+\tanh\frac{\varepsilon_2}{2T}\right)\left(\frac{1}{\varepsilon_1+\varepsilon_2+\omega+i\delta}+\frac{1}{\varepsilon_1+\varepsilon_2-\omega-i\delta}\right)\right.$$
$$\left.+\left(1+\frac{\xi_1\xi_2+\Delta^2}{\varepsilon_1\varepsilon_2}\right)\left(\tanh\frac{\varepsilon_1}{2T}-\tanh\frac{\varepsilon_2}{2T}\right)\left(\frac{1}{\varepsilon_1-\varepsilon_2+\omega+i\delta}+\frac{1}{\varepsilon_1-\varepsilon_2-\omega-i\delta}\right)\right]-\frac{e^2}{mc}N\mathbf{A}(\mathbf{k},\omega).$$

$$\frac{Z(\omega)}{R_n}=2\left(\frac{\omega}{\pi\Delta}\right)^{1/2}\left[\frac{4}{3\pi}\sinh\frac{\omega}{2T}K_0\left(\frac{\omega}{2T}\right)e^{-\Delta/T}-i\right].$$

Strong coupling theory

(Eliashberg JTEP 11 696 1960; Nam Phys Rev 156 470 1967; Marsiglio et al PRB 50 7203 1994)

$$\sigma_1(\nu)=\frac{ne^2}{m}\frac{1}{2\nu}\left(\int_0^D d\omega\left[\tanh\frac{\beta(\omega+\nu)}{2}-\tanh\frac{\beta\omega}{2}\right]g(\omega,\nu)+\int_{-\nu}^{0}d\omega\tanh\frac{\beta(\omega+\nu)}{2}g(\omega,\nu)\right),\qquad(1)$$

$$g(\omega,\nu)=\mathrm{Im}\left(\frac{1-N(\omega)N(\omega+\nu)-P(\omega)P(\omega+\nu)}{\epsilon(\omega)+\epsilon(\omega+\nu)+i/\tau}+\frac{1+N^*(\omega)N(\omega+\nu)+P^*(\omega)P(\omega+\nu)}{\epsilon^*(\omega)-\epsilon(\omega+\nu)-i/\tau}\right)$$

$$\epsilon(\omega)\equiv\sqrt{\tilde\omega^2(\omega+i\delta)-\phi^2(\omega+i\delta)}$$

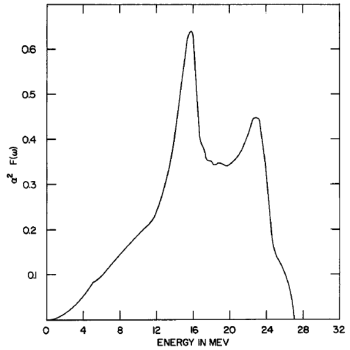
$$N(\omega)\equiv\tilde\omega(\omega+i\delta)/\epsilon(\omega),$$

$$P(\omega)\equiv\phi(\omega+i\delta)/\epsilon(\omega).$$

$$\tilde\omega(\omega)=\omega+i\pi T\sum_{m=0}^{\infty}\frac{\tilde\omega(i\omega_m)}{[\tilde\omega^2(i\omega_m)-\phi^2(i\omega_m)]^{1/2}}[\lambda(\omega-i\omega_m)-\lambda(\omega+i\omega_m)]+i\pi\int_{-\infty}^{\infty}dz\frac{\tilde\omega(\omega-z)}{[\tilde\omega^2(\omega-z)-\phi^2(\omega-z)]^{1/2}}\alpha^2F(z)[N(z)+f(z-\omega)],$$

$$\phi(\omega)=i\pi T\sum_{m=0}^{\infty}\frac{\phi(i\omega_m)}{[\tilde\omega^2(i\omega_m)-\phi^2(i\omega_m)]^{1/2}}[\lambda(\omega-i\omega_m)+\lambda(\omega+i\omega_m)-2\mu^*]+i\pi\int_{-\infty}^{\infty}dz\frac{\phi(\omega-z)}{[\tilde\omega^2(\omega-z)-\phi^2(\omega-z)]^{1/2}}\alpha^2F(z)[N(z)+f(z-\omega)].$$

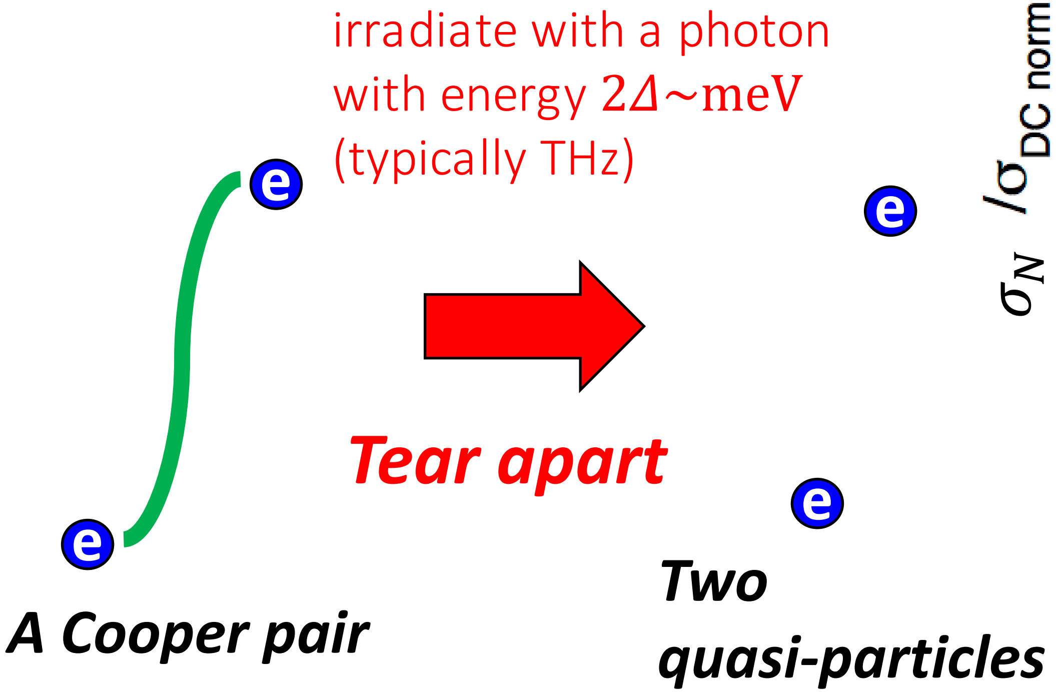
Electron-phonon spectral function $\alpha^2F(\omega)$

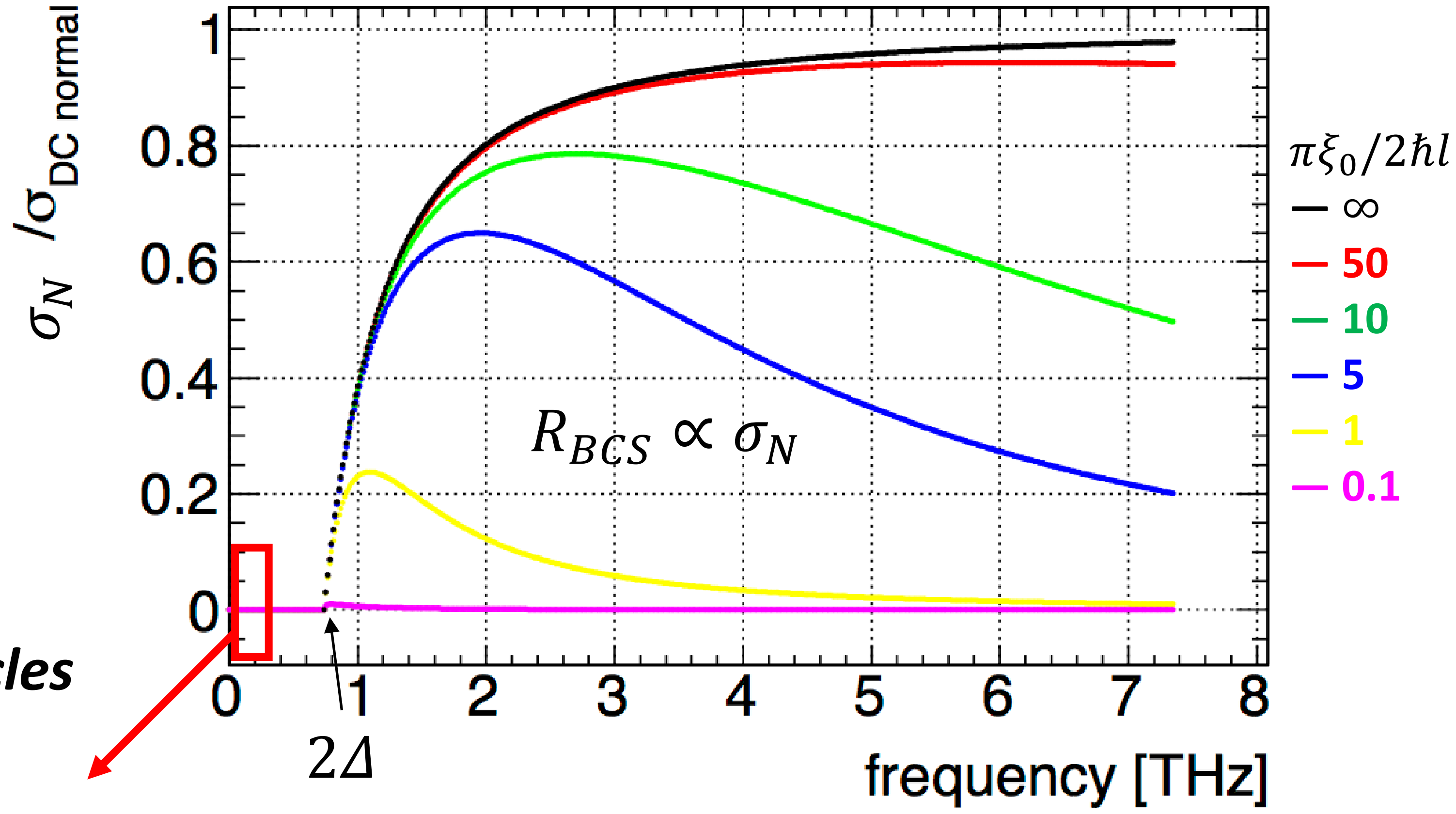


Wolf, J Low Temp Phys 40 19 1980

Be *practical*: features of R_{BCS} from numerical *codes*

- Halbritter, KFK-Ext.03/70-06 (1970), https://publikationen.bibliothek.kit.edu/270004230: Fortran66 code for all (ξ, λ, l)
  Detail phonon-electron interaction is not included → BCS (weak coupling limit) + phenomenological parameter $\alpha = \Delta/k_B T_c$
- Zimmermann et al Physica C 183 99 (1991): Fortran77 code for the London limit $\xi \ll \lambda, l$ of arbitrary purity l
  Good for high frequency

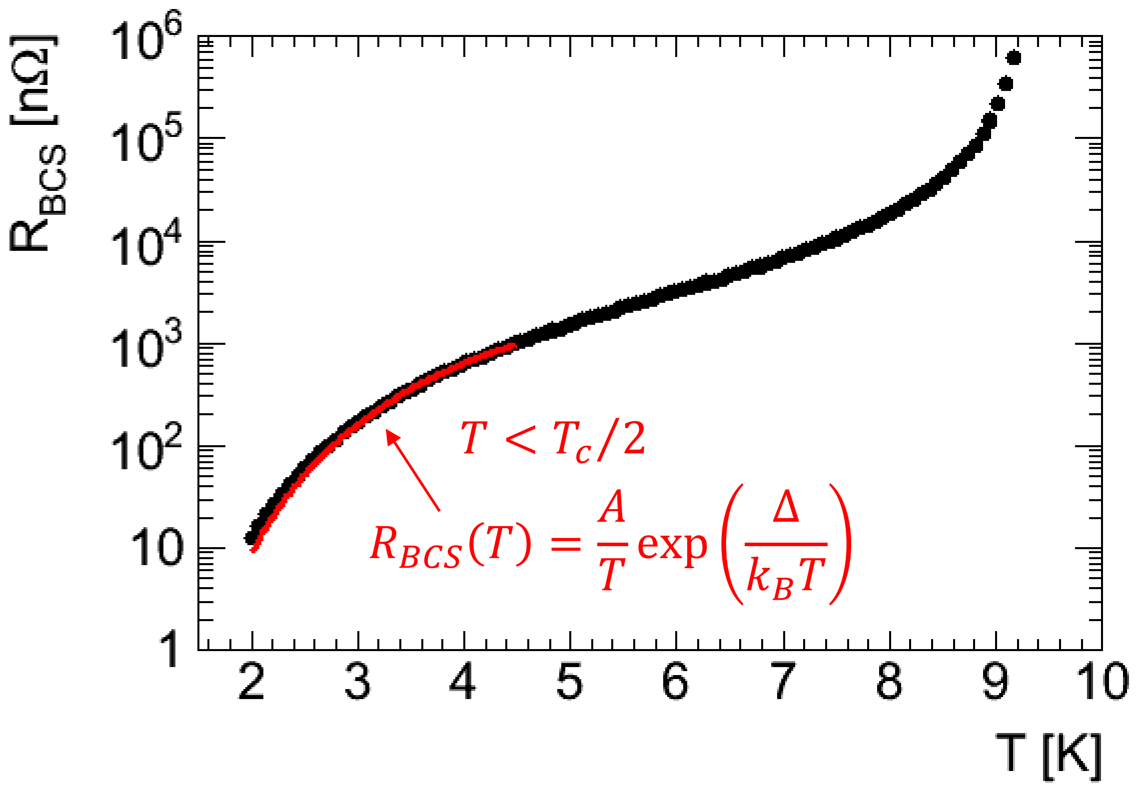




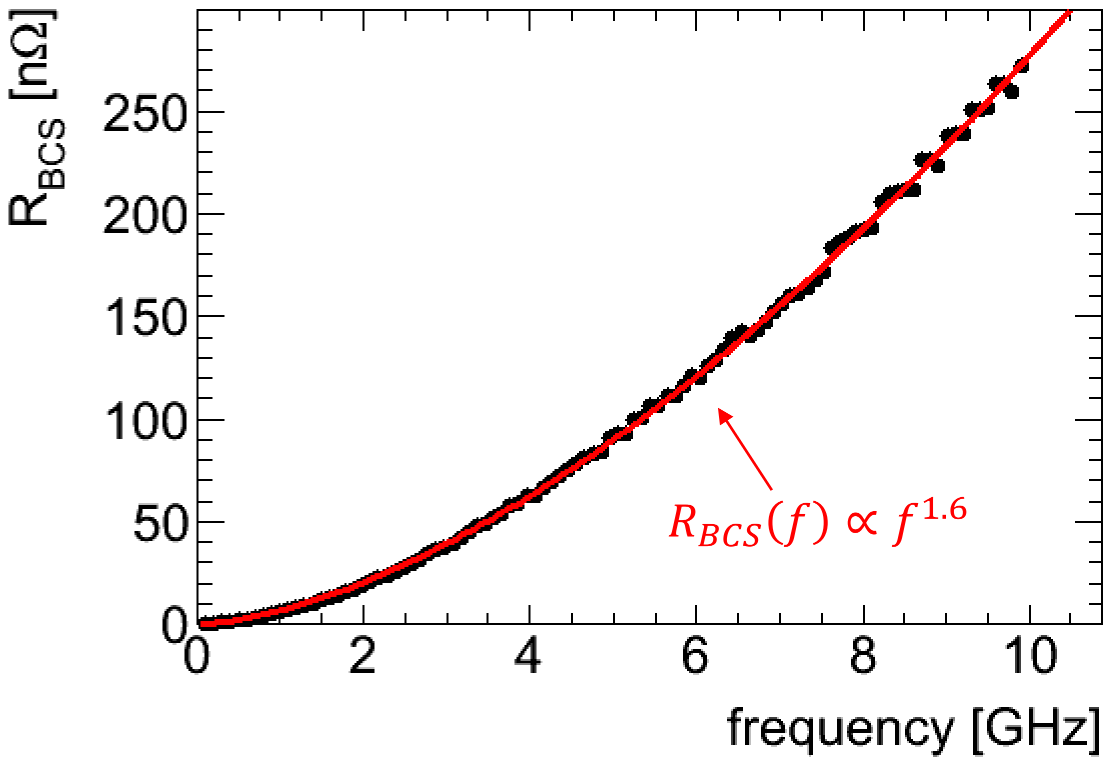
SRF accelerator application is in this region

Numerical calculation of $R_{BCS}(T, f)$

Temperature dependence is exponential



Frequency dependence between $f^{1.5}$ and f^2



Classically derived two-fluid model works fine to explain quantum calculation of BCS

→ **Practically, we can use the two fluid model to interpret data in your lab**

R_{BCS} vs mean free path l

Q7

Why?

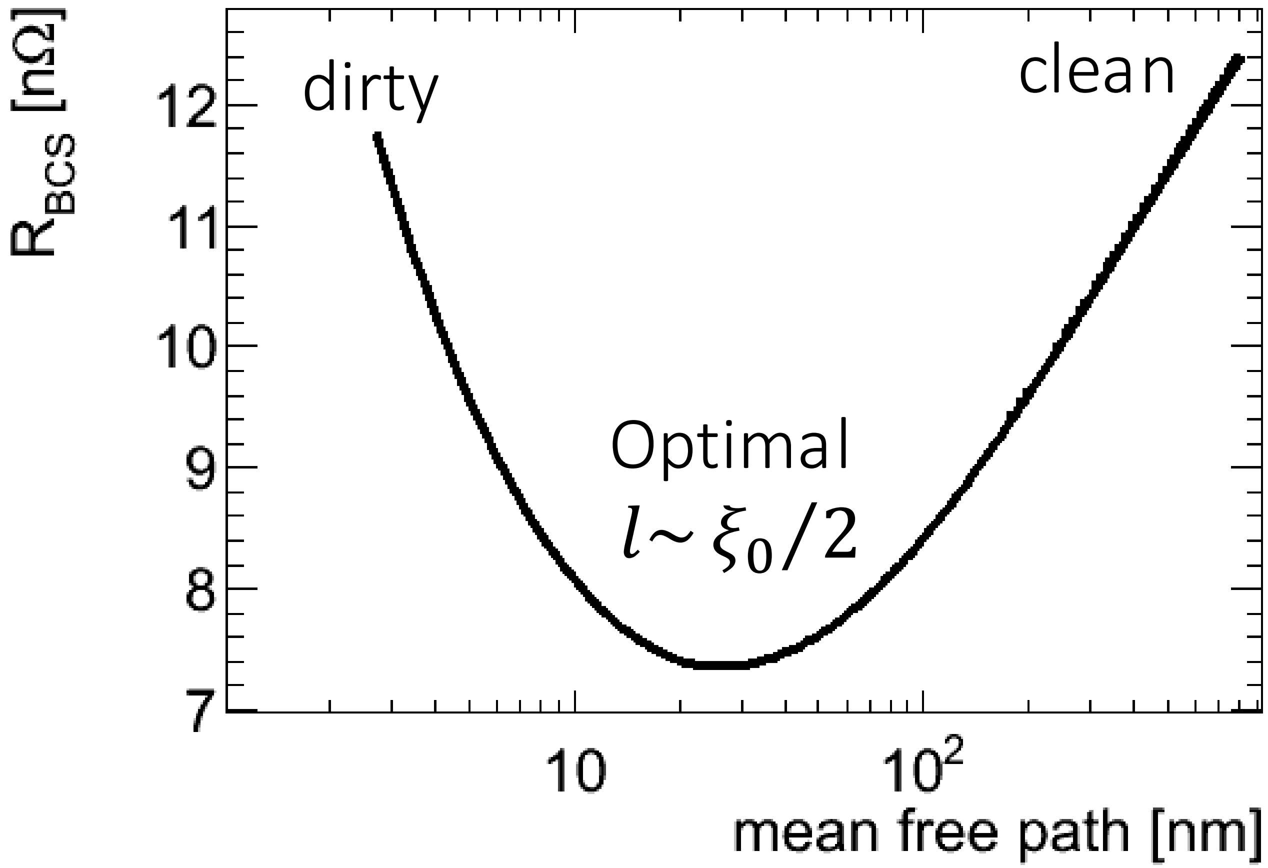
Check "Anomalous skin effect"

$$R_s \sim \frac{\mu_0^2}{2} \lambda_0^3 \sigma_N \omega^2$$

$$\lambda_0 = \lambda_L \sqrt{1 + \frac{\pi \xi_0}{2l}}$$

$$\sigma_N = \frac{e^2 n \tau}{m^*} \propto l$$

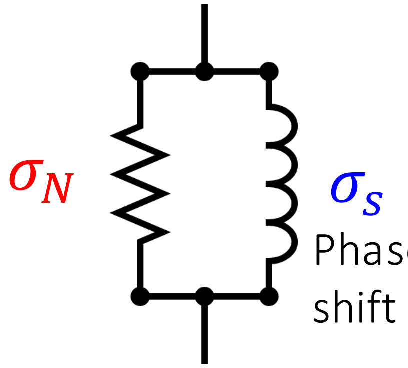
$$\rightarrow R_s \sim l \times \left(1 + \frac{\pi \xi_0}{2l}\right)^{3/2}$$



Counter intuitively, super clean material is not ideal for SRF cavities!

→ Heat treatment, doping, etc to make ***surface*** dirty

$\lambda(T)$ is also predicted by BCS



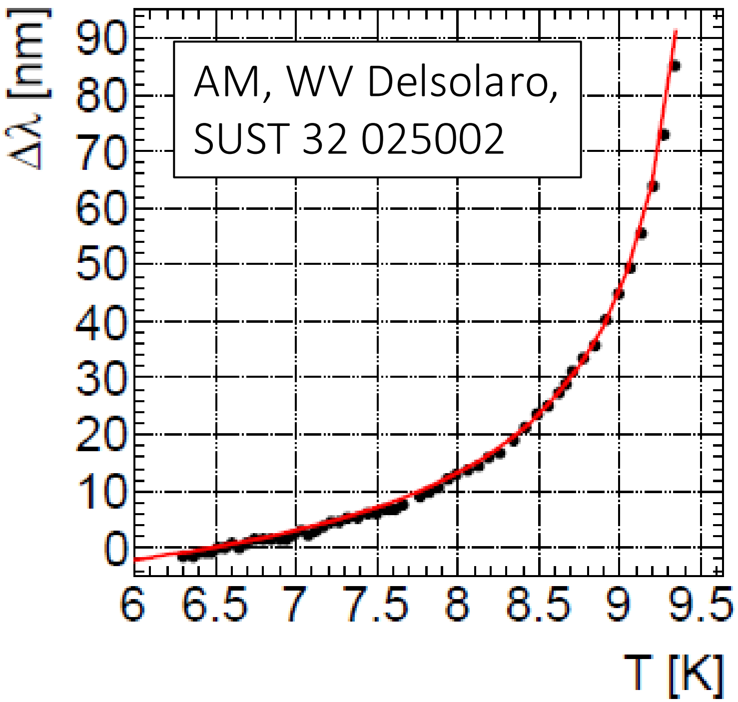
Phase shift $\pi/2$

Surface reactance

$$X_s \equiv \mathrm{Im}\left(\frac{E_x(z=0)}{\int_0^\infty J_x(z)dz}\right) \sim \omega\mu_0\lambda_L$$

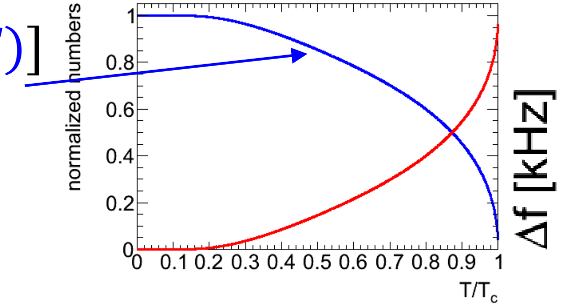
Two fluid circuit model

$$\xrightarrow{\text{BCS}} \lambda(T) = \frac{X_s(T)}{\mu_0\omega}$$



Gorter Casimir expression [two fluid; $\lambda(T) \sim n_S(T)$]

$$\lambda(T) \sim \frac{\lambda_0}{\sqrt{1-(T/T_c)^4}}$$



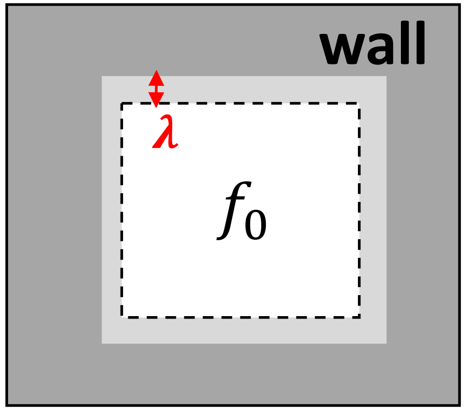
Approximated expression

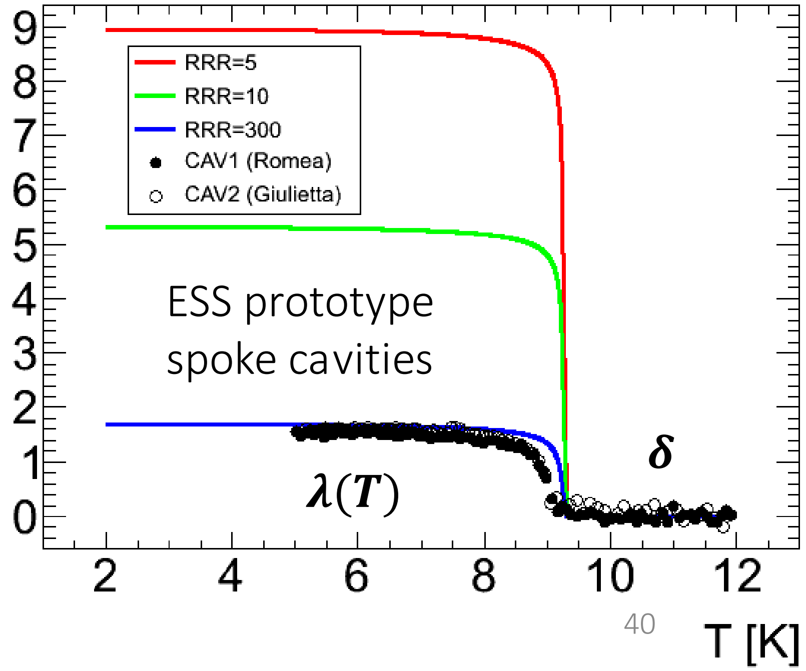
$$\lambda_0 = \lambda_L\sqrt{1+\frac{\pi\xi_0}{2l}}$$

→ Change in penetration depth causes effective change of the cavity size

→ resonance frequency is affected

$\boldsymbol{\Delta f(T) \propto \Delta\lambda(T)}$





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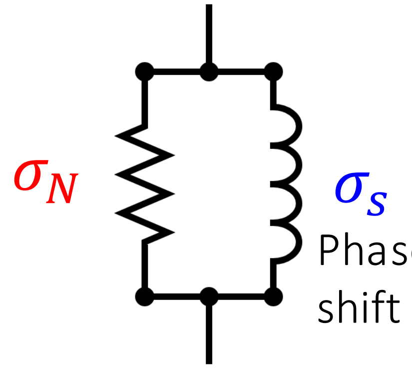

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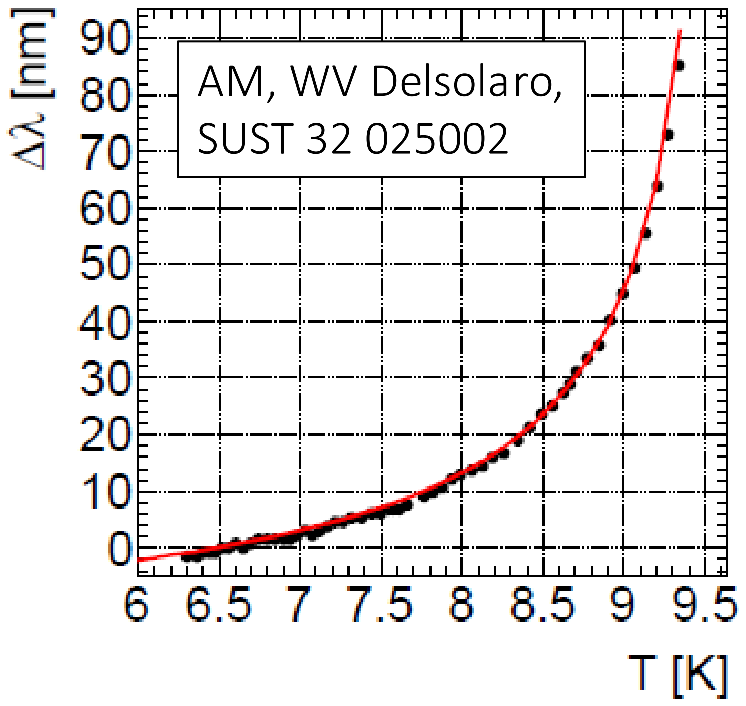
BCS

$$\rightarrow \lambda(T) = \frac{X_s(T)}{\mu_0\omega}$$



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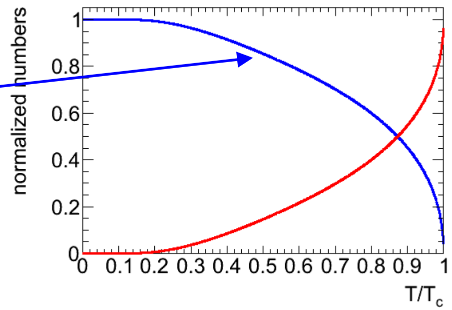

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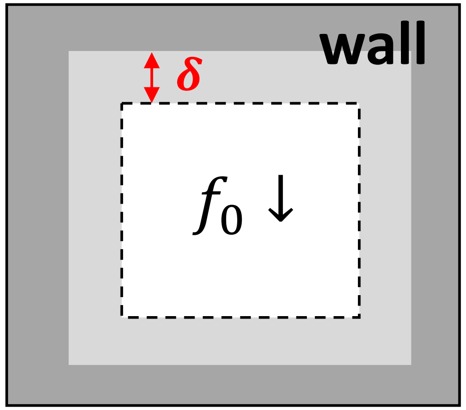
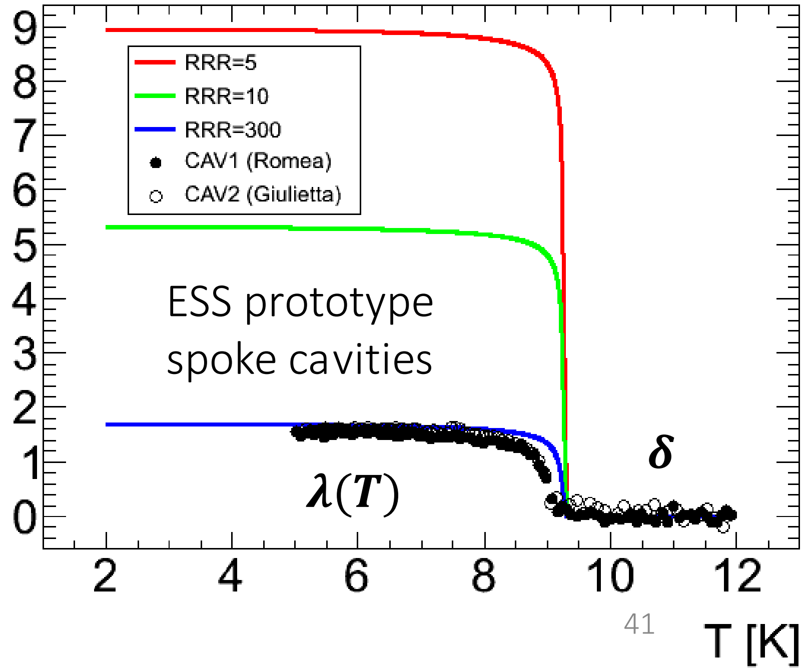
Q8

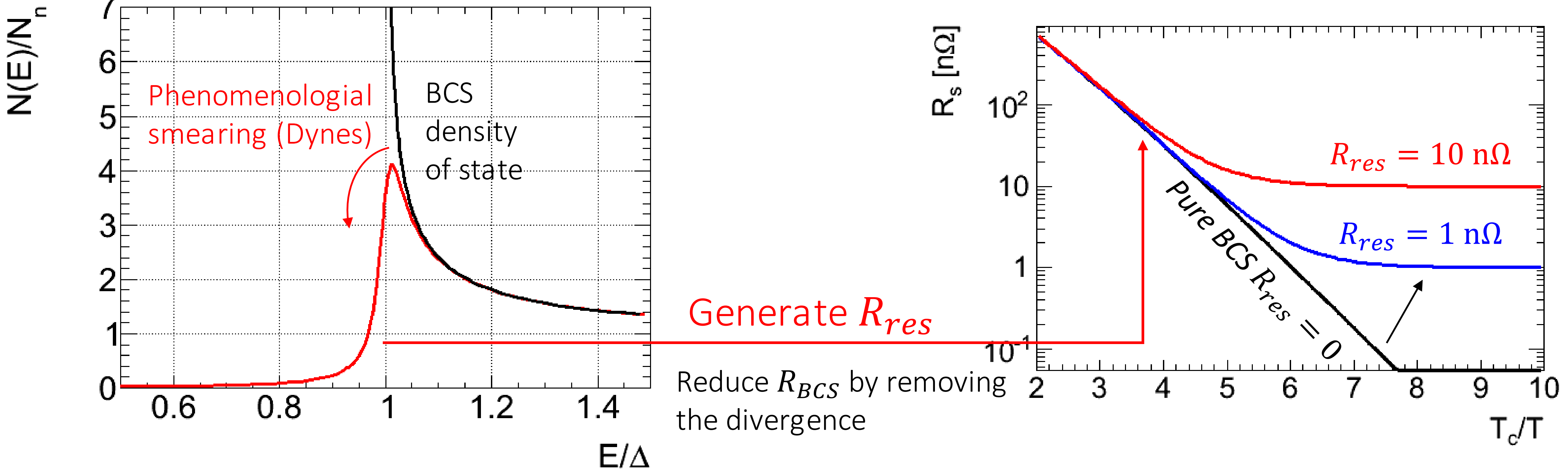
T. Juginger PhD thesis





Smearing of Density of States and residual resistance

In reality $R_s \sim R_{\mathrm{BCS}}(T) + R_{\mathrm{res}}$



Remark: DoS smearing is not the only cause of residual resistance

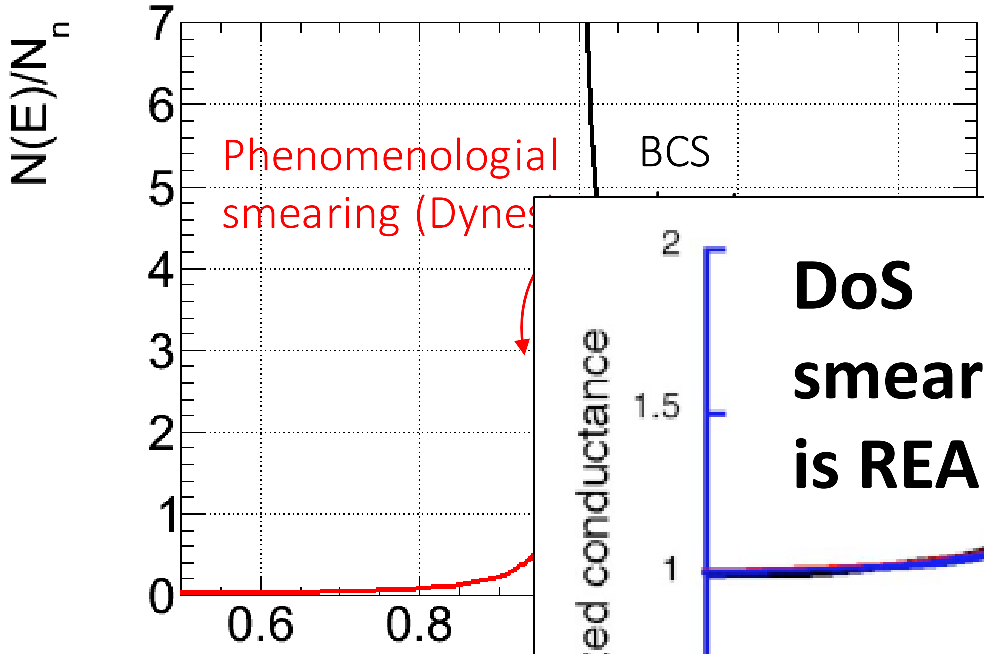
- Lossy oxides?
- Hydride?
- Grain boundaries??
- Influence of magnetic vortex

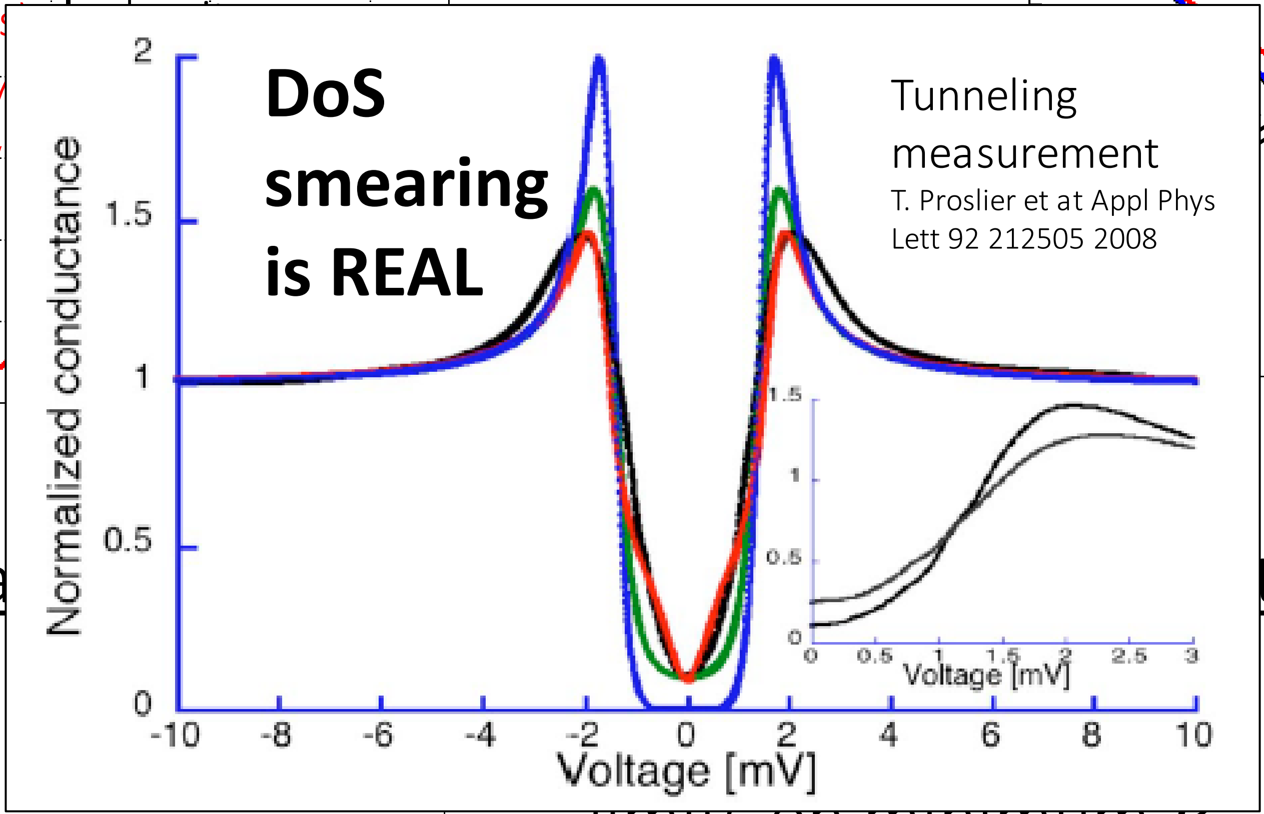
etc...

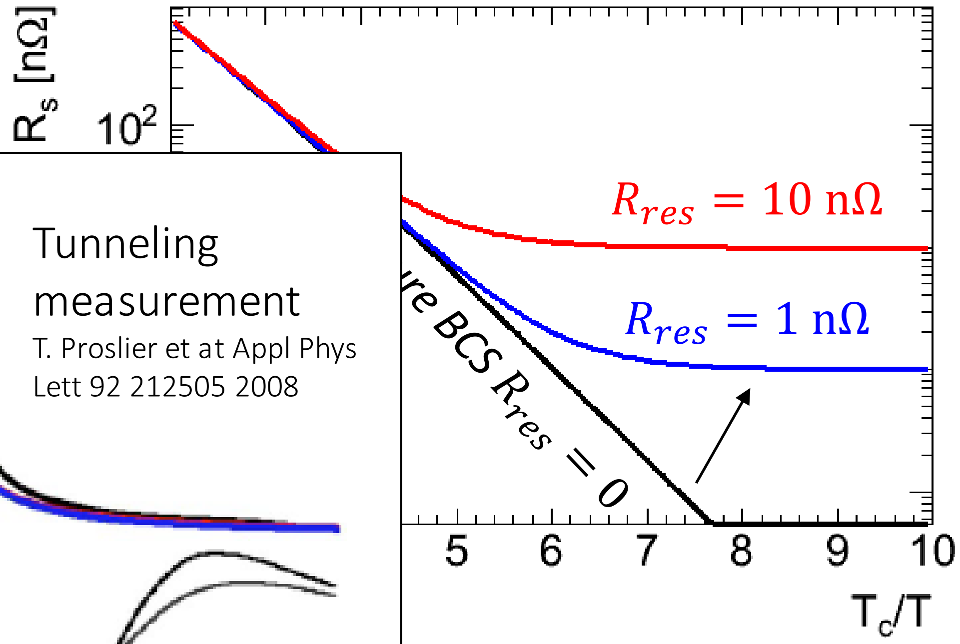
However, the question is ultimate limits on minimum R_s (highest Q) after removing extrinsic effects

Smearing of Density of States and residual resistance

In reality $R_s \sim R_{\mathrm{BCS}}(T) + R_{\mathrm{res}}$







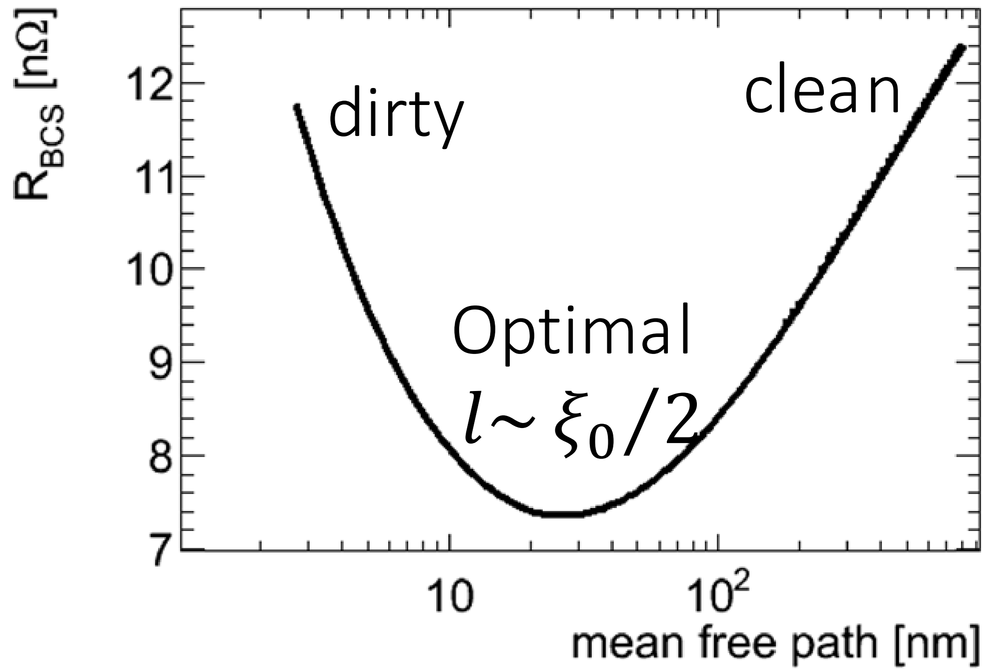
Remark: DoS smea… …tance

- Lossy oxides?
- Hydride?
- Grain boundaries??
- Influence of magnetic vortex

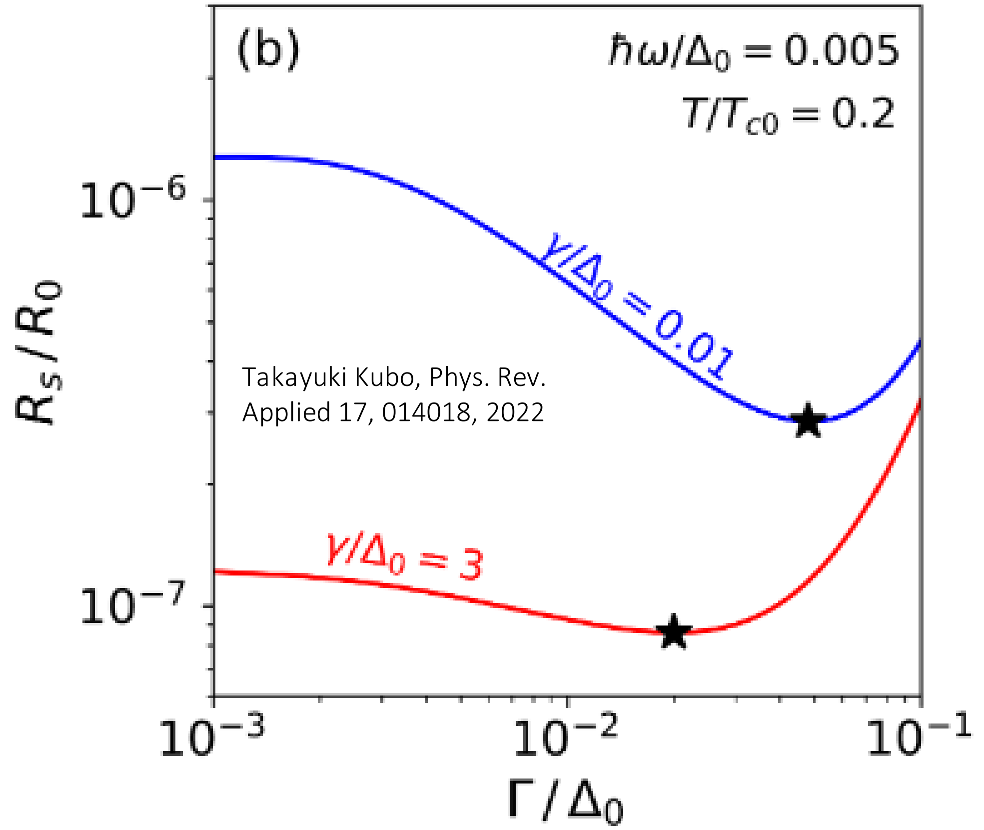
… is ultimate limits on minimum R_s (highest Q) after removing extrinsic effects

Minimum surface resistance from the theory

$R_{\mathrm{BCS}}(T)$ has a minimum as a function of impurity scattering (anomalous skin effect)



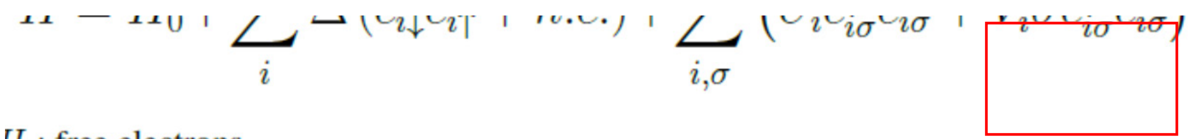
$R_{\mathrm{BCS}}(T) + R_{res}$ has a minimum as a function of Dynes parameter Γ with a given impurity scattering



<u>Fundamental question:</u>

What causes the Dyne's DoS smearing?

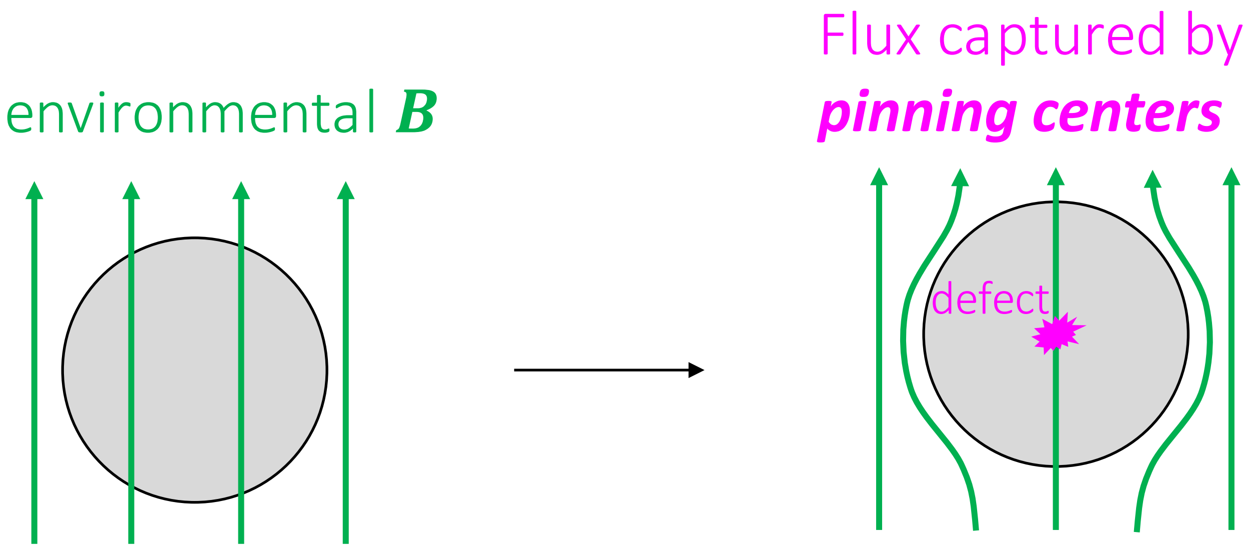
F. Herman: pair-breaking term (?) [PRB 96 014509]

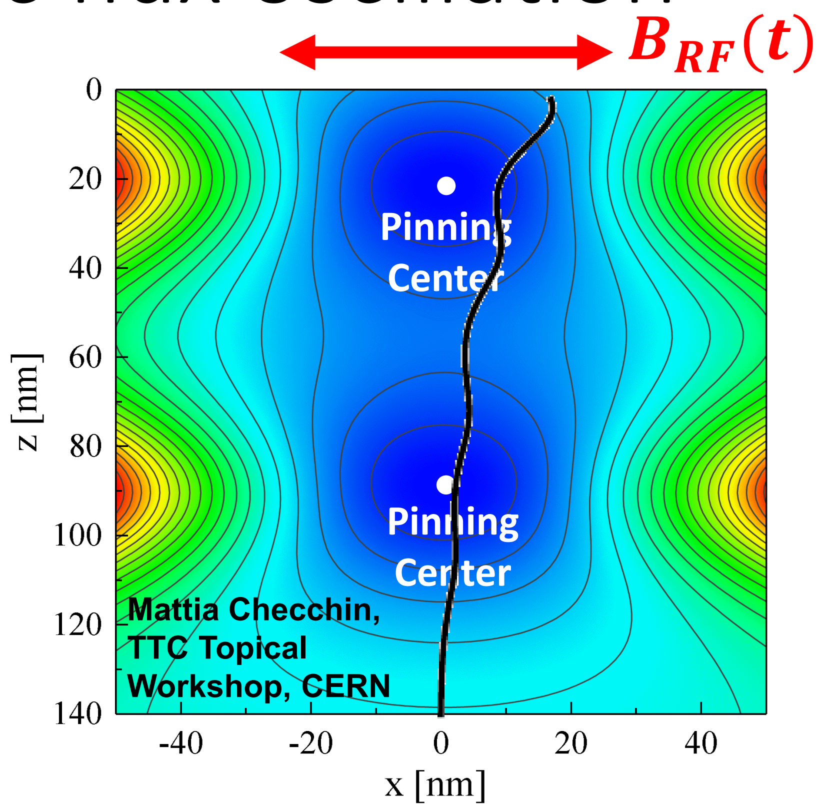


H_0: free electrons.

R_{res} contribution from magnetic flux oscillation

Flux expulsion may not be perfect





Phenomenological equation of motion (Bardeen Stephen)

$$M\frac{\partial^2 \boldsymbol{u}}{\partial t^2} + \eta\frac{\partial \boldsymbol{u}}{\partial t} - \epsilon\frac{\partial^2 \boldsymbol{u}}{\partial z^2} + \nabla U(z,u) = \boldsymbol{J}_{RF}(z,u)\times \boldsymbol{B}_{\mathrm{ext}}$$

Effective inertia | Effective viscosity | Effective tension | **Pinning potential** | **Lorentz force drives flux oscillation**

This flux oscillation can cause substantial power dissipation

Simple approximation

$$M\frac{\partial^2 \boldsymbol{u}}{\partial t^2} + \eta\frac{\partial \boldsymbol{u}}{\partial t} - \epsilon\frac{\partial^2 \boldsymbol{u}}{\partial z^2} + \nabla U(z,u) = \boldsymbol{J}_{RF}(z,u) \times \boldsymbol{B}_{\text{ext}}$$

Static model

Q9

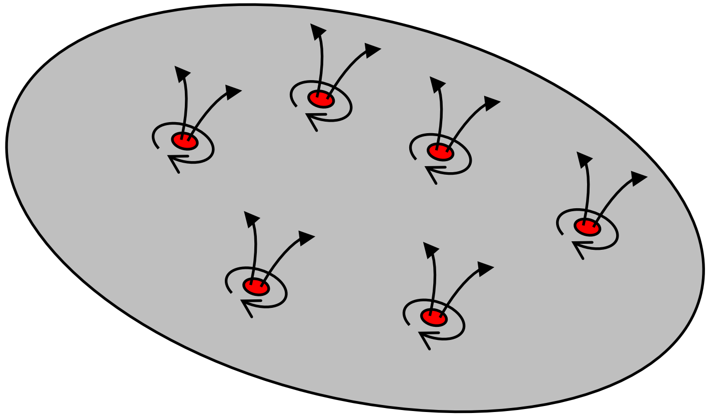
$$R_{\text{mag}} \sim N \times \pi\xi_0^2 \times R_{\text{n}} \sim \frac{B_{\text{ext}}}{2B_{c2}} R_{\text{n}}$$

Flux number density

Normal conducting area

Normal conducting surface resistance

D. Longuevergne, AM arXiv:2009.07007
S, Calatroni and R. Vaglio, PRAB 22, 022001 (2019)



Earth field B_{ext}=50uT
B_{c2}~400 mT (Nb)
R_n~1.3 mΩ at 1.3 GHz (Nb)

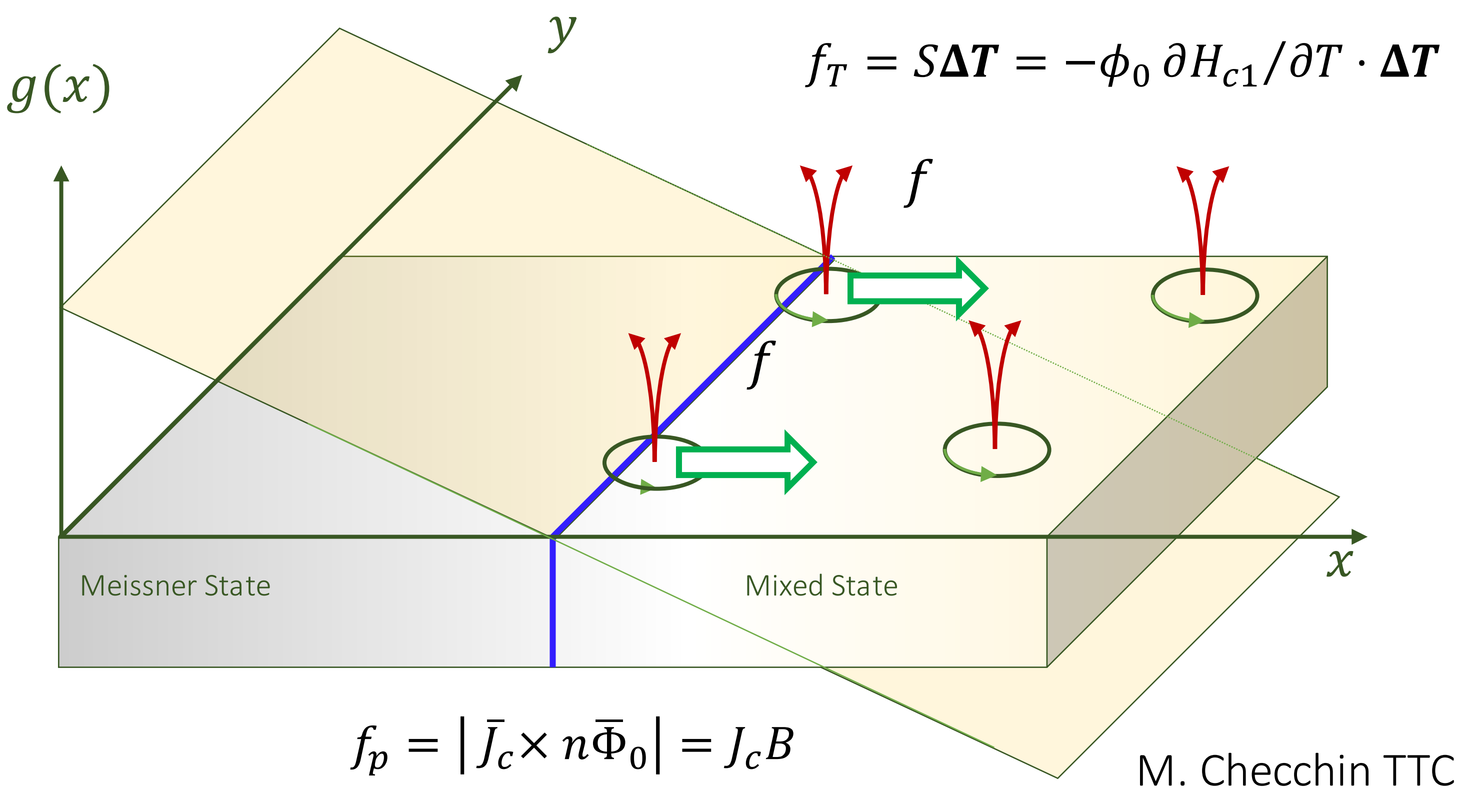
$$R_{\text{mag}} \sim 80\text{n}\Omega > R_{BCS}(2K) \sim 10\text{n}\Omega$$

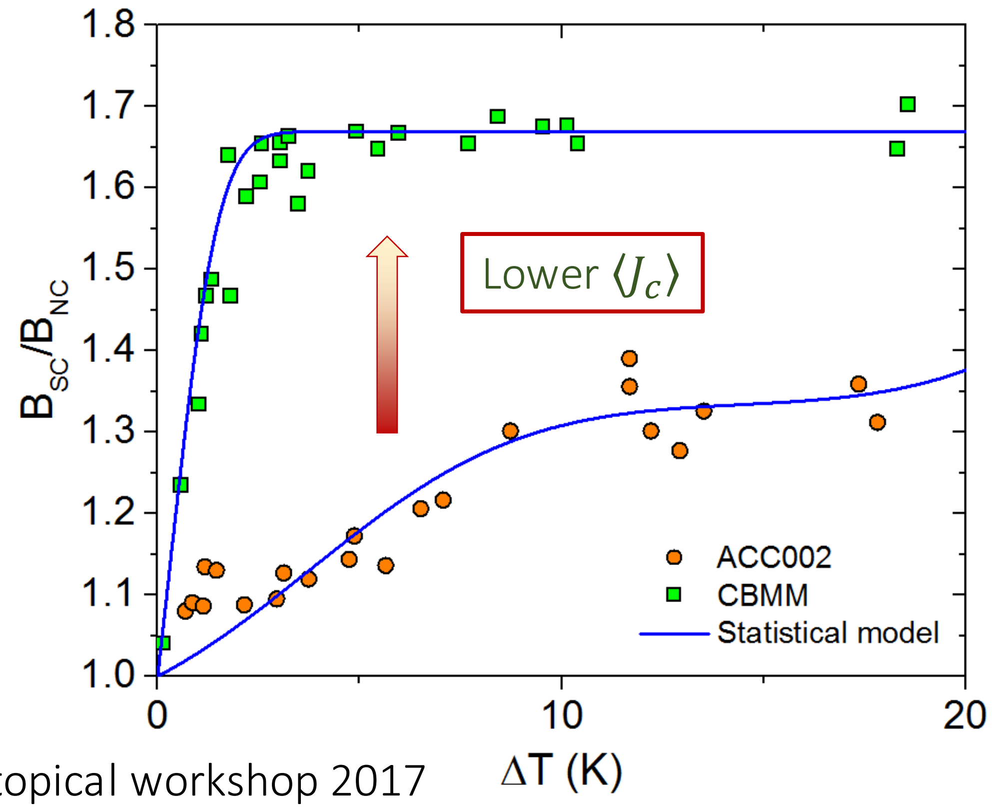
A cavity can be spoiled!

Solutions

1. A good **magnetic shield** (earth field 50uT → < 1uT) ***Lecturer by Nicolas Bazin***
2. Expel more fluxes at phase transition
3. (Reduce sensitivity of the flux oscillation against RF)

Flux expulsion at the phase transition from NC to SC

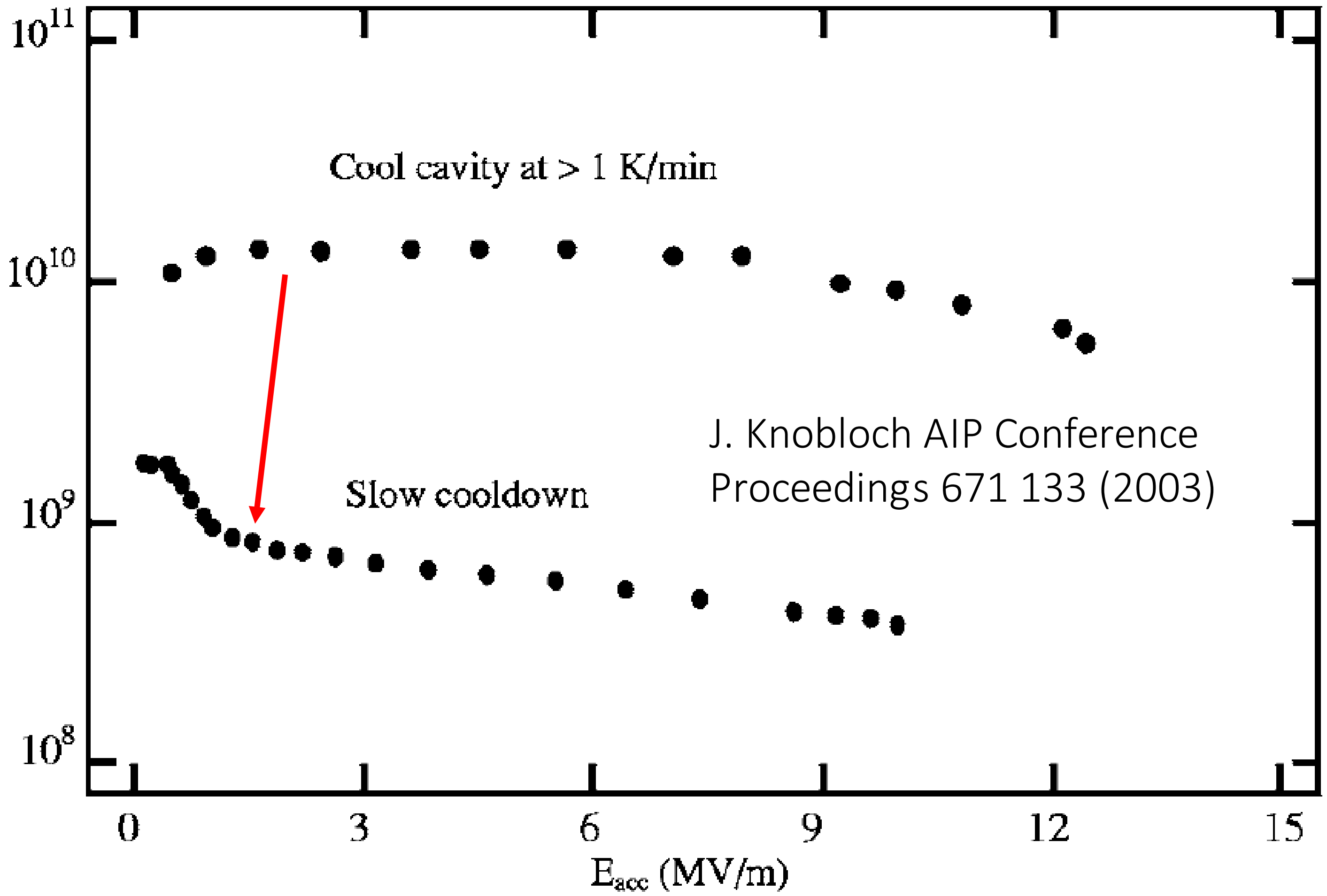




M. Checchin TTC topical workshop 2017

- Balance between thermodynamic force f_T and pinning force f_p in the mixed state $[B_{c1}(T_c) < B_{ext} < B_{c2}(T_c)]$
- Higher thermal gradient → higher expulsion efficiency
- Statistical assumption in trapping efficiency → Material difference (J_c) reproduced
  → Cooling down with higher thermal gradient is a standard receipt in LCLS-II at SLAC

Q-disease: Nb hydride formed during slow cooling down



J. Knobloch AIP Conference Proceedings 671 133 (2003)

Solution

1. Anneal the cavity **600-900 C** to degas hydrogen
2. Avoid slow cooling down around dangerous temperature 75-150K

Seldom appears in modern cavities but be careful

Annealing in the different recipes

xNy doping

1. High temperature degassing 800 °C (~ 3hours) in UHV
2. Nitrogen injection at the end of process (~ 25 mTorr) hold for 'x' mins
3. Evacuate furnace and temperature hold for 'y' mins
4. Furnace cooldown to room temperature in UHV
5. Electropolishing (5-10 μm)
6. High Pressure Rinse (HPR), clean room asselbly and rf test

N-infusion

1. HPR and Nb caps installed in clean room
2. High temperature degassing 800 °C (~ 3hours) in UHV
3. Furnace cooldown to (120-200 °C) in UHV
4. Nitrogen injection ~ 25 mTorr hold for 24-48 hours
5. Furnace cooldown to room temperature in UHV
6. High Pressure Rinse (HPR), clean room asselbly and rf test

2-step baking (1.3 GHz)

1. HPR and Nb caps installed in clean room
2. High temperature degassing 800 °C (~ 3hours) in UHV
3. 75C for a few hours in UHV
4. 120C for 48 hours in UHV
5. Furnace cooldown to room temperature in UHV
6. High Pressure Rinse (HPR), clean room asselbly and rf test

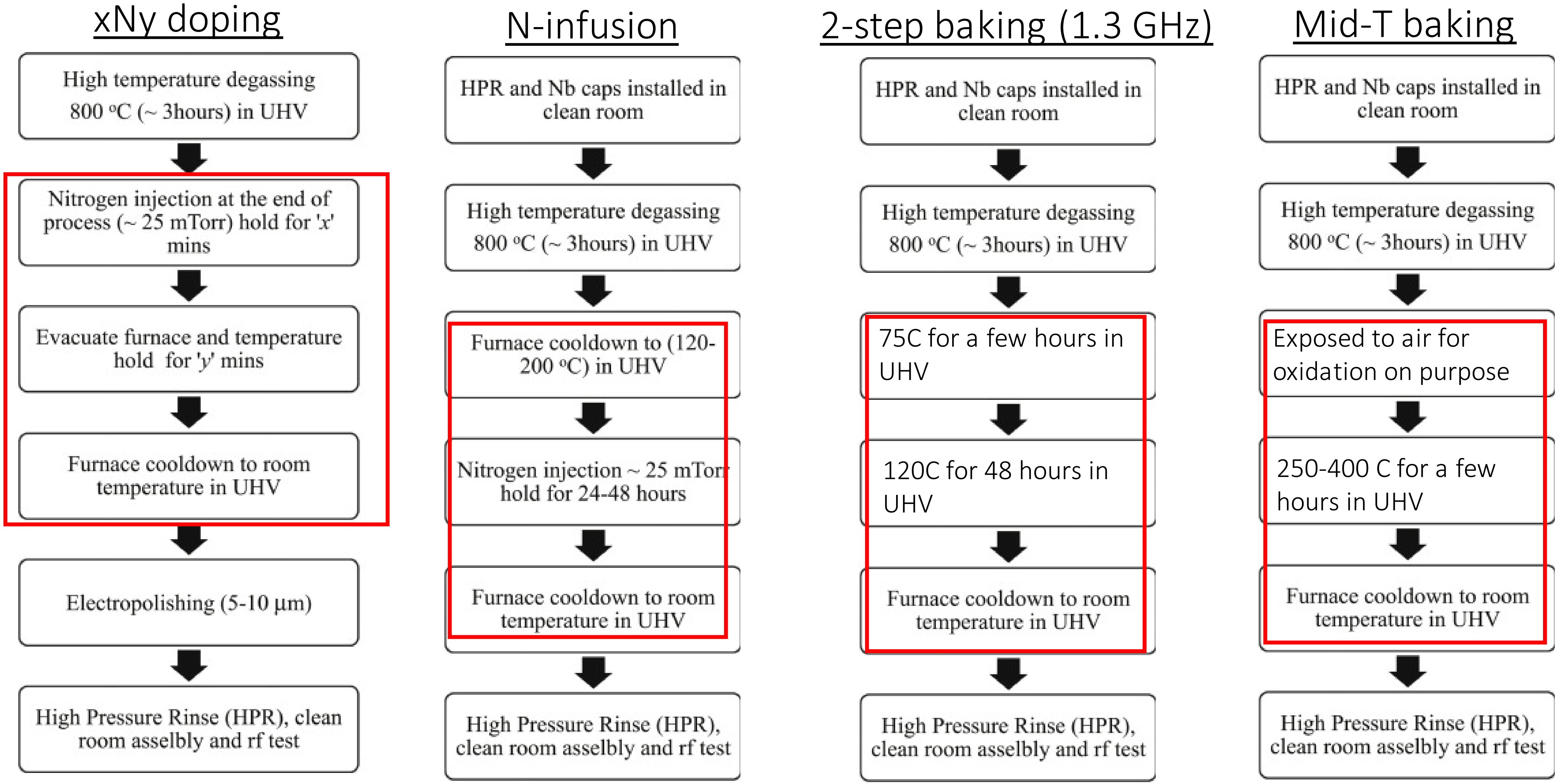
Mid-T baking

1. HPR and Nb caps installed in clean room
2. High temperature degassing 800 °C (~ 3hours) in UHV
3. Exposed to air for oxidation on purpose
4. 250-400 C for a few hours in UHV
5. Furnace cooldown to room temperature in UHV
6. High Pressure Rinse (HPR), clean room asselbly and rf test

P. Dhakal "Nitrogen doping and infusion in SRF cavities: A review" Phys Open 5 100034 2020

P. Sha et al. *Appl. Sci.* **12**, 546 (2022)

Doping/baking → high-Q and even high-G



P. Dhakal "Nitrogen doping and infusion in SRF cavities: A review" Phys Open 5 100034 2020

P. Sha et al. *Appl. Sci.* **12**, 546 (2022)

Outline

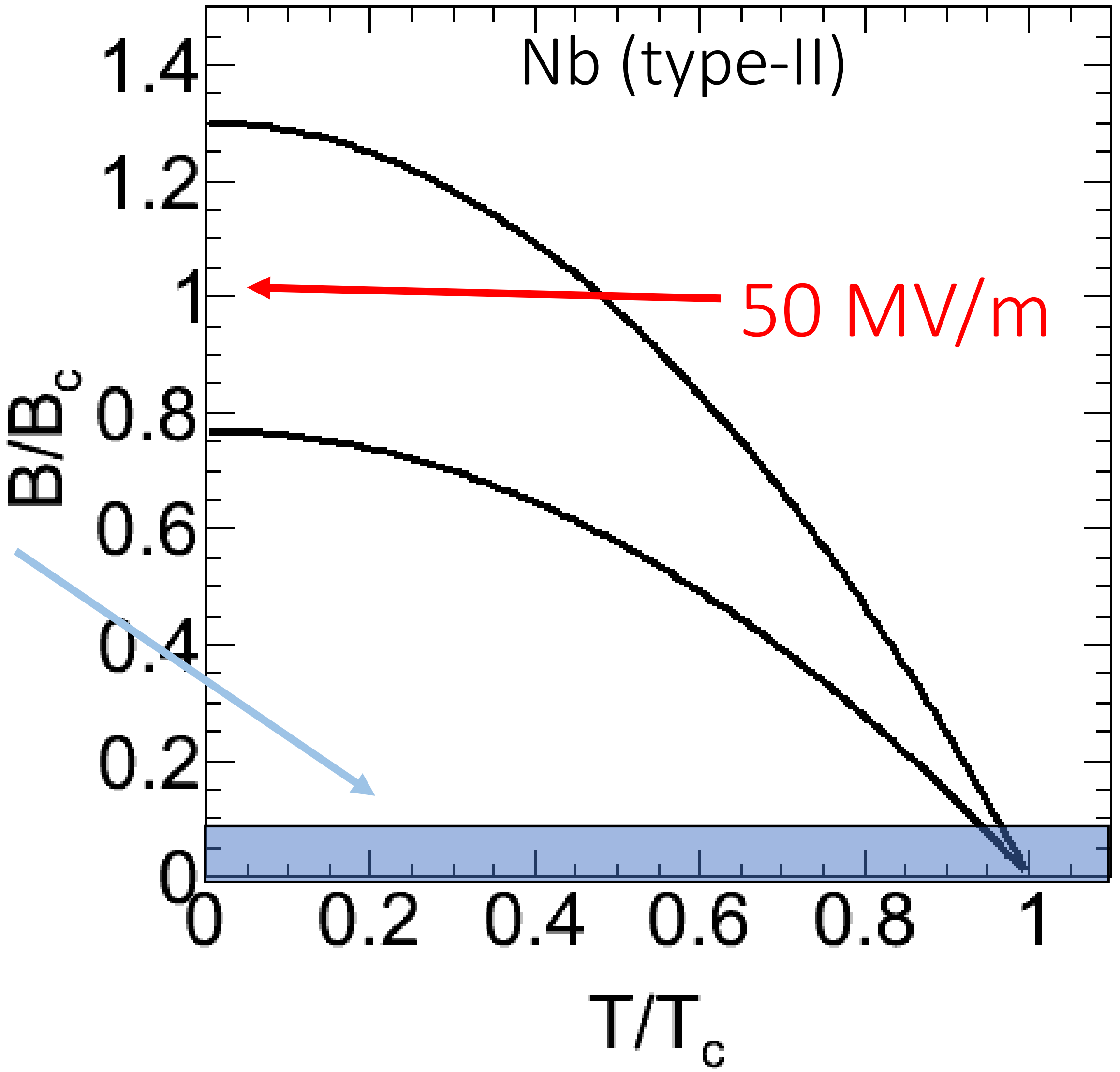
- Introduction: why superconducting RF?
- Finite surface resistance of superconductors
  - Superconductors in equilibrium
  - BCS resistance
  - Residual resistance
- Field limitations
  - Fundamental limit
  - Practical limits
- Niobium as a cavity material
  - Required feature
  - Beyond niobium

Remark: validity of linear response theory

Mattis-Bardeen formula
($f \ll 2\Delta$, $T < T_c/2$)

$$R_{BCS} \propto \frac{\omega^{1.5-2.0}}{T} \exp\left(-\frac{\Delta}{k_B T}\right)$$

is valid for low RF field ($B_{RF} \ll B_c$) because it is 1st order perturbation (linear response)



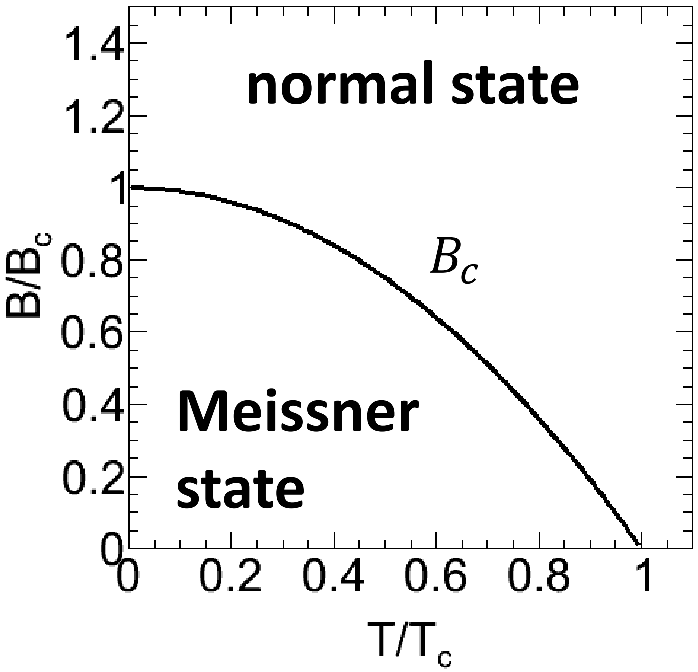
However, state-of-the-art cavities reach 50 MV/m i.e. $B_{RF} \sim B_c$

→ Fundamental challenge in condensed matter physics

Under strong but *static* magnetic field

Type-I

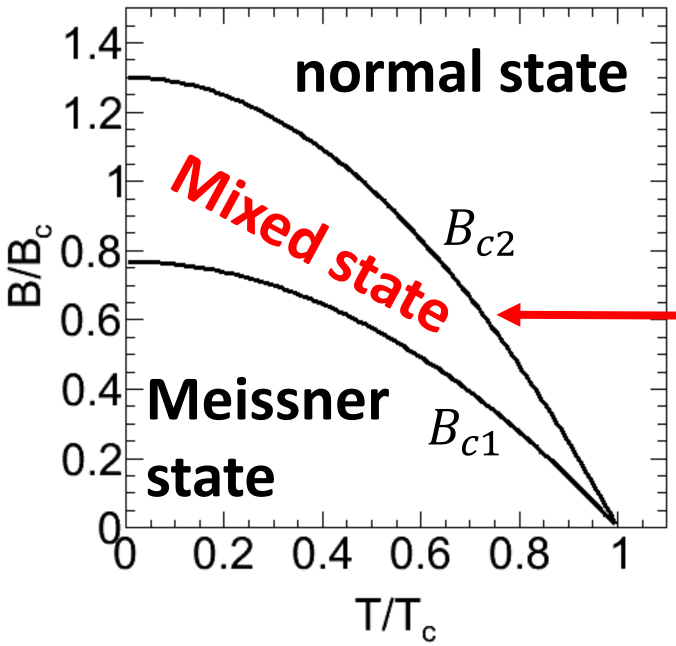
$$\kappa = \frac{\lambda}{\xi} < \frac{1}{\sqrt{2}} = 0.71$$



$$\kappa_{Pb} \sim \frac{28\text{ nm}}{71\text{ nm}} \sim 0.40$$

Type-2

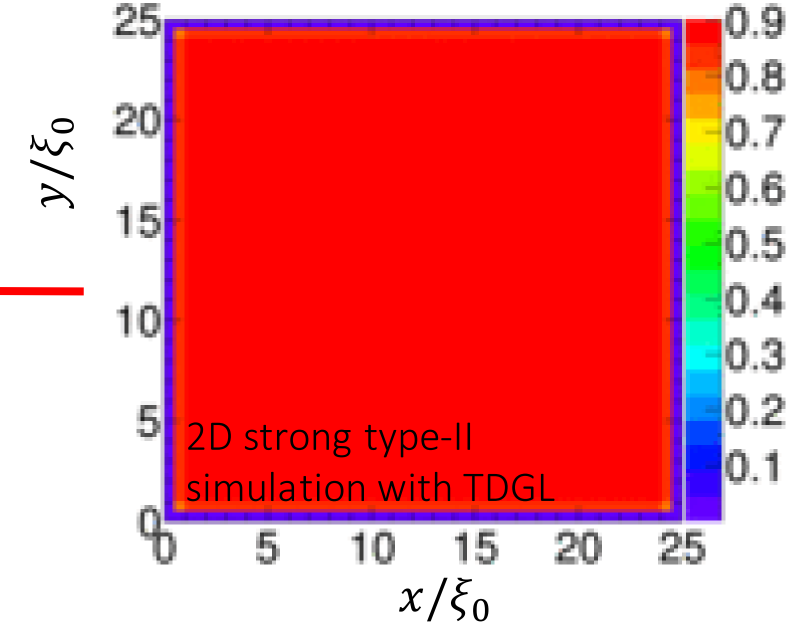
$$\kappa = \frac{\lambda}{\xi} > \frac{1}{\sqrt{2}} = 0.71$$



$$\kappa_{Nb} \sim \frac{36\text{ nm}}{39\text{ nm}} \sim 0.92$$

Stabilized by NC/SC boundary energy

$$\frac{1}{2\mu_0}\left(\xi_0 B_c^2 - \lambda_L B^2\right) < 0 \text{ for } B > B_{c1}$$



Without pinning centers, type-II ***traps*** magnetic flux if $B > B_{c1}$

Does type-II superconductor dissipate too much power from flux entry & oscillation? Are type-II superconductors ***useless*** for SRF?

1st order phase transition can be *metastable*

Super-cooling of water: $T < 0$ C but still liquid

https://tenor.com/view/diy-science-hack-ice-water-gif-3448836

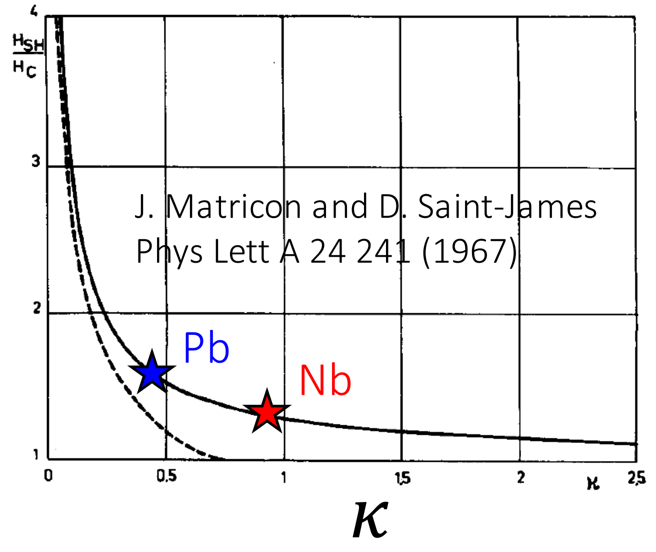
SC phase transition with a ***magnetic field*** is a 1st order phase transition

→ **$B > B_{c1}$ can be a metastable super-heating state**

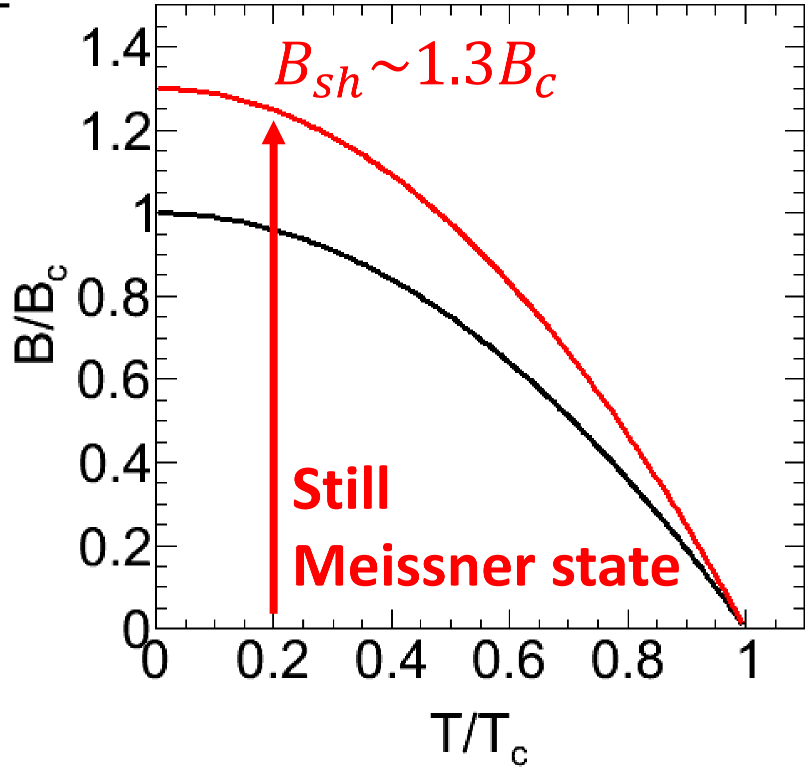
Relevant critical field for SRF: superheating field

Ginzburg-Landau equation

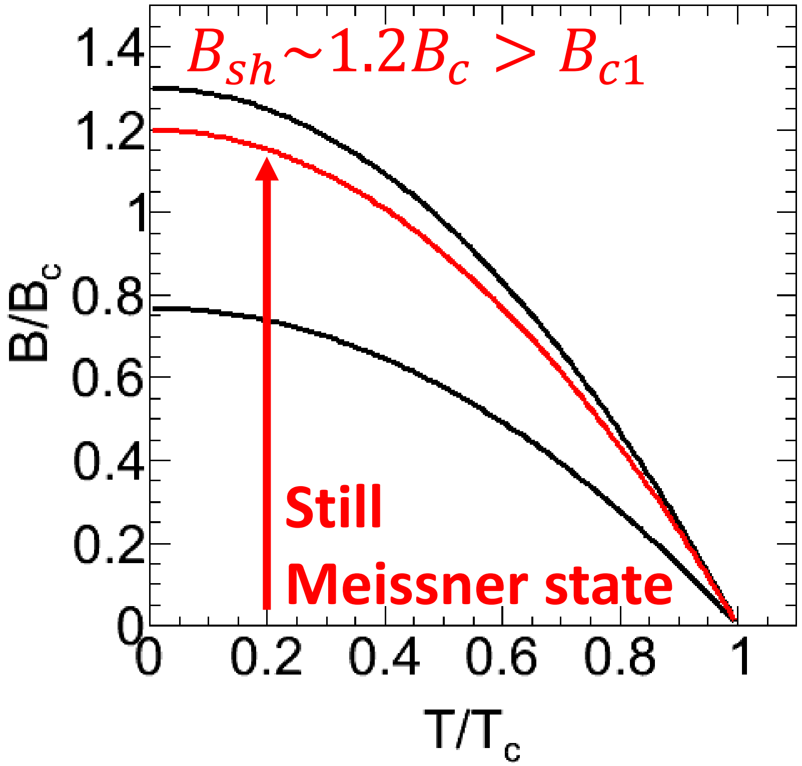
$B_{sh} > B_c$ in general



Pb (type-I)



Nb (type-II)



Go'rkov showed that BCS theory reproduces Ginzburg Landau equation around $\boldsymbol{T \rightarrow T_c}$

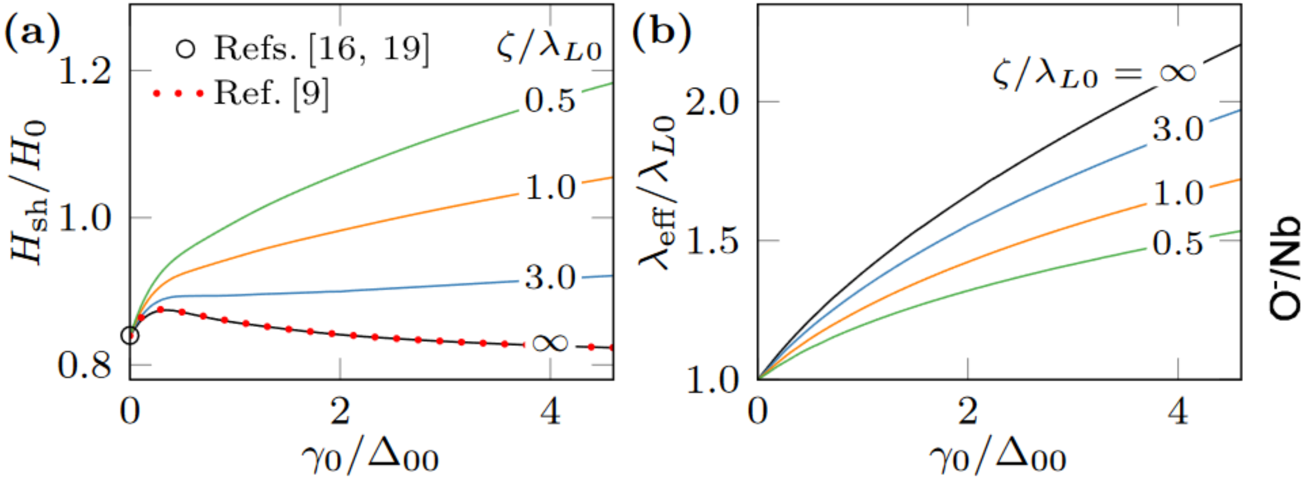
→ The validity of this B_{sh} at $T < T_c$ deserves discussion

Quasi-classical formalism, influence of impurity, multilayer coating to further enhance B_{sh}, nonlinear $R_s(B_{RF})$…

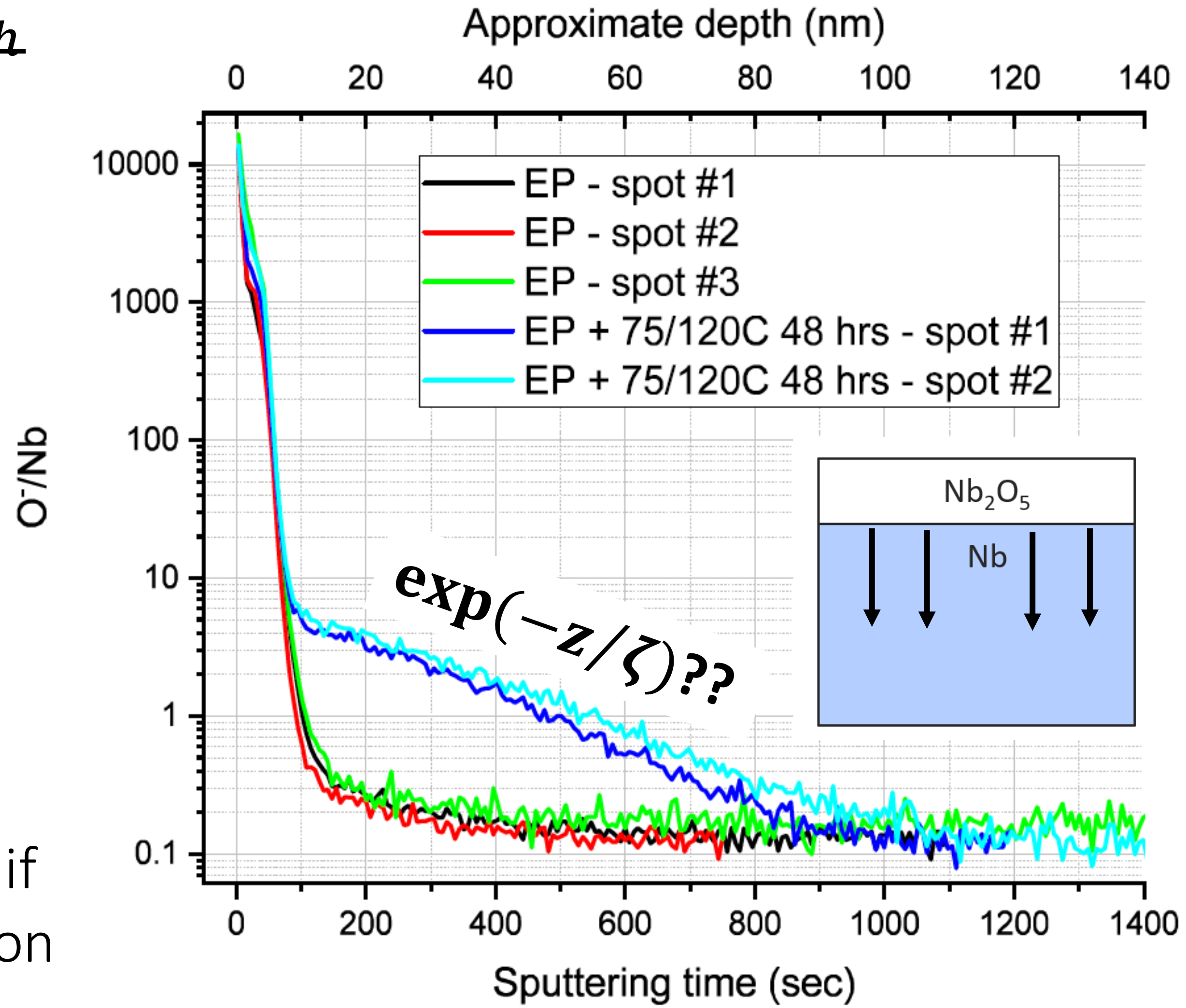
Higher/lower gradient by low-T / 2-step baking

Inhomogeneous dislocations impacts H_{sh}

Parameter of inhomogeneity $\exp(-z/\zeta)$



Impurity content   Impurity content



→ Higher superheating fields can be achieved if surface impurity has inhomogeneous distribution

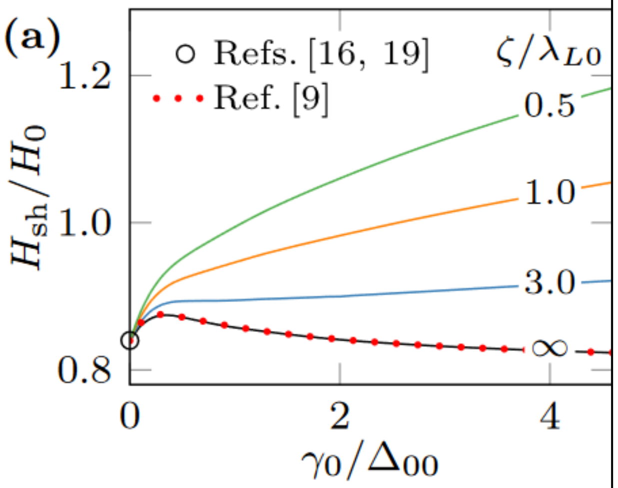
D. Bafia LCWS2023

Vudtiwat Ngampruetikorn and J. A. Sauls Phys. Rev. Research 1, 012015(R) 2019

Higher/lower gradient by low-T / 2-step baking

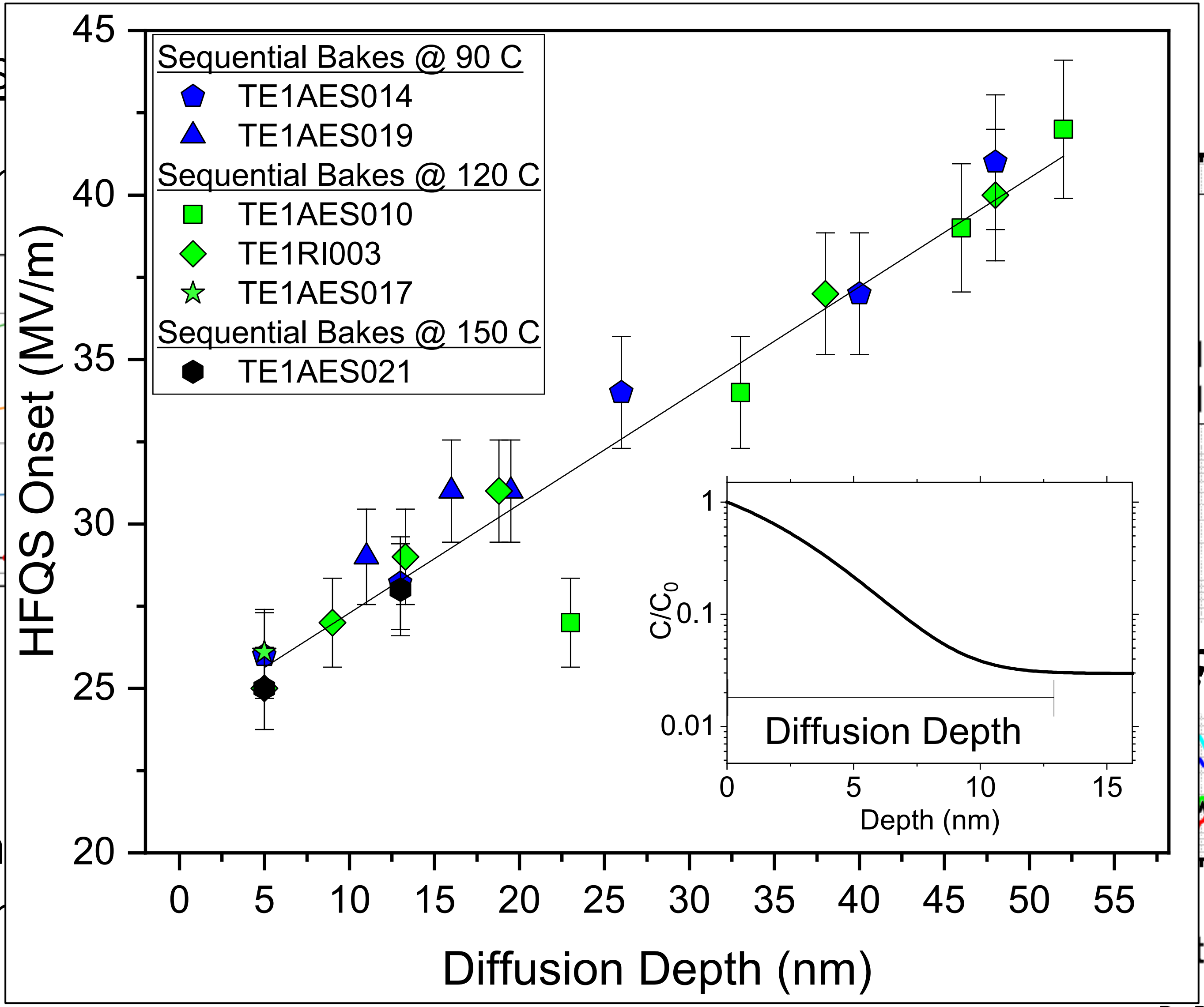
Inhomogeneous dis

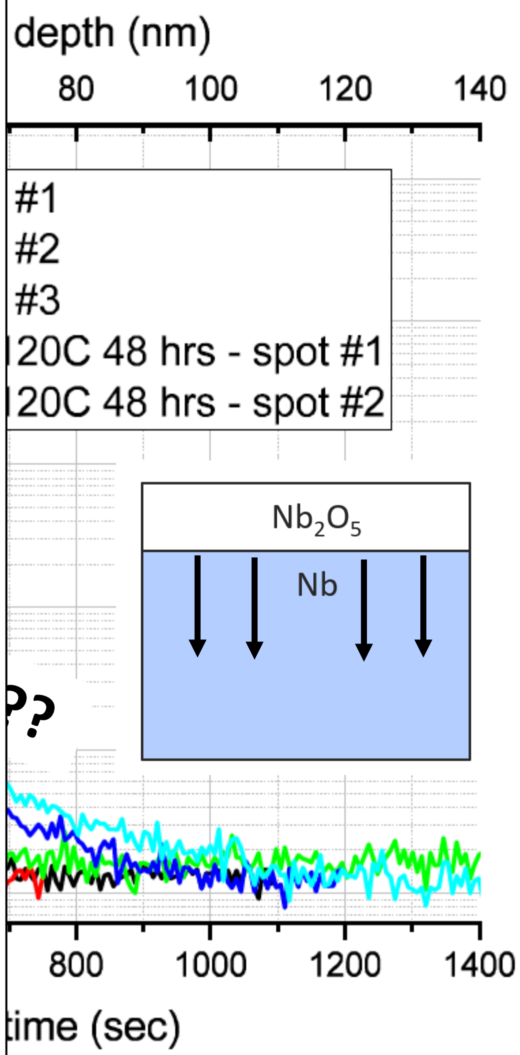
Parameter of inhor



Impurity content

→ Higher superheatin
surface impurity has ir





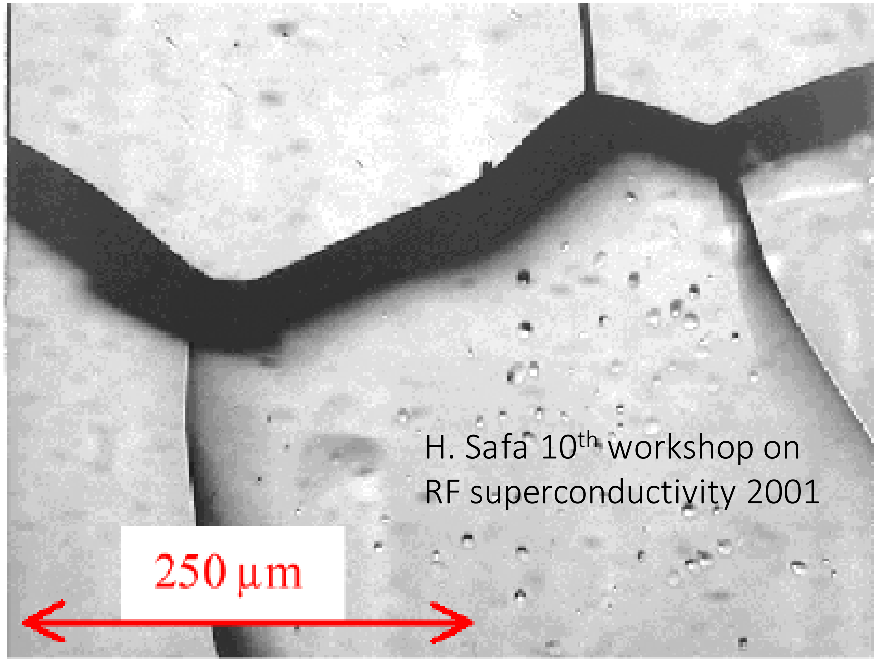
D. Bafia LCWS2023

Vudtiwat Ngampruetikorn and J. A. Sauls Phys. Rev. Research 1, 012015(R) 2019

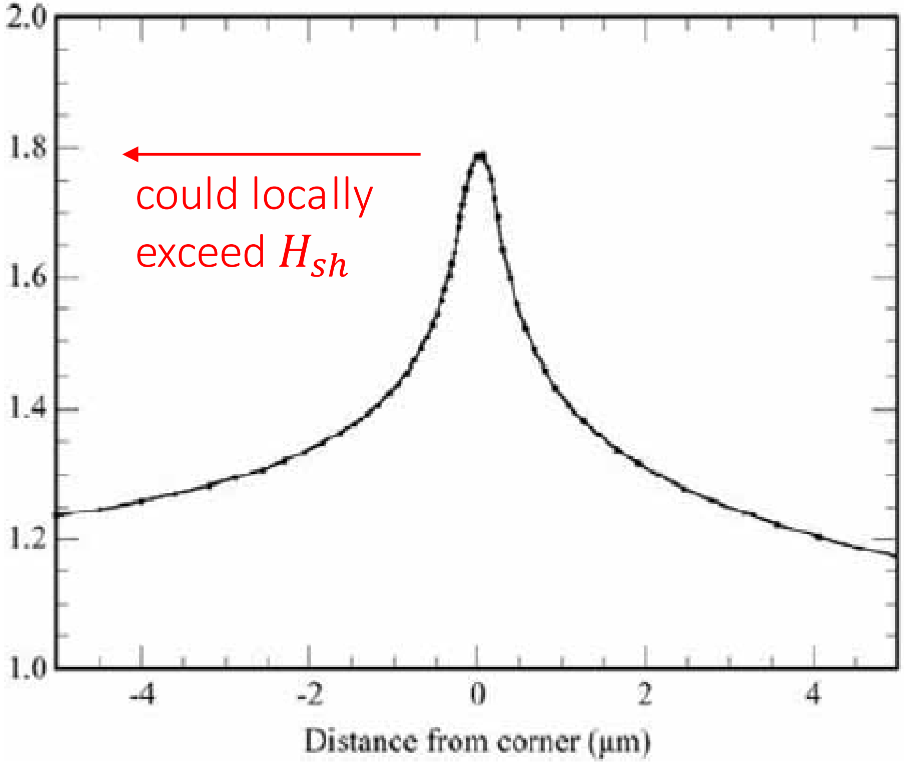
Practical quench limits: one example

Local defect or field enhancement

Standard BCP Chemistry on niobium :
Sharp boundary edges are clearly visible



Calculated magnetic field enhancement
on a 100 µm x 10 µm step



→ Choice of chmical etching method
(Buffer Chemical Polishing or Electro Polishing)

Lecture by L. Popielarski

Quench limit and high-field Q-slope is an open research area

Thermal breakdown

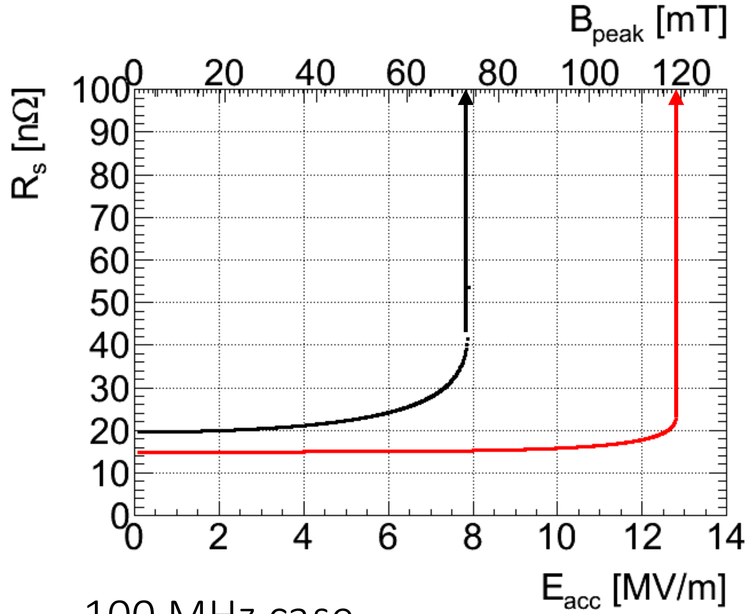
BCS resistance

$$\boldsymbol{R_s} \sim \frac{A\omega^2}{\boldsymbol{T}} \exp\left(-\frac{\Delta}{k_B \boldsymbol{T}}\right)$$

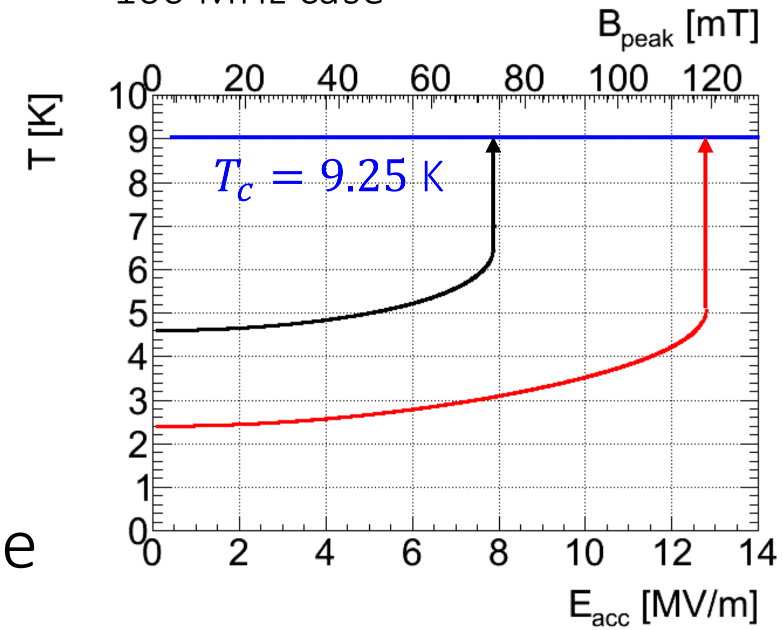
Joule heating: $\boldsymbol{P} = \frac{1}{2}\boldsymbol{R_s} H^2$

$$\boldsymbol{\Delta T} = \boldsymbol{R_{th}} \boldsymbol{P}$$

- Exponential temperature dependence can cause catastrophic positive feedback in temperature



100 MHz case



Defects enhance thermal breakdown

- Thermal conductivity λ
- Thermal resistance R_{th}

$$\lambda = \frac{1}{R_{th}}$$

BCS resistance

$$R_s \sim \frac{A\omega^2}{T} \exp\left(-\frac{\Delta}{k_B T}\right)$$

Thermal feedback

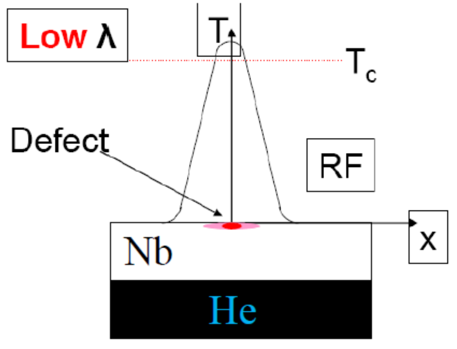
Joule heating: $P = \frac{1}{2} R_s H^2$

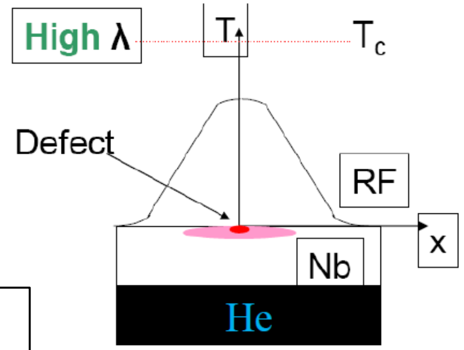
$$\Delta T = R_{th} P$$

Local quench spot

$$R_s \sim 1\ \mathrm{m}\Omega$$

Either cooled down again or trigger global quench





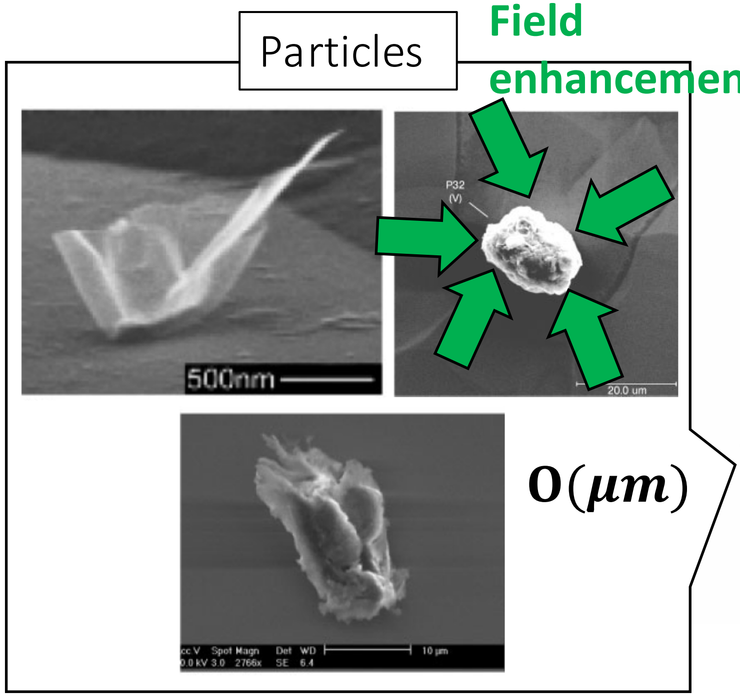
Q10

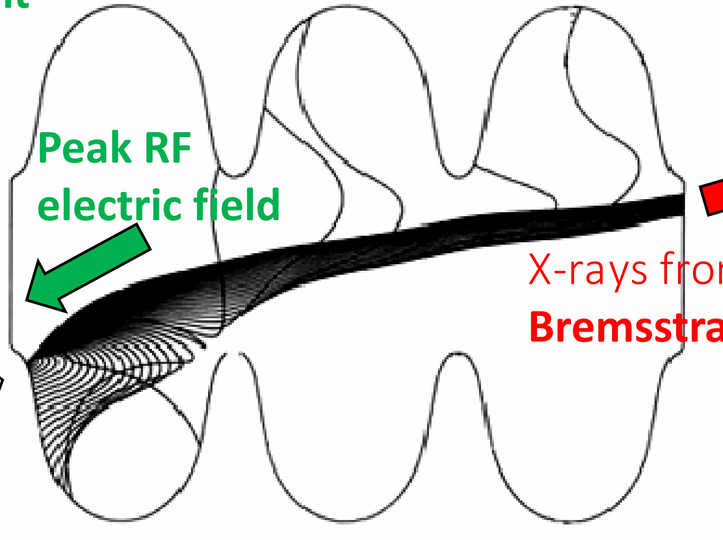
What about the thermal resistance at the Nb/He interface →Kapitza resistance

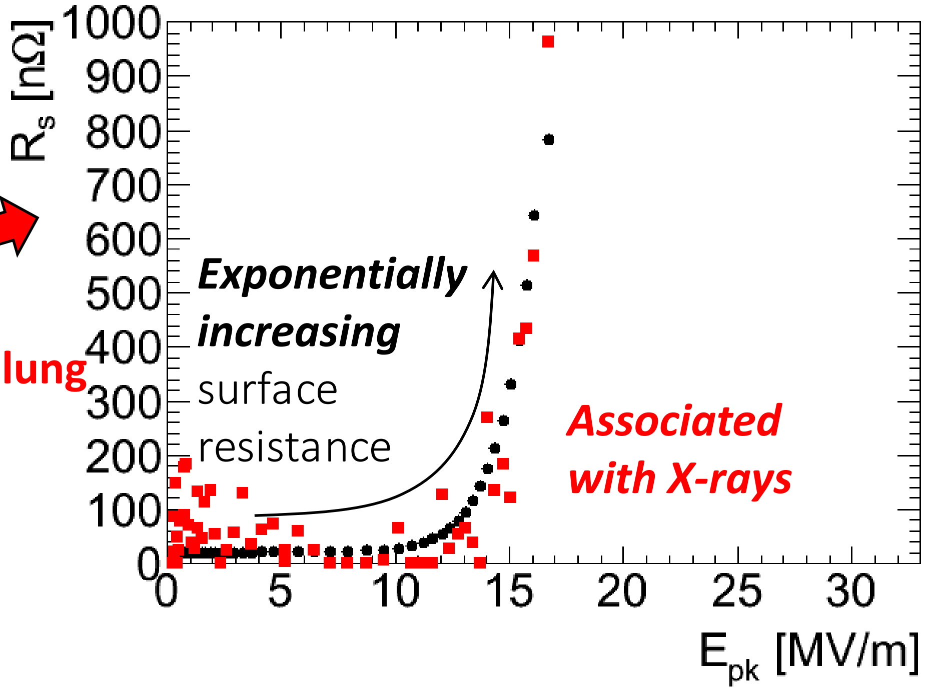
Defect, bad thermal resistance R_{th} can enhance thermal breakdown (or Q-switch)

→ **defect-free and good thermal conductance is a key of SRF cavities**

Field emission: discharge due to electron tunneling



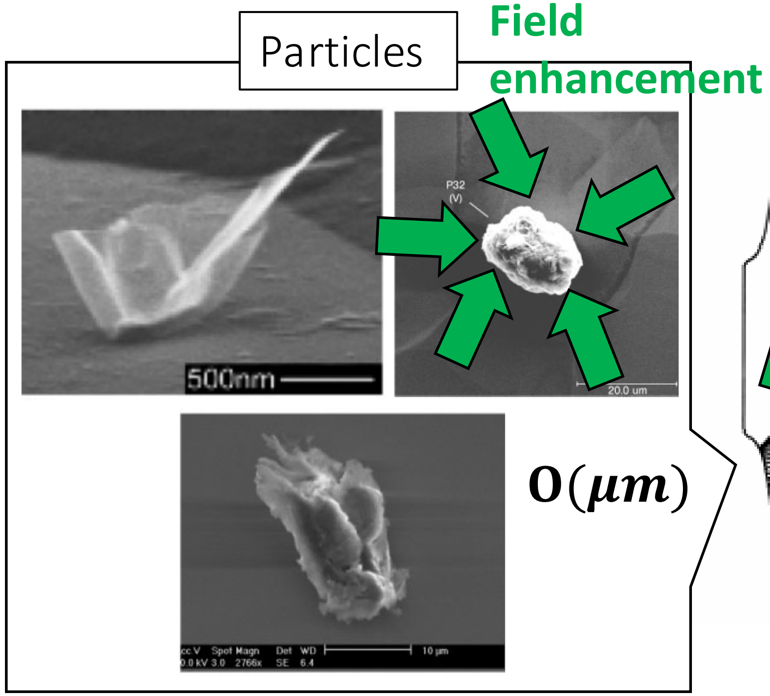


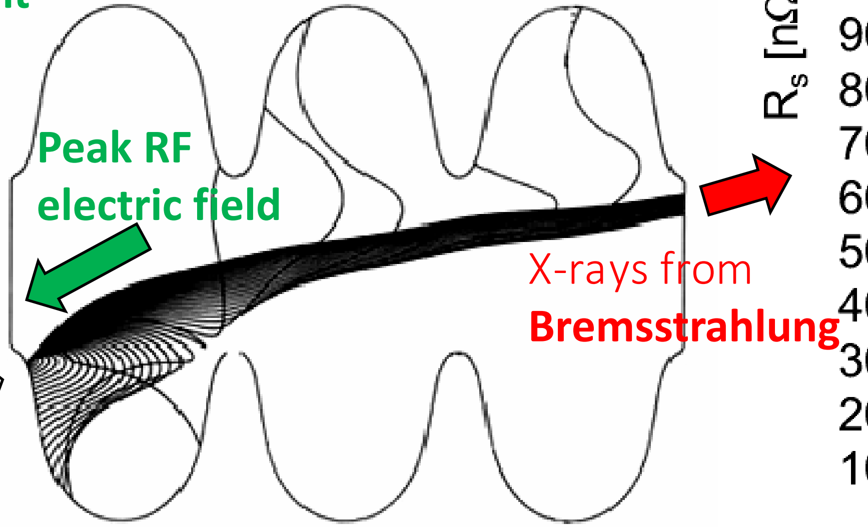


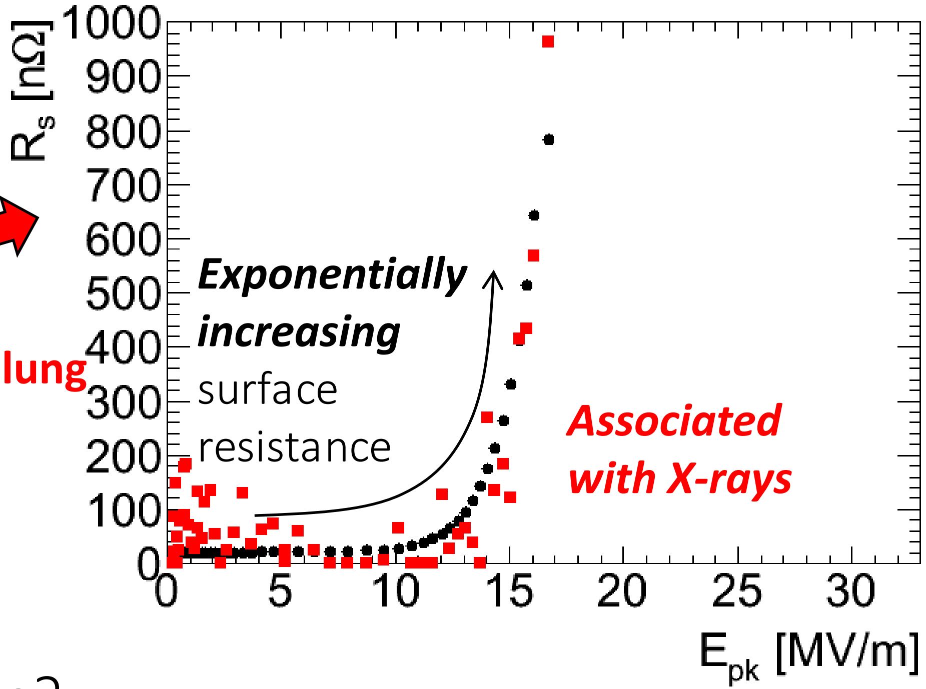
Tunneling current by Fowler-Nordheim

$$J \propto \exp\left(-6.53 \times 10^6 \frac{\phi^{3/2}}{\beta E}\right)$$

Field emission: discharge due to electron tunneling







Tunneling current by Fowler-Nordheim

$$J \propto \exp\left(-6.53 \times 10^6 \frac{\phi^{3/2}}{\beta E}\right)$$

How to mitigate?

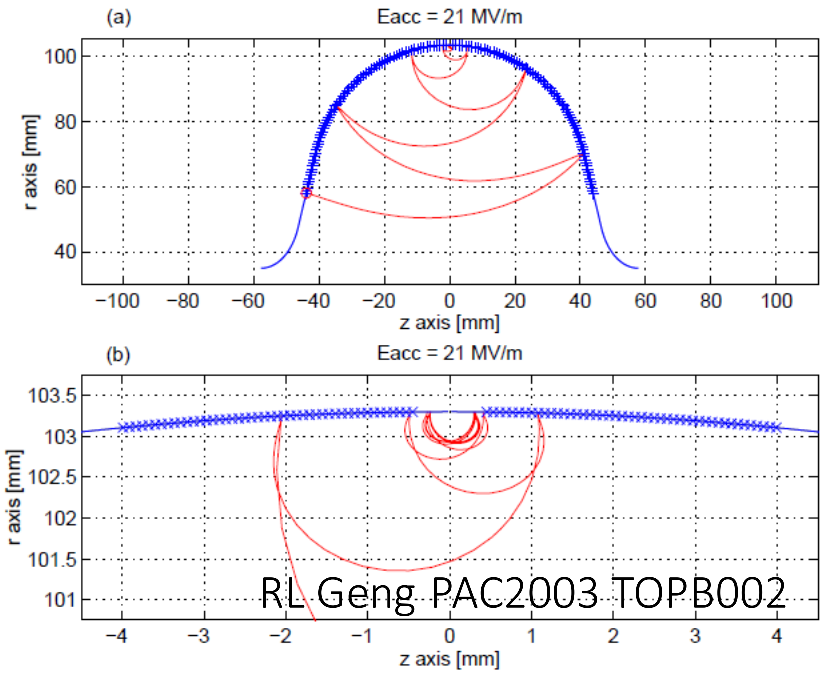
1. Increase surface work function ϕ~4-5 eV (plasma processing)
https://www.osti.gov/servlets/purl/1234337
2. Decrease peak electric field (cavity design)
3. Decrease field enhancement (clean fabrication, He processing)

Practical challenge in SRF projects with large number of cryomodules and cavities

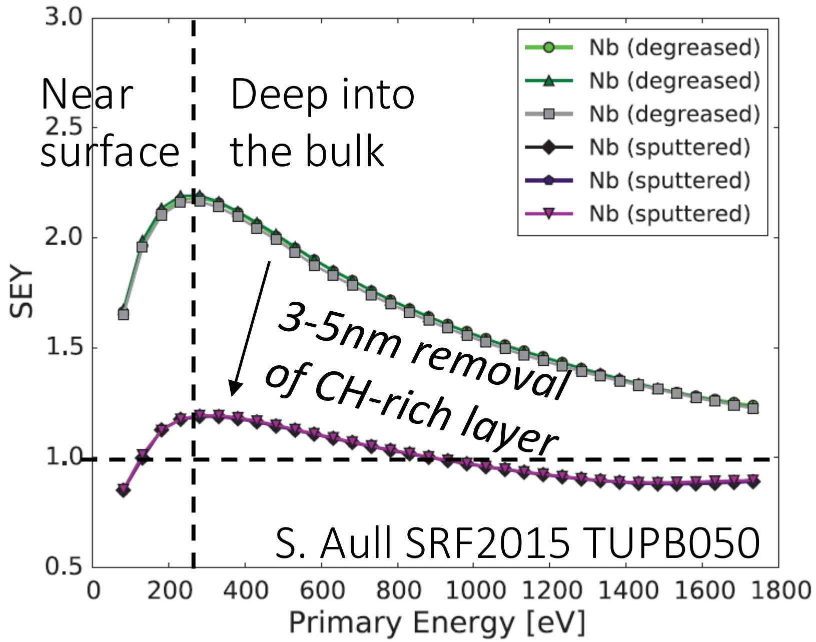
Lectures by Rongli Gen, N. Bazin, W. Hartung, M.

Multipacting: resonant avalanche of secondary electrons

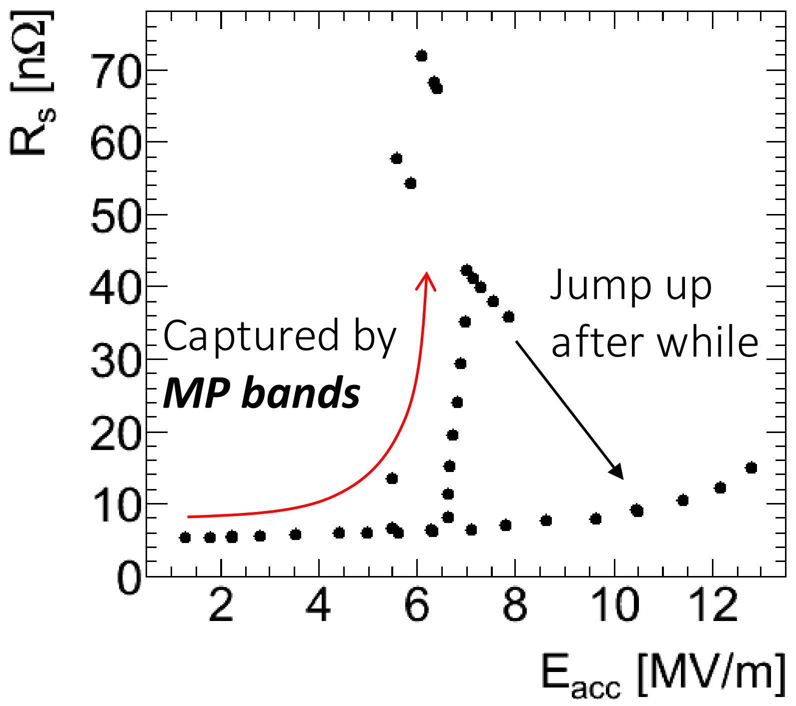
Resonance and geometry



Secondary Electron Yield



Example: ESS double spoke



Multipacting is annoying but ***conditionable*** in properly designed Nb cavities

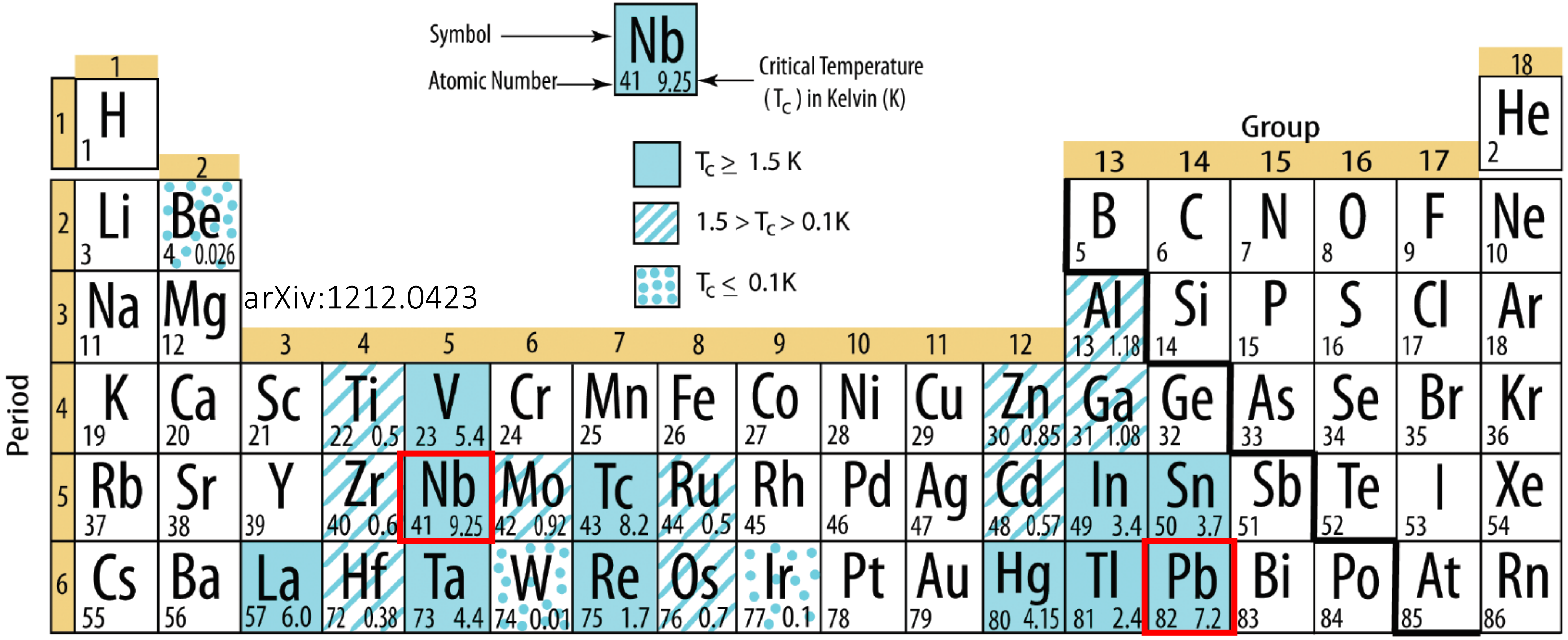
- Sending RF in the MP band
- Jump up to outside the band within a few hours or one day
- Repopulated after thermal cycles

Low-T baking is often performed to get rid of water from the surface

Outline

- Introduction: why superconducting RF?
- Finite surface resistance of superconductors
  - Superconductors in equilibrium
  - BCS resistance
  - Residual resistance
- Field limitations
  - Fundamental limit
  - Practical limits
- Niobium as a cavity material
  - Required feature
  - Beyond niobium

Table of superconductors of pure elements



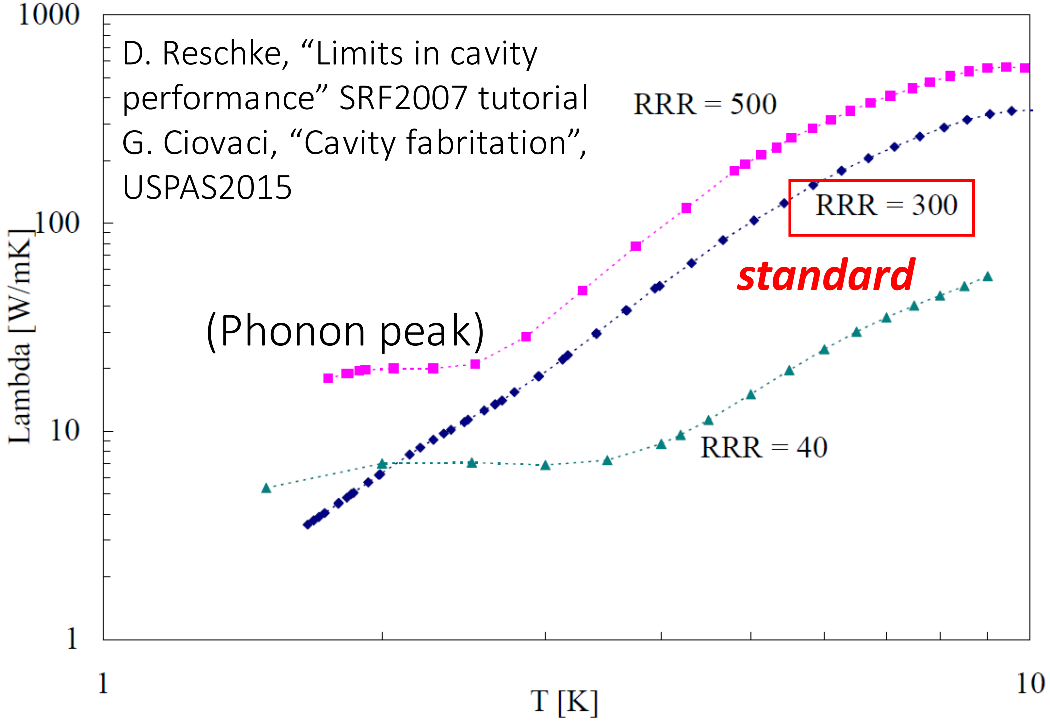
[a] Period 7, and the ***f*** elements in period 6, with the exception of lanthanum, La, are not shown.

Pb is toxic and soft → Nb is the standard for SRF cavities

Nb: $T_c = 9.25$ K, $B_c = 200$ mT

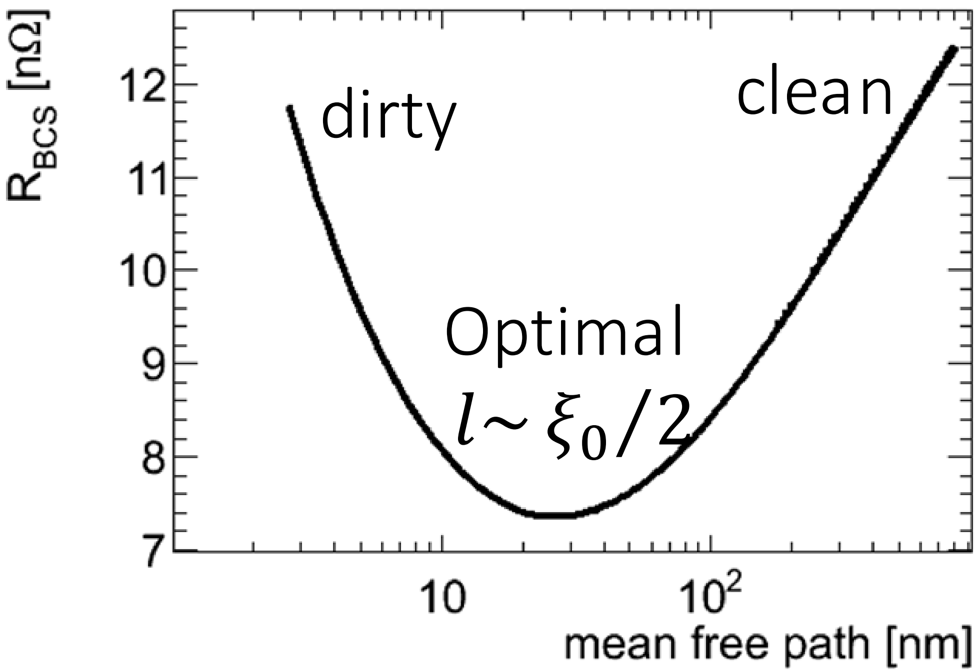
Issue of Nb: thermal conductivity vs surface resistance

$\lambda(4.2\text{K}) \sim 0.25 \times \boldsymbol{RRR}$ W/(m K)



$l \sim 2.7 \times \boldsymbol{RRR}$ nm

B.B. Goodman et G. Kuhn, J. Phys. France **29**, 240-252 (1968)

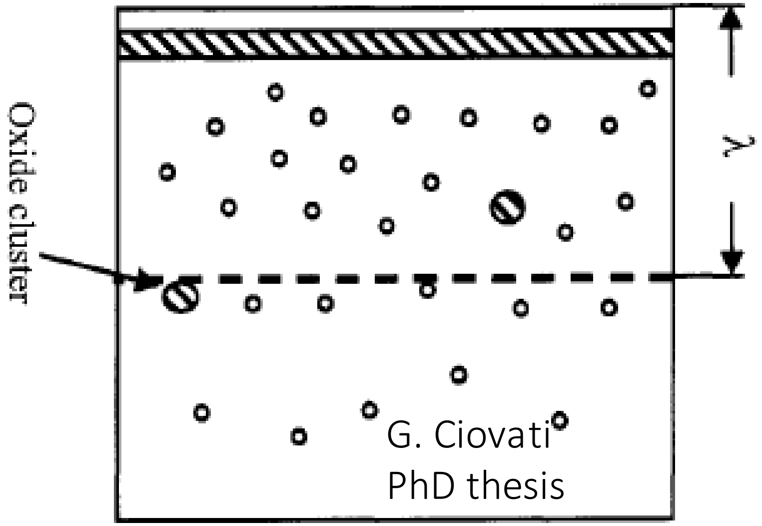


1. **Clean** bulk for thermal conductivity
   - RRR~300: $530/kg
   - RRR~30: $130/kg
2. Sufficiently **dirty** surface for lower BCS resistance
   - R_{res} can be worse

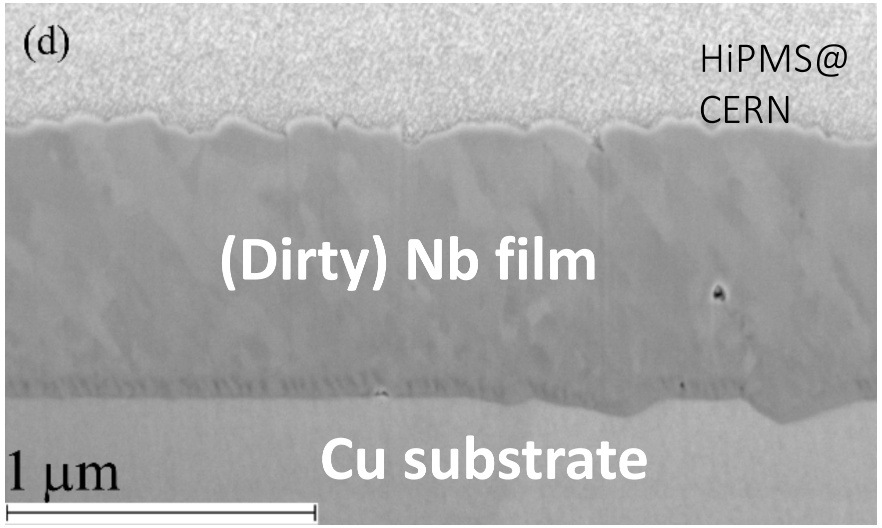
These two requirements contradict with each other

How to achieve **clean** bulk and **dirty** surface

<u>Heat treatment, doping,...</u>



<u>Nb film</u>



Hyper-low R_{BCS}, sensitive R_{mag},
anti-Q-slope, a lot of mysteries

Very low R_{BCS}, insensitive R_{mag},
Q-slope, ... a lot of mysteries

We have been developing **recipes** but why and how are generally missing

One of the research frontiers for new SRF cavities

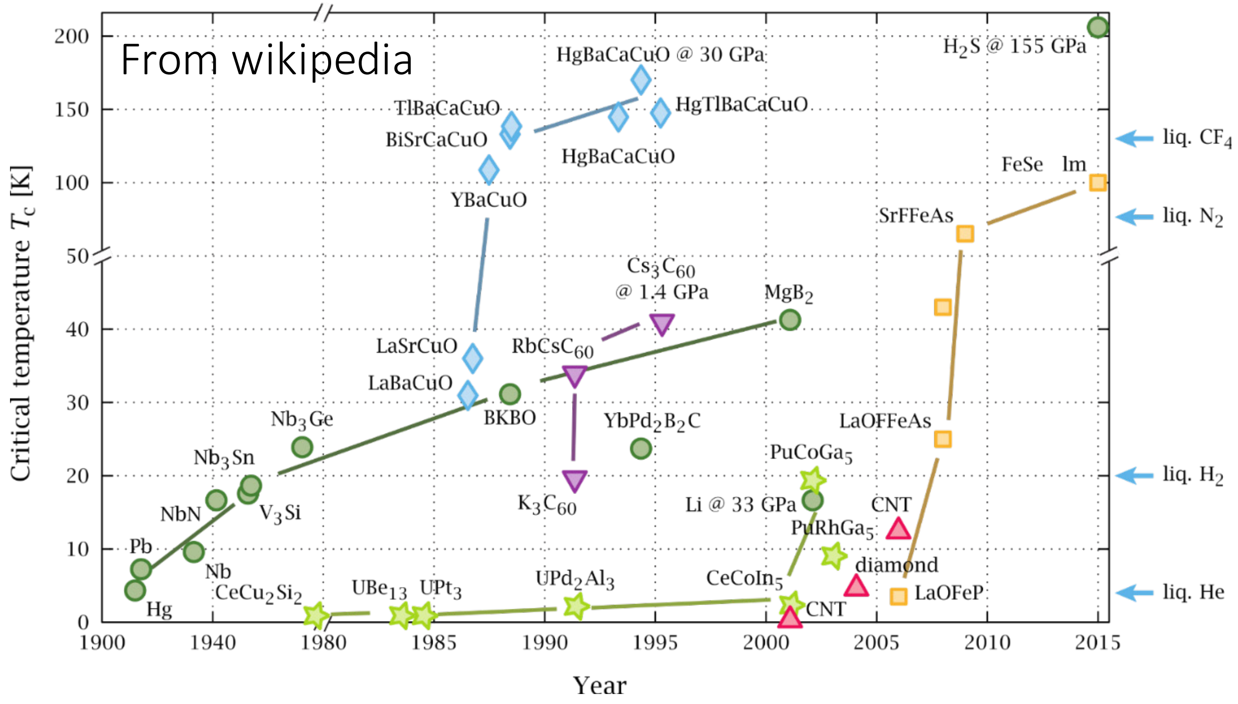
Lecture by
S. Posen and
A.-M. VALENTE-FELICIANO

How about alloys?

Material	$\lambda(T=0)$ [nm]	$\xi(T=0)$ [nm]	$\mu_0 H_{sh}$ [mT]	T_c [K]	$\Delta/k_B T_c$
Nb	50	22	219	9.2	1.8
Nb_3Sn	111	4.2	425	18	2.2
MgB_2	185	4.9	170	37	0.6-2.1
NbN	375	2.9	214	16	2.2

S. Posen PhD thesis

$$R_{BCS}(T) = \frac{A}{T}\exp\left(-\frac{\Delta}{k_B T_c}\frac{T_c}{T}\right)$$



<u>Mechanically brittle</u>

Difficult to fabricate cavity structures

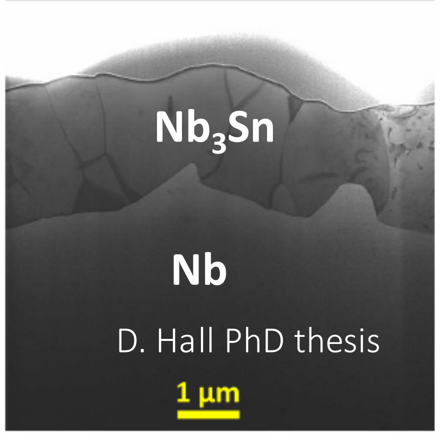
→ coating?

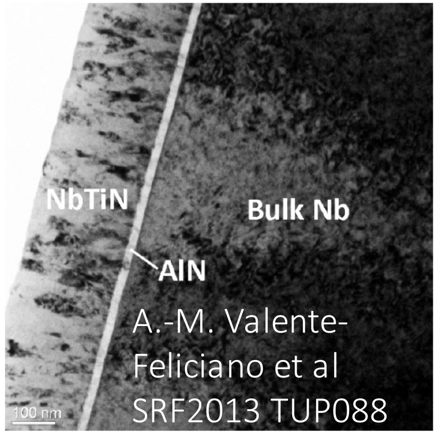
<u>Thermal conductivity</u>

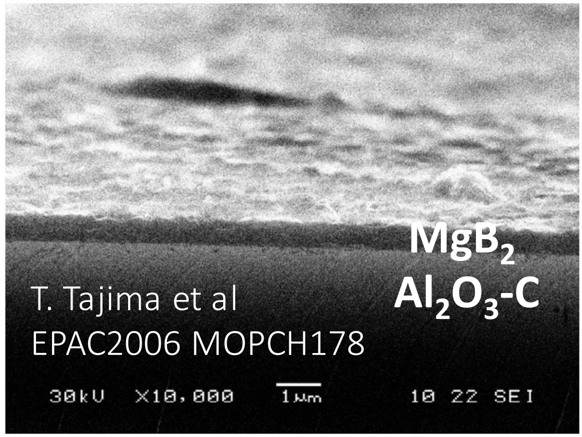
Much worse than Nb → Just a film?

<u>Short ξ_0</u>

Flux penetration through grain boundaries → Protective layer?

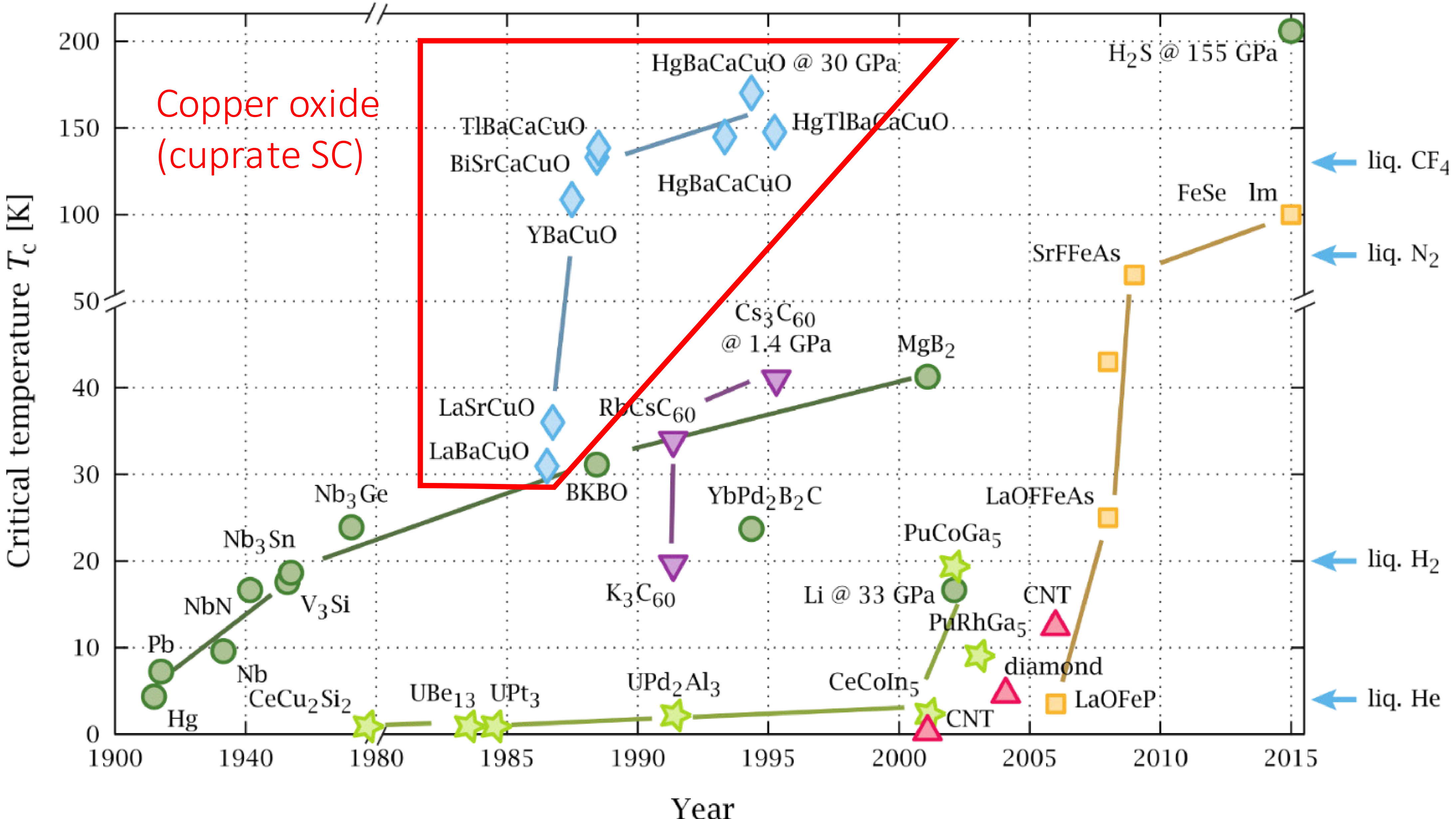






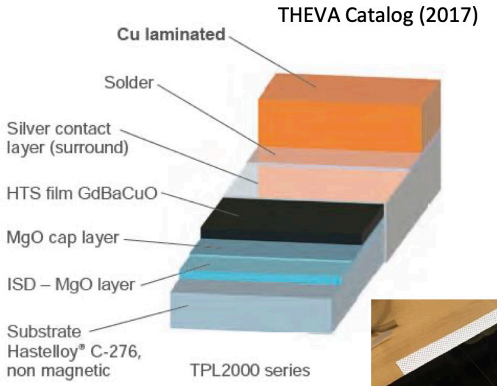
Another research frontier for new SRF cavities

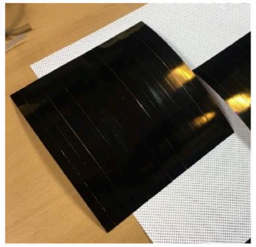
How about High Tc Superconductors (HTS)?

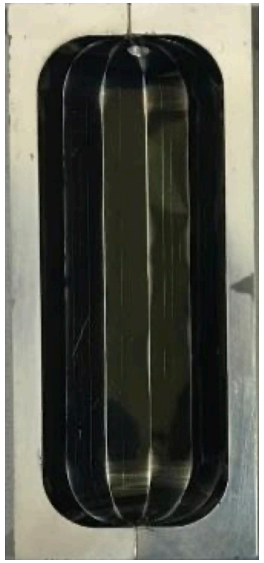


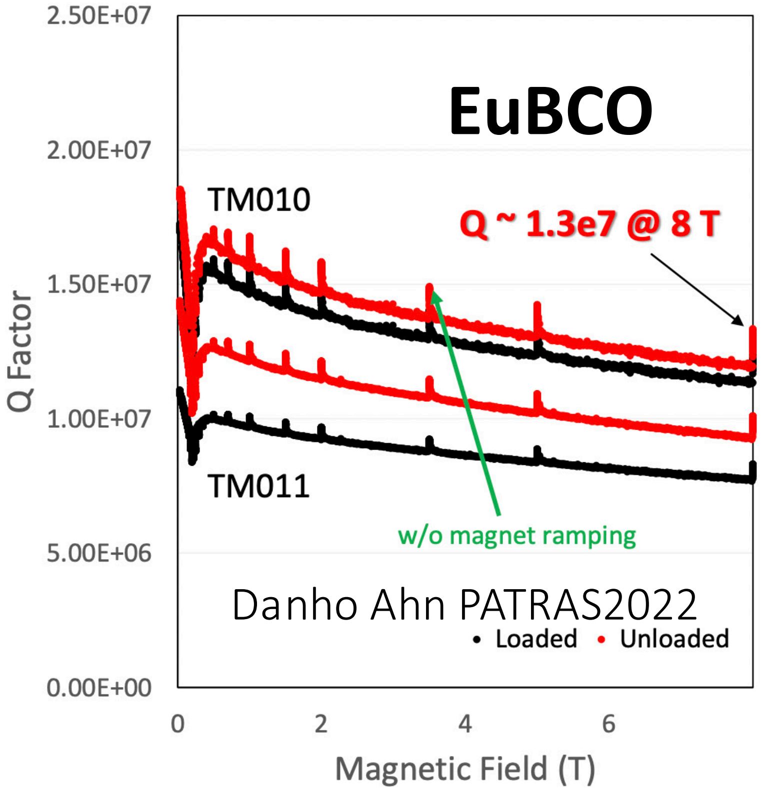
HTS SRF cavities under static magnetic field

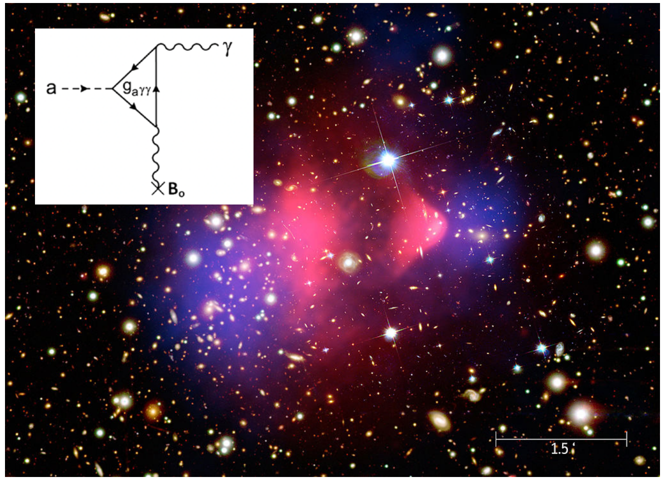
cuprate tapes on copper cavities



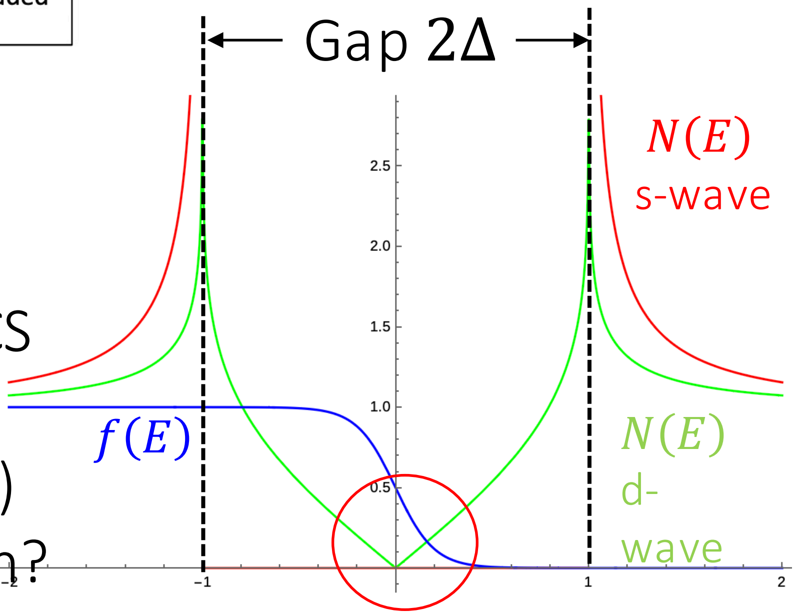








- Excellent Q is obtained under strong **static** magnetic fields
  - Good for dark matter axion search
- The fundamental cause of superconductivity is different from BCS
  - BCS: phonon mediated Cooper pairs
  - HTS cuprates: not yet known but spin-related Cooper pairs(?)
- Gapless (d-wave) → too many quasi-particles for SRF application?



Summary: answer to the first three questions

1. What are the fundamental origins of finite RF loss in SRF cavities?
    1. Thermally activated quasi-particles at finite temperature act like normal conducting electrons and cause a loss in RF
    2. Even at absolute zero temperature, residual resistance exists due to several different mechanisms, such as flux oscillation and subgap state's effect, whose ultimate origins are not wholly understood
2. What are the fundamental and practical limitations of the field inside SRF cavities?
    1. Superheating field, which exceeds thermodynamic critical fields in equilibrium state, would give a fundamental limitation
    2. Practically, the field level can be limited by various phenomena, including thermal quench, field emission, Q-slope, …
3. What is the requirement for material and why niobium?
    1. On top of the material property as a superconductor, niobium is mechanically good to fabricate cavity structure and can have sufficient thermal conductivity
    2. New materials beyond niobium is a frontier research field of SRF community

References 1/2: textbook and reviews

- Standard textbooks on SRF
  - H. Padamsee et al "RF superconductivity for accelerators", 2nd edition, WILEY-VCH (2008)
  - H. Padamsee "RF superconductivity", WILEY-VCH (2009)
- Reviews on SRF
  - J. P. Turneaure et al "The surface impedance of superconductors and normal conductors: the Mattis-Bardeen theory", J. Supercond. 4, 341-355 (1991)
  - A. Gurevich "Theory of RF superconductivity for resonant cavities", Supercond. Sci. Technol. 30 034004 (2017)
- Introduction to solid state physics (before second quantization)
  - N. W. Ashcroft and N. D. Mermin, "Solid State Physics" Thomson Learning (1976)
- Introduction to superconductivity + minimal knowledge on condensed matter physics (but lack of SRF…)
  - S. Fujita and S. Godoy "Quantum statistical theory of superconductivity", Springer, (1996)
- Dictionary of superconductivity
  - M. Tinkham "Introduction to superconductivity", 2nd edition, Dover (2004)
- More advanced textbook on superconductivity
  - N. Kopnin "Theory of Nonequilibrium Superconductivity", Oxford Science Publications (2001)

References 2/2: selected papers related to this lecture

- BCS resistance
  - J. Bardeen, L. N. Cooper, and J. R. Schrieffer,Phys. Rev.108, 1175 (1957). [Matrix elements for static magnetic field were calculated here]
  - D. C. Mattis and J. Bardeen, Phys. Rev.111,412 (1958). [1st order perturbation of RF response and nonlocality, substituting matrix elements modified for RF]
  - J. Halbritter, Z. Physik266, 209 (1974) [Fermi‘s golden rule applied for constant martix element and two fluid ***approximation***]
  - J. Halbritter, KFK-Ext.03/70-06 (1970) [FORTRAN66 code for BCS resistance of f<Δ/2 and arbitrary ξ_0, λ_L, l]
- Residual resistance due to flux oscillation
  - J. Bardeen and M. J. Stephen, Phys. Rev.140, A1197(1965). [Phenomenological model to describe trapped flux as a string]
  - J. I. Gittleman and B. Rosenblum, Phys. Rev. Lett.16,734 (1966). [driven-damped ordinary differential equation for flux oscillation driven by Lorentz force]
  - M. Checchin, M. Martinello, A. Grassellino, A. Roma-nenko, and J. F. Zasadzinski, Supercond. Sci. Technol.30, 3 (2017). [application of Gittleman & Rosenblum for SRF cavities]
  - A. Gurevich and G. Ciovati, Phys. Rev. B87, 054502 (2013). [keeping tension term and solved partial differential equation instead]
- Quench field
  - J. Matricon and D. Saint-James Phys Lett A 24 241 (1967). [solving Ginzburg-Landau equation to estimate superheating field]
  - F. P.-J. Lin and A. Gurevich, Phys. Rev. B 85, 054513 (2012). [solving Eilenberger equations to estimate superheating field in arbitrary impurity]
  - Vudtiwat Ngampruetikorn and J. A. Sauls, Phys. Rev. Research 1, 012015(R) (2019). [including inhomogeneity at the surface]

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Cross-over of particle physics and condensed matter physics

PHYSICAL REVIEW VOLUME 122, NUMBER 1 APRIL 1, 1961

Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I*

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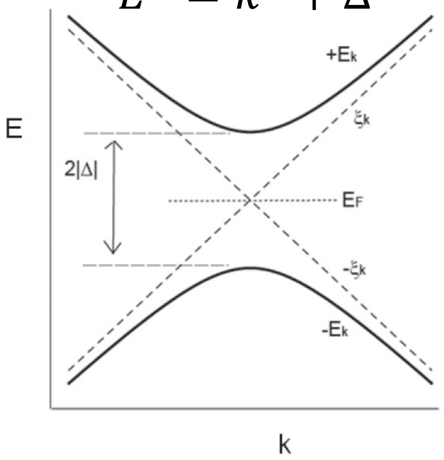
The Enrico Fermi Institute for Nuclear Studies and the Department of Physics, The University of Chicago, Chicago, Illinois

(Received October 27, 1960)

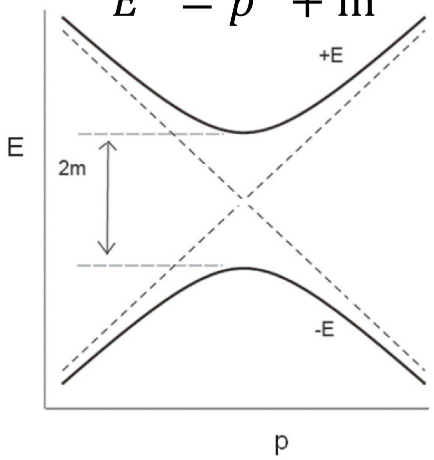
Superconductivity

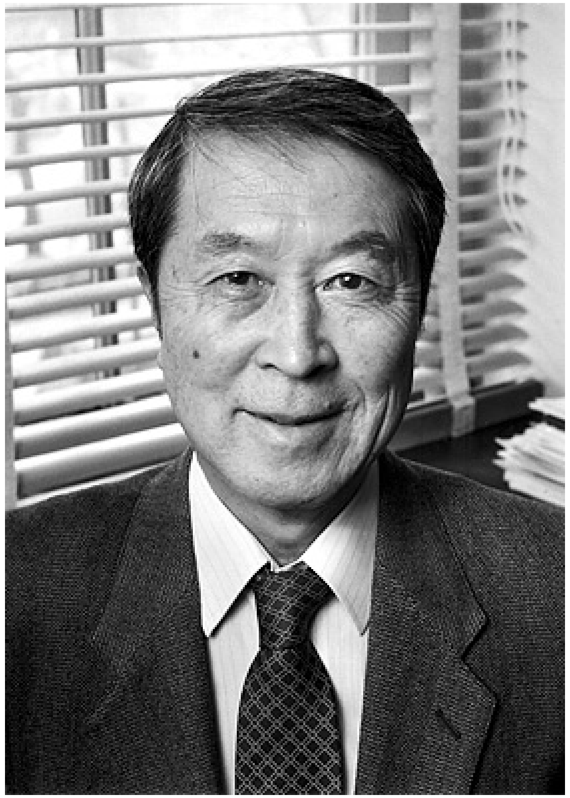
$$E^2 = k^2 + \Delta^2$$



Particle physics

$$E^2 = p^2 + \mathrm{m}^2$$





Yoichiro Nambu

The vacuum is similar to the superconducting state

Particle mass = superconducting gap (gauge symmetry is broken in the ground state)

→ Chiral symmetry breaking, Higgs mechanism, Electroweak theory

Spontaneous gauge symmetry breaking

<u>Ginzburg-Landau theory ($T \to T_c$ of BCS theory, $\Psi = \Delta$)</u>

$$F = (\nabla \times A)^2 + \frac{\hbar^2}{4m_e}|(\nabla + ieA)\Psi|^2 + \frac{g}{4}\left(|\Psi|^2 - v^2\right)^2 \qquad \sim\phi^4 \text{ theory}$$

EM energy　Scaler Kinetic energy　Scaler potential

Excitation around potential minimum v at fixed gauge (Unitary gauge)

$$\Psi(\mathrm{x}) \to v + \phi(x)$$

Kinetic term

$$|(\nabla + ieA)\Psi|^2 = |\nabla\phi|^2 + e^2v^2|A|^2 + \cdots$$

Gauge field gains mass: Nambu-Goldston mode is absorbed by photon

$$e^2v^2|A|^2 \equiv m_v|A|^2$$

$$\left(\nabla^2 - m_v^2\right)A = 0$$

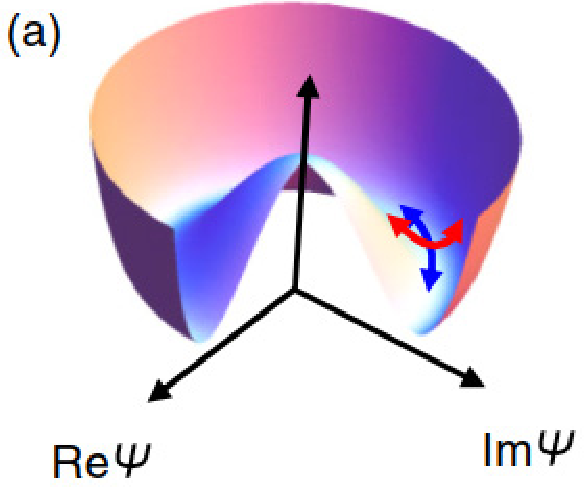
Massive vector boson eq.
↔London eq.

→ Massive photon → finite interaction length: penetration depth

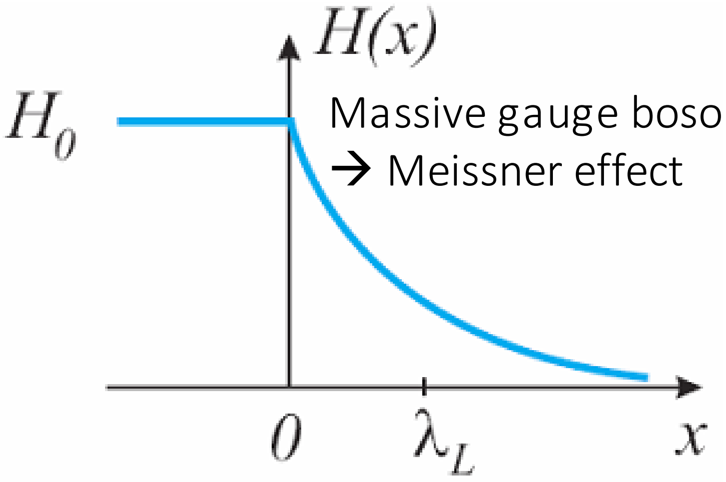
$$\lambda_L = \frac{1}{m_v}$$

Higgs mode ϕ has a mass $m_s = v\sqrt{g}$: coherence length

$$\xi_0 = \frac{1}{m_s}$$



R. Matsunaga et al PRL 111 057002 (2013)



Massive gauge boson
→ Meissner effect

(ξ_{GL}, λ_{GL}) in Ginzburg Landau theory

BCS-Gor'kov → GL around T_c

$$\xi_{GL}(T) = 0.739\left[\xi_0^{-2} + 0.882(\xi_0 l)^{-1}\right]^{-1/2} R^{-1/2}\left(1 - \frac{T}{T_c}\right)^{-1/2}$$

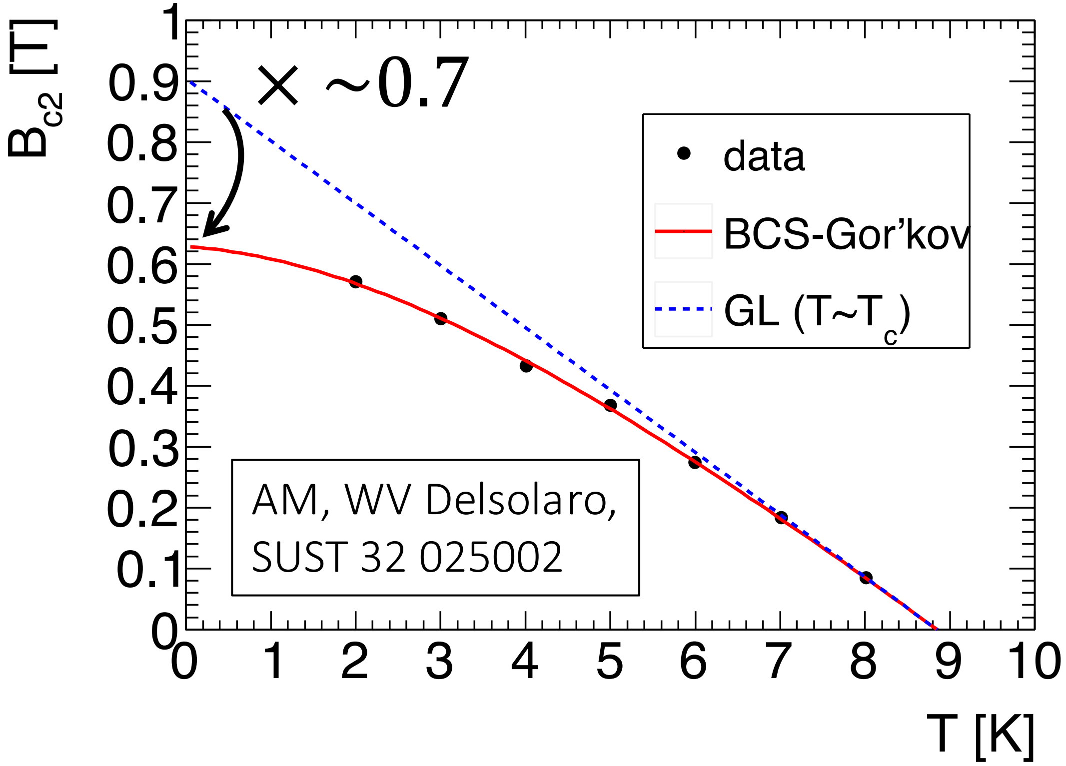
$$\lambda_{GL}(T) = 2^{-1/2}\lambda_L\left[1 + \frac{0.882\xi_0}{l}\right]^{1/2} R^{1/2}\left(1 - \frac{T}{T_c}\right)^{-1/2}$$

$$\kappa_{GL} \equiv \frac{\lambda_{GL}(T)}{\xi_{GL}(T)} = 0.957\frac{\lambda_L}{\xi_0}\left(1 + \frac{0.882\xi_0}{l}\right)R^{-1} \sim \frac{\lambda_L}{\xi_0}$$

$$1 = R(0) < R(l) < R(\infty) = 1.17$$

$$B_{c2}(T) = \frac{\Phi_0}{2\pi\xi_{GL}(T)^2}$$

→ $B_{c2}(T \to T_c) \propto 1 - (T/T_c)$



T. P. Orlando, E. J. McNiff, Jr., S. Foner, and M. R. Beasley, Phys. Rev. B 19, 454 (1979)

Superconductor is *protected* against *parallel* magnetic fields

<u>Solving London equation with the image force term</u>

(To fulfill boundary condition)

$$\nabla^2 H(x,z) - \frac{1}{\lambda^2} H(x,z) = -\frac{\phi_0}{\mu_0 \lambda^2}\left[\delta(x)\delta(z-z_0) - \delta(x)\delta(z+z_0)\right]$$

Results in two terms

1. External field term which attracts the parallel flux

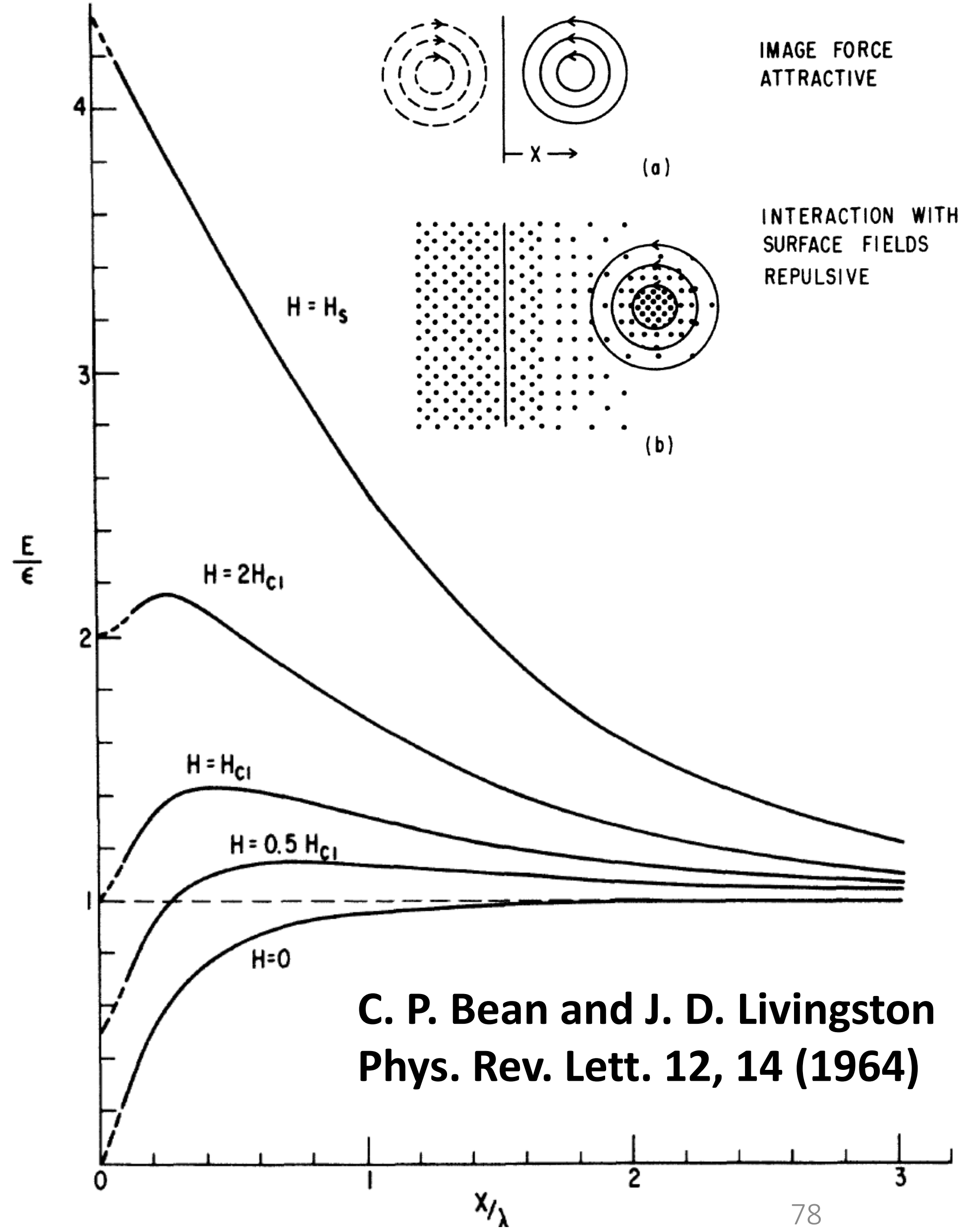
$$f_1 = \frac{\phi_0 H_0}{\lambda} \exp\left(-\frac{z_0}{\lambda}\right)$$

2. Image force term which expels the parallel flux

$$f_2(x) = \frac{\phi_0}{2\pi\mu_0\lambda^3} K_1\left(\frac{2z_0}{\lambda}\right)$$

(one particular solution using 2D Green function)

The 2nd term dominates even at $H > H_{c1}$ but to be defeated by the 1st term Above $H > H_s \sim \frac{\phi_0}{4\pi\xi\lambda} \sim \frac{H_c}{\sqrt{2}}$ the surface barrier disappears but this is still lower than superheating field H_{sh} estimated from GL theory



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