Outstanding Issues in RF Superconductivity
What can theory tell us?

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(presented by Hasan Padamsee)

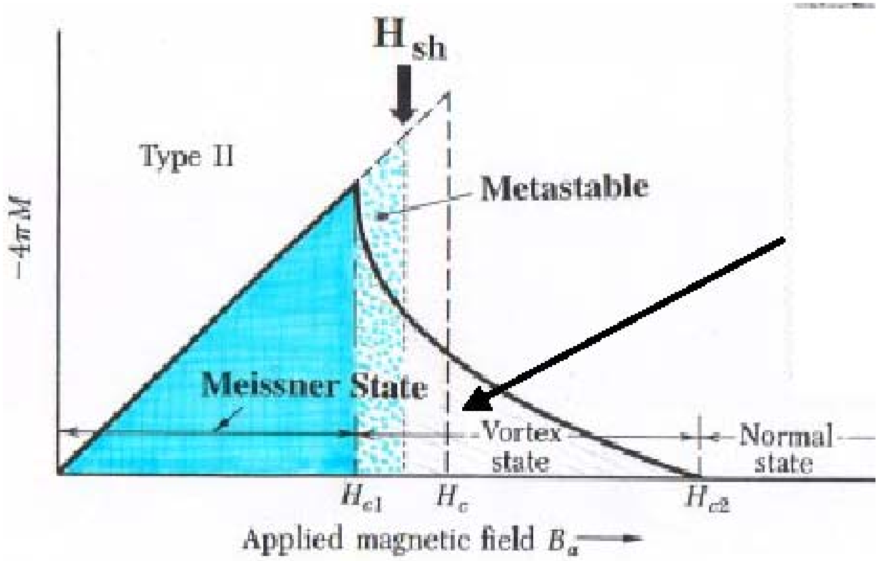
I. Accelerators, RF cavities, and the superheating field
II. Superconductivity review
III. Theories of superconductivity
- Ginzburg-Landau *(& questionable variations)*
- BCS/Gorkov
- Eilenberger

IV. Metastability and Nucleation: Why superheating?
V. Tentative results: Numerics, Eilenberger $\boldsymbol{H_{sh}}(\boldsymbol{T})$ at high $\boldsymbol{\kappa}$
VI. On-going and proposed work

Outstanding Question

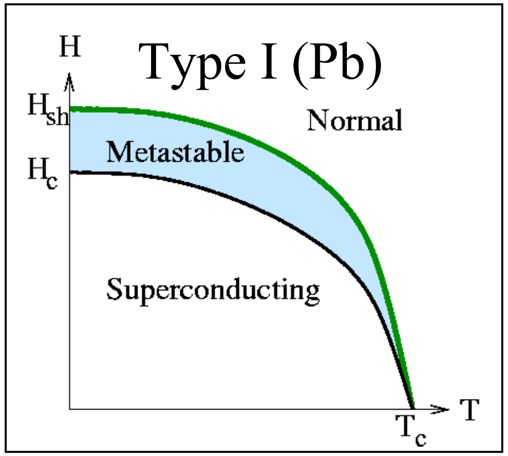
- What is the RF critical magnetic field?
- Is it
  - H_{c1}, H_c, H_{sh}?
  - How does it depend on temperature?
  - How does it depend on Ginzburg-Landau parameter $\kappa = \lambda/\xi$?
  - Nb: $\kappa \sim 1$, Nb3Sn: $\kappa \sim 20$..

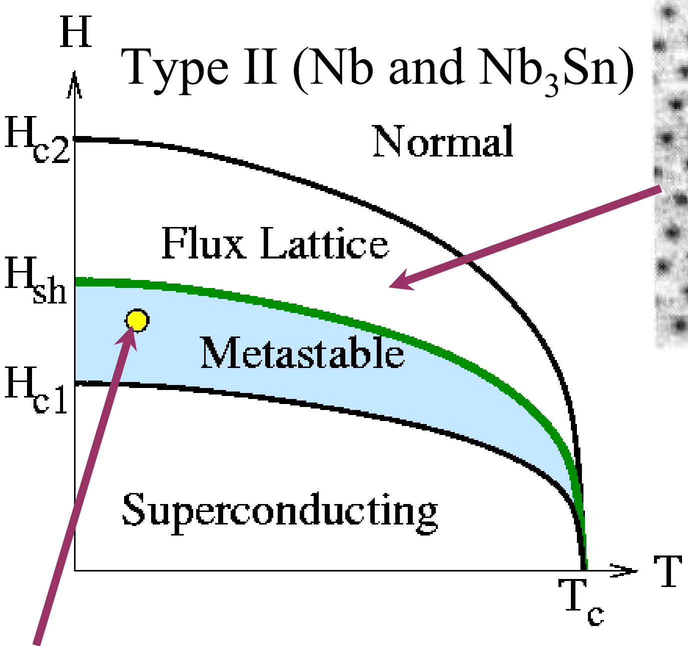
DC Critical Fields

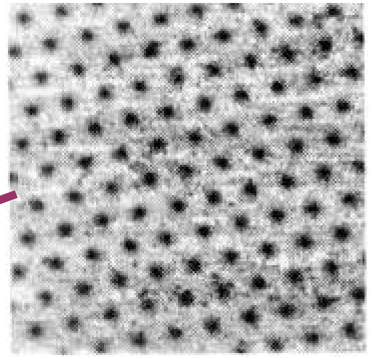


II. Review: Superconductors and fields

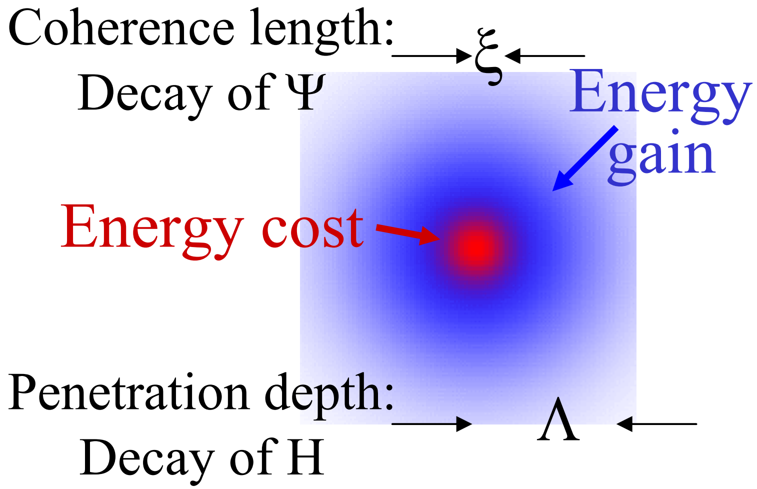
Schematic phase diagrams







RF cavity operating conditions
Vortex nucleation slower than GHz



Type II superconductors

- $\Lambda > \xi$
- Magnetic flux lattice $H > H_{c1}$

Can we calculate the phase diagram for H_{sh}?

H_{sh} in RF Fields

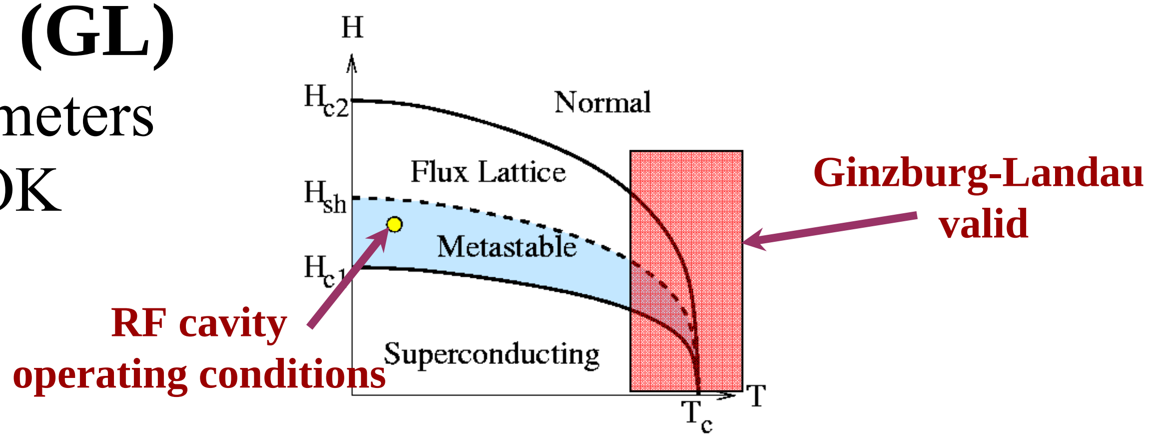
- In RF, fields change rapidly, nanoseconds.
- If the time it takes to nucleate fluxoids is long compared to the rf period (10^{-9} s)
- There is a tendency for the meta-stable superconducting state to persist up to
- $H_{sh} > H_{c1}$

III. Theories of superconductivity

Validity versus complexity

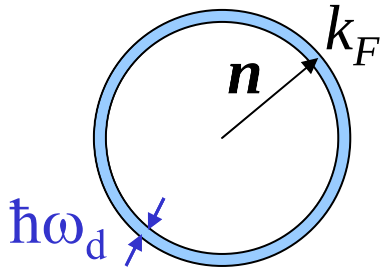
Ginzburg-Landau (GL)

- ψ(r), H(r) order parameters
- Spatial dependence OK
- ***Valid only near T_c***



BCS theory

- Mean-field, pairing k, -k within $\hbar\omega_d$
- Excellent for traditional superconductors
  - Strong-coupling Eliashberg
  - May not apply to high T_c (thermal fluctuations, no microscopics)
- $H_{c1}(T)$, $H_{c2}(T)$ done, $H_{sh}(T)$ hard
  - Surface makes inhomogeneous
  - Needs Gorkov theory, Green's functions $F(t_1,t_2,\mathbf{r_1}, \mathbf{r_2})$, $G(t_1,t_2,\mathbf{r_1}, \mathbf{r_2})$

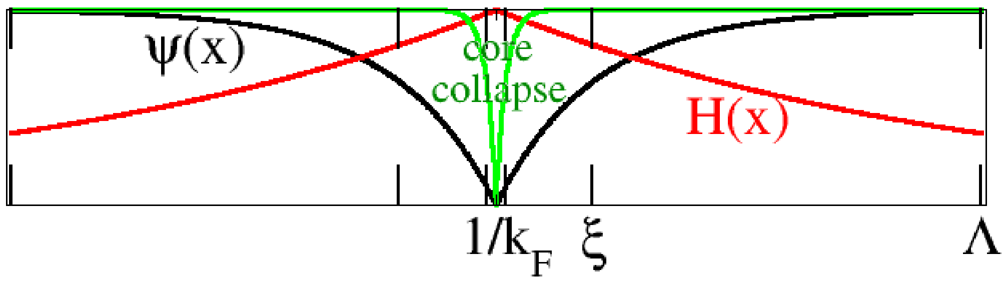


Theories of superconductivity

Validity versus complexity

Eilenberger Equations

- *Valid at all temperatures*
- Assumes Δ(r), H(r) vary slowly over atomic scale $1/k_F$
- Solve for regular and anomalous Green's function f, g
- Many coupled equations (f, g depend on Matsubara frequencies ω and spherical harmonics of Fermi surface **n**)
- Otherwise analogous to linear stability in Ginzburg-Landau
- Vortex core collapse??



$$\left[\omega + \boldsymbol{n}\cdot\left(\boldsymbol{\nabla} - i\frac{2\pi\xi_0}{\phi_0}\boldsymbol{A}(\boldsymbol{r})\right)\right] f(\omega,\boldsymbol{n},\boldsymbol{r}) = \Delta(\boldsymbol{r})g(\omega,\boldsymbol{n},\boldsymbol{r})$$

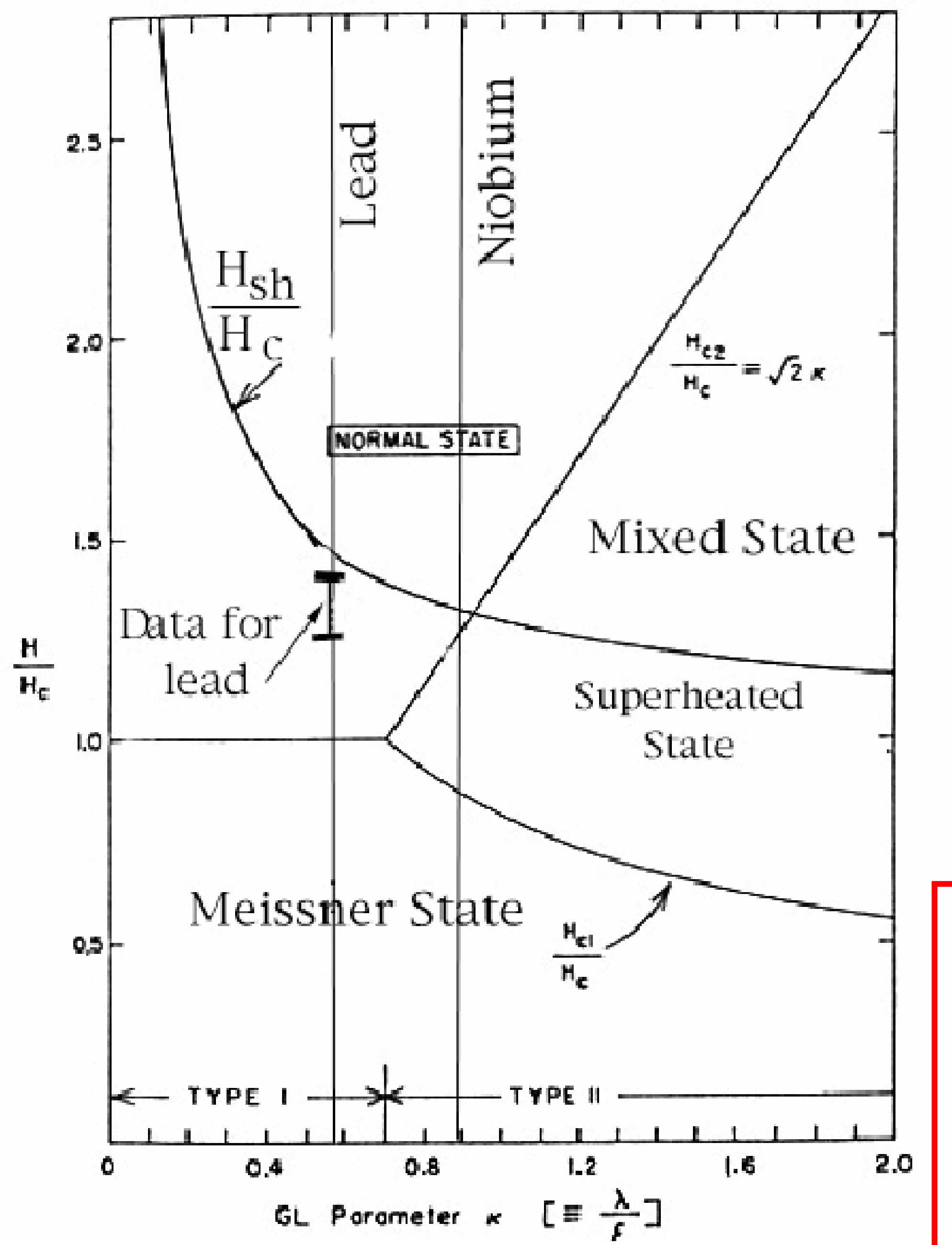
$$\left[\omega - \boldsymbol{n}\cdot\left(\boldsymbol{\nabla} + i\frac{2\pi\xi_0}{\phi_0}\boldsymbol{A}(\boldsymbol{r})\right)\right] \bar{f}(\omega,\boldsymbol{n},\boldsymbol{r}) = \Delta^\dagger(\boldsymbol{r})g(\omega,\boldsymbol{n},\boldsymbol{r})$$

$$\Delta(\boldsymbol{r})\log\left(\frac{T}{T_c}\right) + 2\pi T\sum\left[\frac{\Delta(\boldsymbol{r})}{\omega} - \int\frac{d\boldsymbol{n}}{4\pi}f(\omega,\boldsymbol{n},\boldsymbol{r})\right] = 0$$

$$\frac{2\pi\xi_0}{\phi_0}\boldsymbol{\nabla}\times\boldsymbol{H} + i\left(\frac{\xi_0}{\lambda_0}\right)^2 2\pi T\sum\int\frac{d\boldsymbol{n}}{4\pi}3\boldsymbol{n}g(\omega,\boldsymbol{n},\boldsymbol{r}) = 0$$

$$g^2(\omega) + f(\omega)\bar{f}(\omega) = 1$$

GL Superheating Field



H_{sh} is defined as the maximum permissible value of the applied field, which satisfies Ginzburg-Landau (GL) equations.

Matricon and Saint-James solved GL equations numerically for the one-dimensional case where half of the space is occupied by a superconductor.

$$H_{sh} \approx \frac{0.89}{\sqrt{\kappa_{GL}}} H_c \qquad \text{for } \kappa_{GL} \ll 1$$

$$H_{sh} \approx 1.2 H_c \qquad \text{for } \kappa_{GL} \approx 1$$

$$H_{sh} \approx 0.75 H_c \qquad \text{for } \kappa_{GL} \gg 1.$$

T. Yogi measured $H_{sh} > H_{c1}$ for alloys Sn-In and In-Bi over a range of κ values

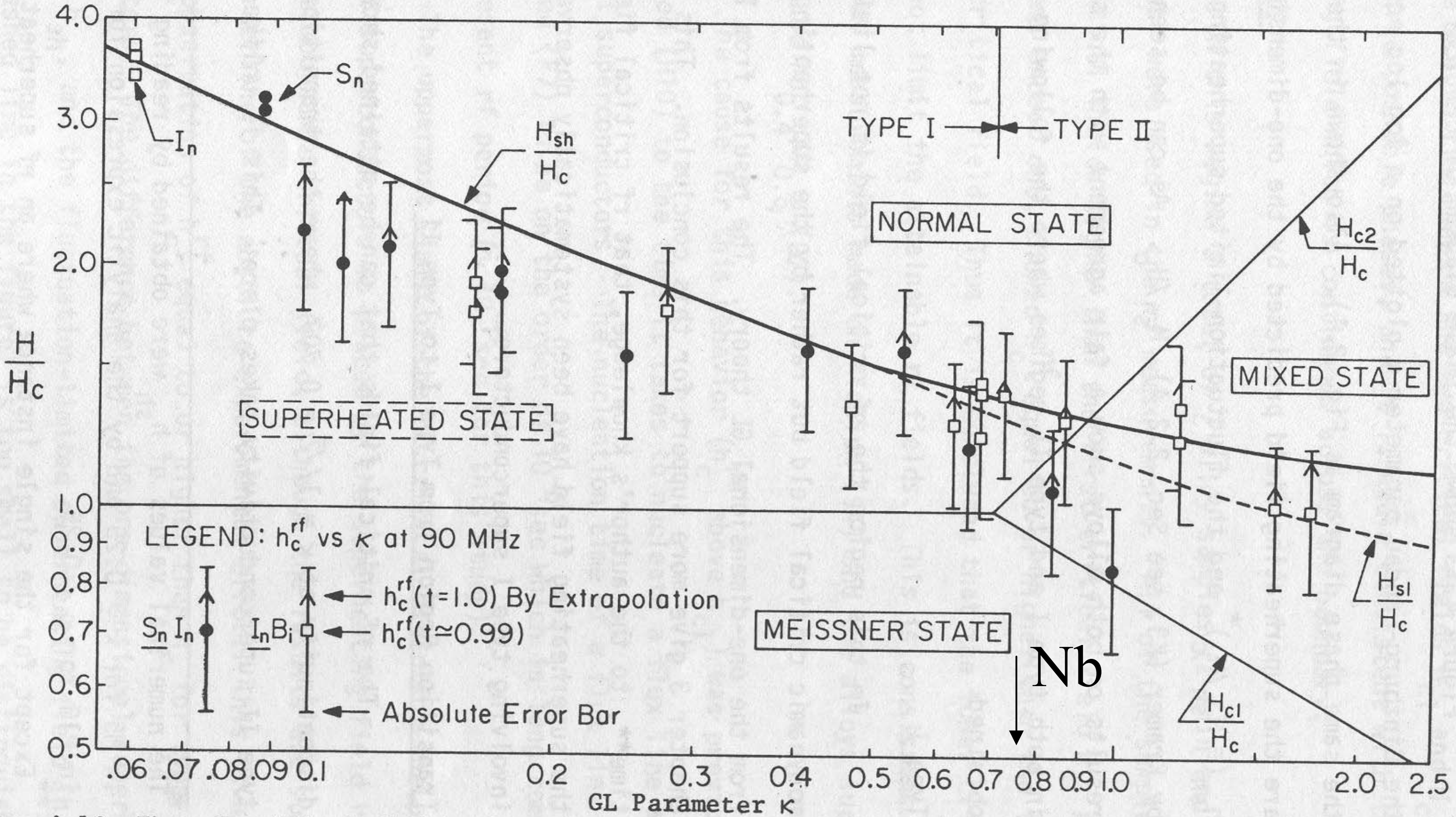


Fig. 4.13 The rf critical fields that were actually measured (near $t = 0.99$) for all SnIn and InBi alloy samples are shown as a function of the GL parameter κ. Also, the values extrapolated to T_c, when possible, are shown by arrows originating from the data points.

Variations: Energy Balance Arguments To Estimate H_{sh} For a Planar Boundary Between N and S Phase (Started by Yogi)

In the process of phase transition, a boundary between N and SC must be nucleated.

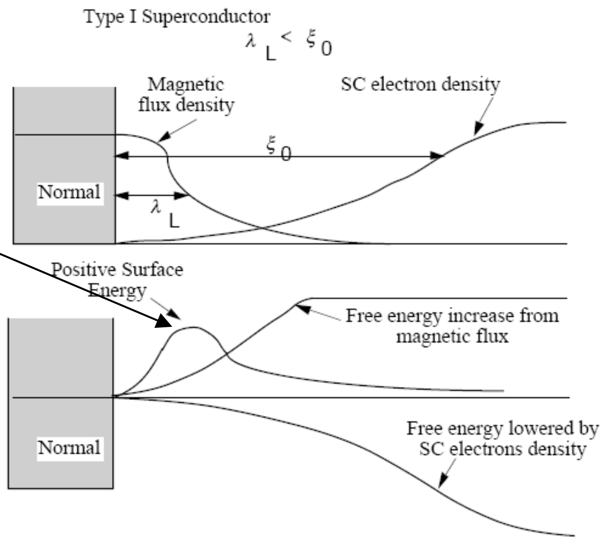
At a planar boundary, the free energy per unit volume <u>increases</u> by $\mu_0 Ha^2 \lambda/2$ over the penetration depth (λ_L) due to the diamagnetism; work is done to exclude the magnetic flux

and <u>falls</u> by $\mu_0 Hc^2 \xi_0 /2$ over the coherence length due to the increase of the super-electron density.

In a Type I superconductor, the positive surface energy suggests that, dc fields, the Meissner state can persist metastably beyond the thermodynamic critical field, up to the superheating field, H_{sh}.

At this field, the surface energy per unit area vanishes:

$$\frac{\mu_0}{2}\left(H_c^2\xi - H_{sh}^2\lambda\right) = 0, \quad H_{sh} = \frac{1}{\sqrt{\kappa_{GL}}}H_c.$$



Which looks temptingly close to the GL result for Type I

Yogi and Saito extended the energy balance argument to other dimensional forms of nucleation such as a line nucleation (~ vortex nucleation).

The diamagnetic energy is given by

$$\mu_0 \pi H^2 \lambda_{GL}^2 / 4$$

and the condensation energy is

$$-\mu_0 \pi H^2 \xi_{GL}^2 / 4$$

Balancing the two contributions, the superheating field is

$$H_{sh} = \frac{\xi_{GL}}{\lambda_{GL}} H_c = \frac{1}{\kappa_{GL}} H_c .$$

Issues with this Energy Balance Approach

- Nothing in the energy balance argument discusses meta-stability, which is the key aspect for H_{sh}
- As an energy-balance argument, the vortex nucleation model gives an upper bound on the *equilibrium* critical field for vortex penetration, which is related to H_{c1}.
- The line nucleation model is useful in the context of nucleation on in-homogeneities on the scale of the coherence length,
  - but not as a fundamental limit for uniform, flat, pure superconductors.

Problems

- Saito also introduces Hsh-rf = √2 Hsh-dc
  - Do we need √2 for a phase transition field ?

$$H_{c,rf}(t) = \sqrt{2}\frac{1}{\kappa_{GL}(0)}H_c(0)\left(1-t^4\right),$$

- For example,
  - if Hrf (T = 0 K) = 1800 Oe = Hsh (T = 0),
  - then, Hsh (dc) at zero temperature = 1270 Oe
  - which is << Hc_1 ~1800 Oe from magnetization curves !!

Review: Theoretical predictions of superheating field for Ideal surface

- Current theories for H_{sh} used in the accelerator community are
- GL (Planar Nucleation)
- Yogi & Saito (Line Nucleation)

H_{sh} Oe	Ginzburg Landau	E_{acc} GL MV/m	Line nuc.*
Nb	2300	64	1370
Nb_3Sn	3900	108	260
MgB_2	6200	172	1650

*Without the $\sqrt{2}$ factor

Can we theoretically calculate the maximum possible H_{sh} for perfect samples of practical materials (Nb, Nb_3Sn, MgB2) at realistic operating temperatures (2K)?

*

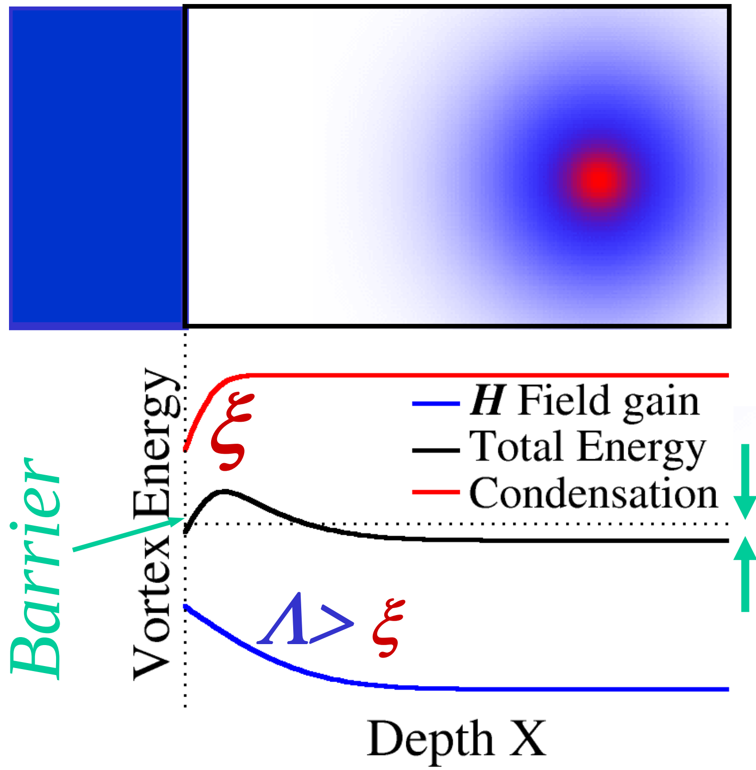
*How to correctly calculate **Hsh**?*

- Field where barrier vanishes
- Linear stability analysis also *determines* the correct vortex array
- At large $\boldsymbol{\kappa}$ and *T~Tc*, 1-D analysis gives ***Hsh*** = 0.745 ***Hc*** (as discussed)
- At lower ***T***, we need the ***Eilenberger equations***
  - (Non-local, Green's functions, …)

Metastability threshold and H_{sh}

Why is there a barrier to vortex penetration?

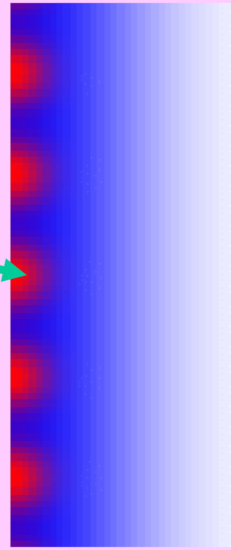
Why a superheating field?



Costly core ξ enters first; gain from field Λ later

How to calculate H_{sh}*?*

- Field where barrier vanishes
- Linear stability analysis *determines* nucleation mechanism: vortex array
- GL Nb, H_{sh}*~2400Oe, E~63MV/m*
- But GL@large κ, $H_{sh} = 0.745\ H_c$,



"Line nucleation"

- Yogi, Saito $H_{sh} \sim H_c/\kappa$ discouraging
- Via "energy balance", no barrier calculation
- Does not work for large κ,

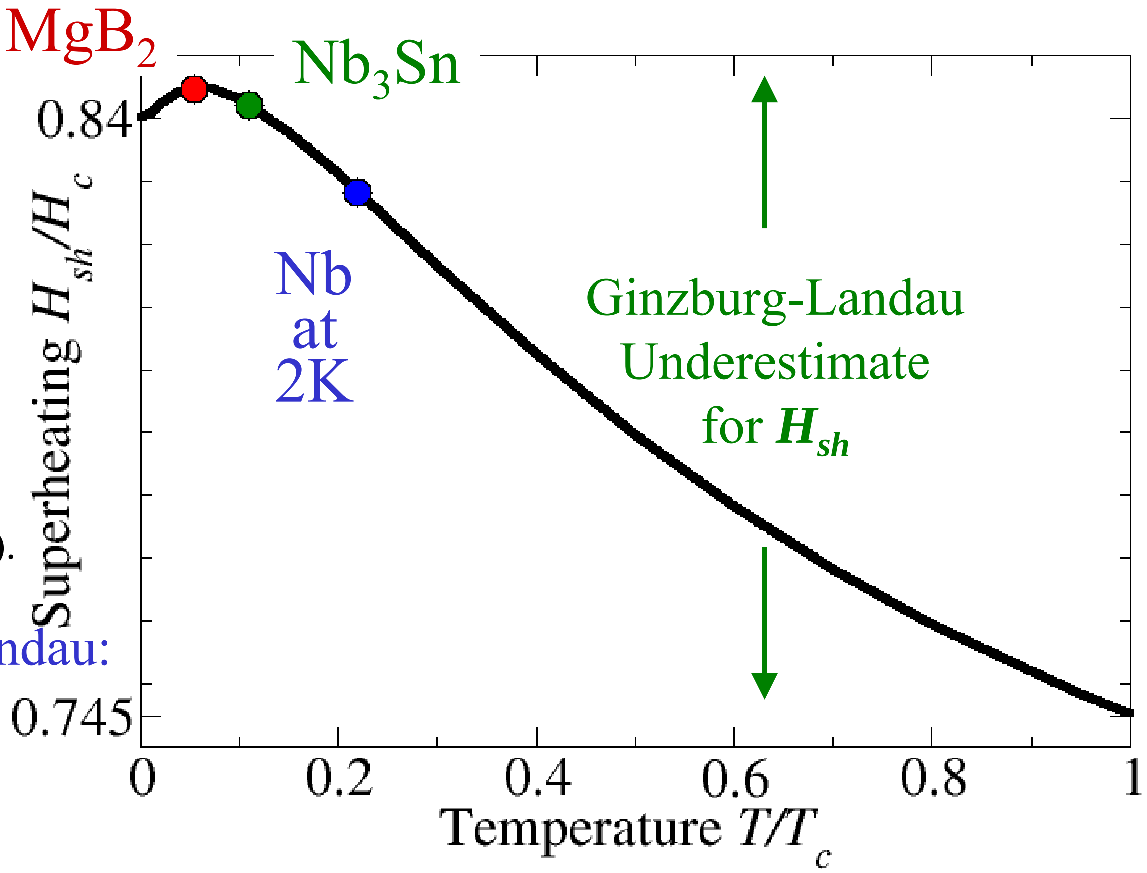
$$H_{sh} < H_{c1} = H_c \ln(\kappa)/(\sqrt{2}\,\kappa)$$

- Correct balance theory gives H_{c1} not H_{sh}

Preliminary Eilenberger Results !

Superheating field $H_{sh}(T)$ from the Eilenberger Equations
And large κ (so not applicable for Nb)

13% larger H_{sh} at low T than Ginzburg-Landau estimate !



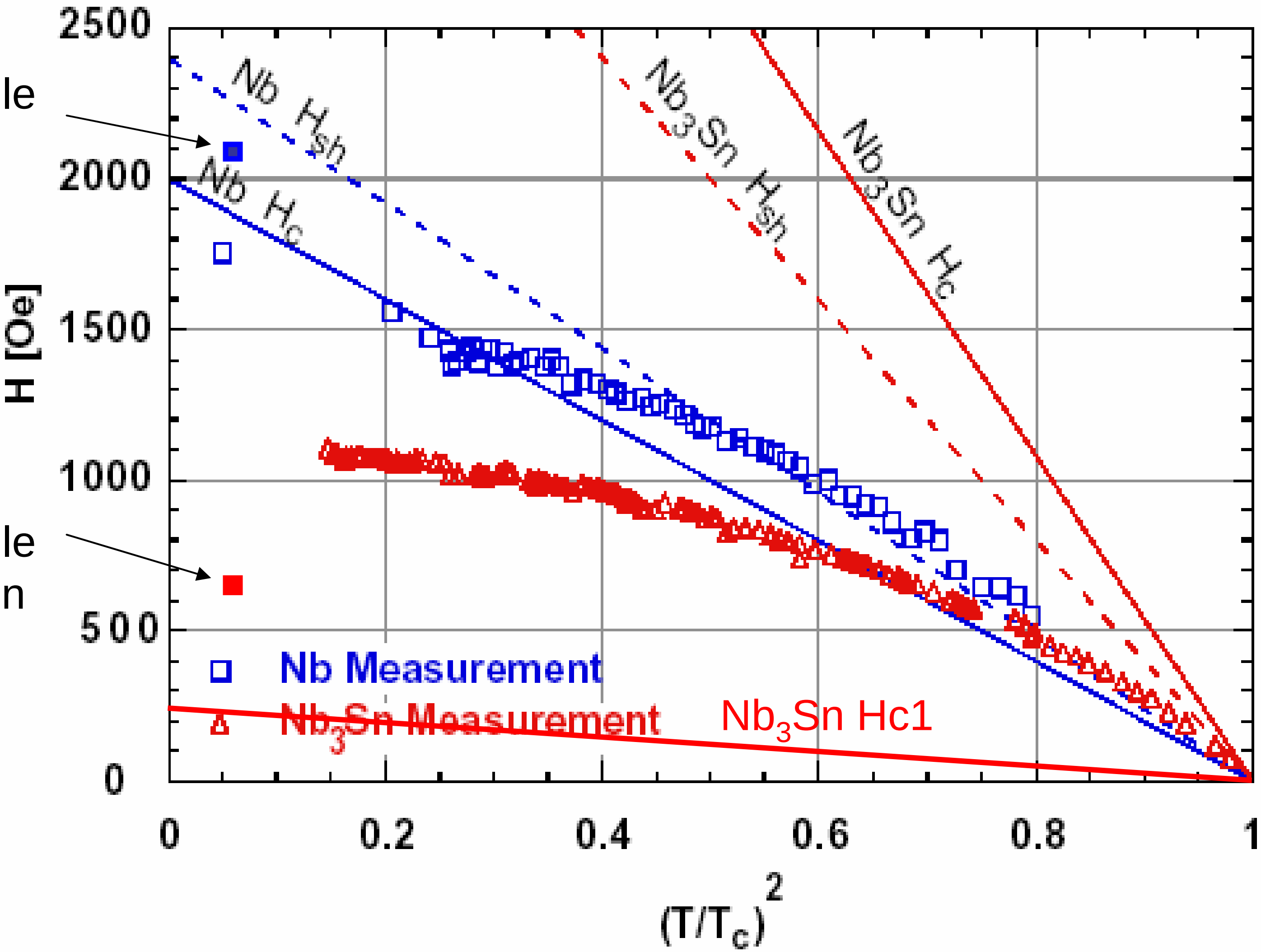
- T=0, Eilenberger: $H_{sh}/H_c = 0.84$
- In agreement with
  - V. P. Galaiko, JETP 23, 475 (1966).
- *In agreement with* Ginzburg-Landau: ***near Tc*** $H_{sh}/H_c = 0.745$
- T-dependence: Catelani 2007

Eilenberger predicts E_{acc} ~ 120 MV/m for perfect Nb
and 200 MV/m for perfect MgB2 !!

Experimental Status (1996) At Cornell T. Hays Measured the RF Critical Field for : Nb_3Sn Using High Pulse Power (Calibrated results with Nb)

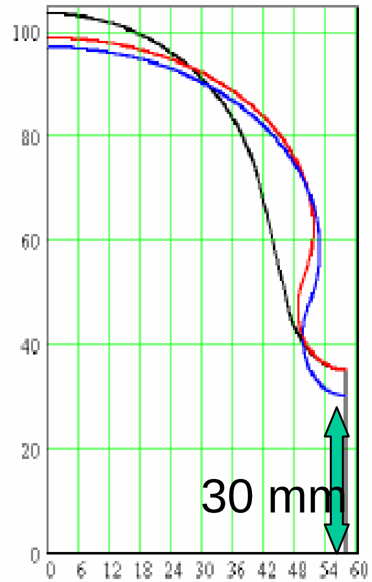
Best single cell Nb cavity result

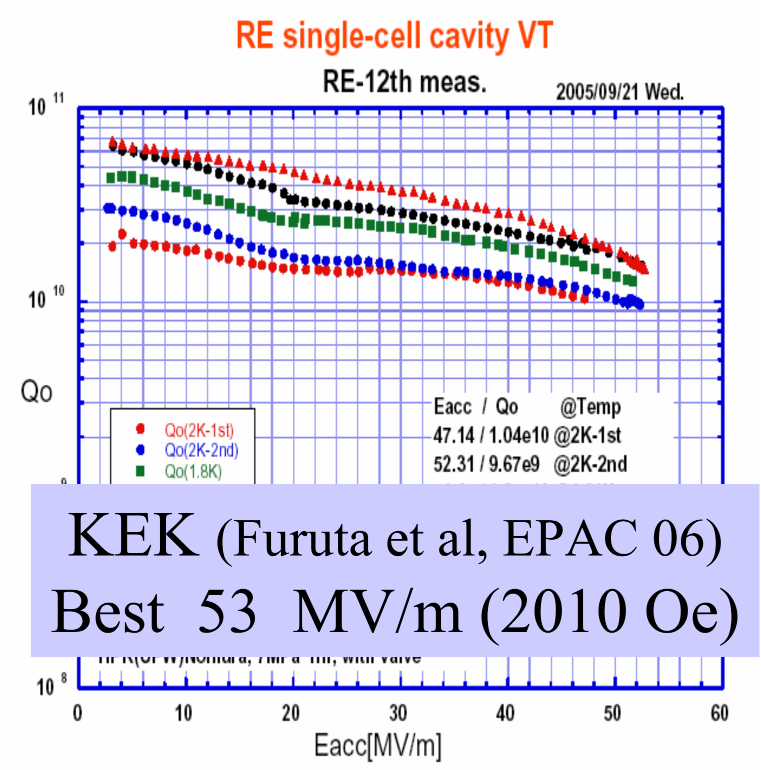
Best single cell Nb_3Sn cavity result

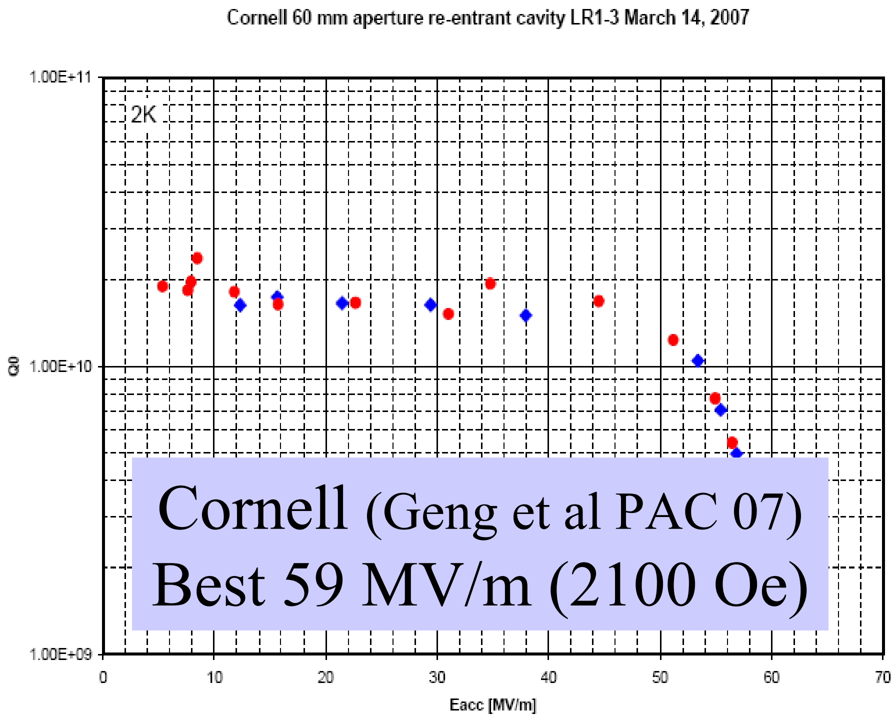


Cornell Collaboration with KEK

- Two Re-entrant Shape Single Cell Cavities
  - H_{pk} = 38, 36 Oe/MV/m
- Cavities built at Cornell, treated and tested at KEK
- # 1 Best 53 MV/m (2010 Oe) at KEK,
- #2 Best 59 MV/m (2100) Oe at Cornell





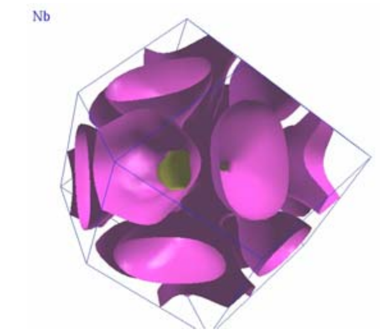


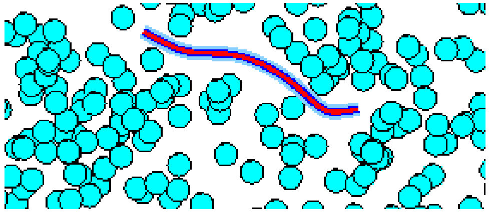
Proposed continuation

Interesting theoretical issues of importance for H_{sh}

Incorporate Fermi surface anisotropy
  (important for Nb): single crystal best surface?

Nucleation theory on atomic-scale disorder
  (rare disorder fluctuations dominate: instanton methods). Small ξ more sensitive?

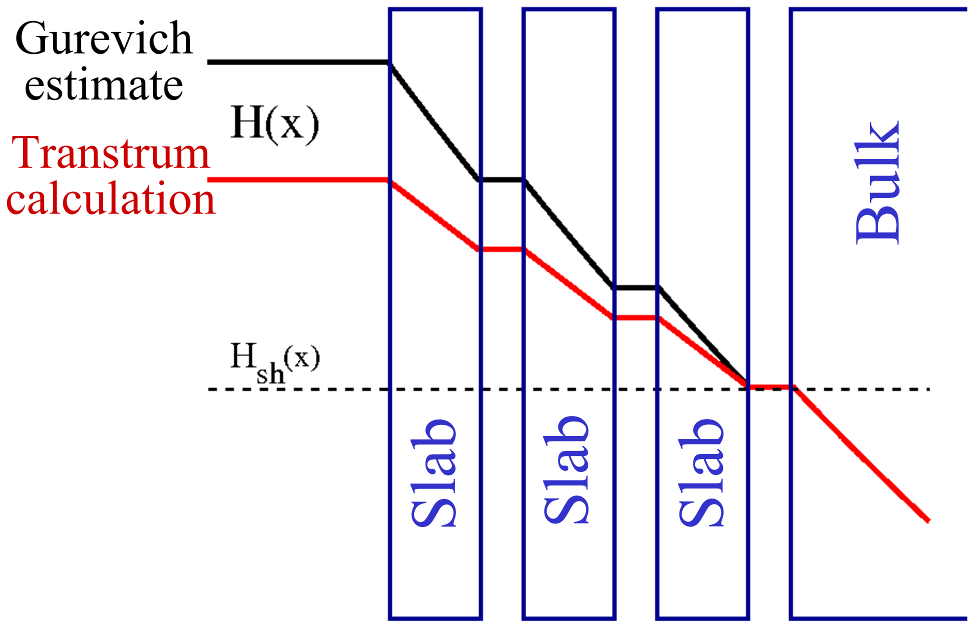




Nucleation theory on macroscale inhomogeneities
  (3D critical droplet calculations: nudged elastic band)

Experimental characterization of dominant losses and failure modes
  (hot spots & low-field Q-slope, nucleation & high-field Q-slope)

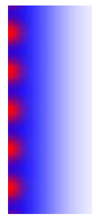
Thin slabs within Ginzburg-Landau



Thin slabs in parallel fields have enhanced $\boldsymbol{H_{c1}}$ (little flux expulsion, so little cost to ψ). Gurevich proposes to use slabs to protect bulk superconductor, ***H(x)*** below $\boldsymbol{H_{sh}}$ at surface (Interesting theory questions about the AC dynamics arise from Gurevich's proposal.)

Elegant calculation of $\boldsymbol{H_{sh}}$ for thin film

First variation of free energy:
  $\psi(z)$, $\mathbf{H}(z)$ (1D solution)
Second variation, wavevector k:
eigen*function* analysis
  $\delta\psi(z)\exp(i\,k\,y)$,
  $\delta\mathbf{H}(z)\exp(i\,k\,y)$



$\boldsymbol{H_{sh}}$ from first zero eigenvalue

- thin film, Tinkham/Gurevich
- thick film = bulk $\boldsymbol{H_{sh}}(\boldsymbol{\kappa})$

Conclusion

- Preliminary new calculation from basic superconductivity Eilenberger equations gives
  - $H_{sh} = 0.84\ H_c$ at $T = 0$ K and
  - $H_{sh} = 0.745\ H_c$ at $T = T_c$ in agreement with GL
- Encouraging for perfect Nb_3Sn and perfect MgB_2
- More work on the way to predict effect of real defects like grain boundaries….