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Lowest Longitudinal and Transverse Resistive-wall wake and impedance for nonultrarelativistic beams

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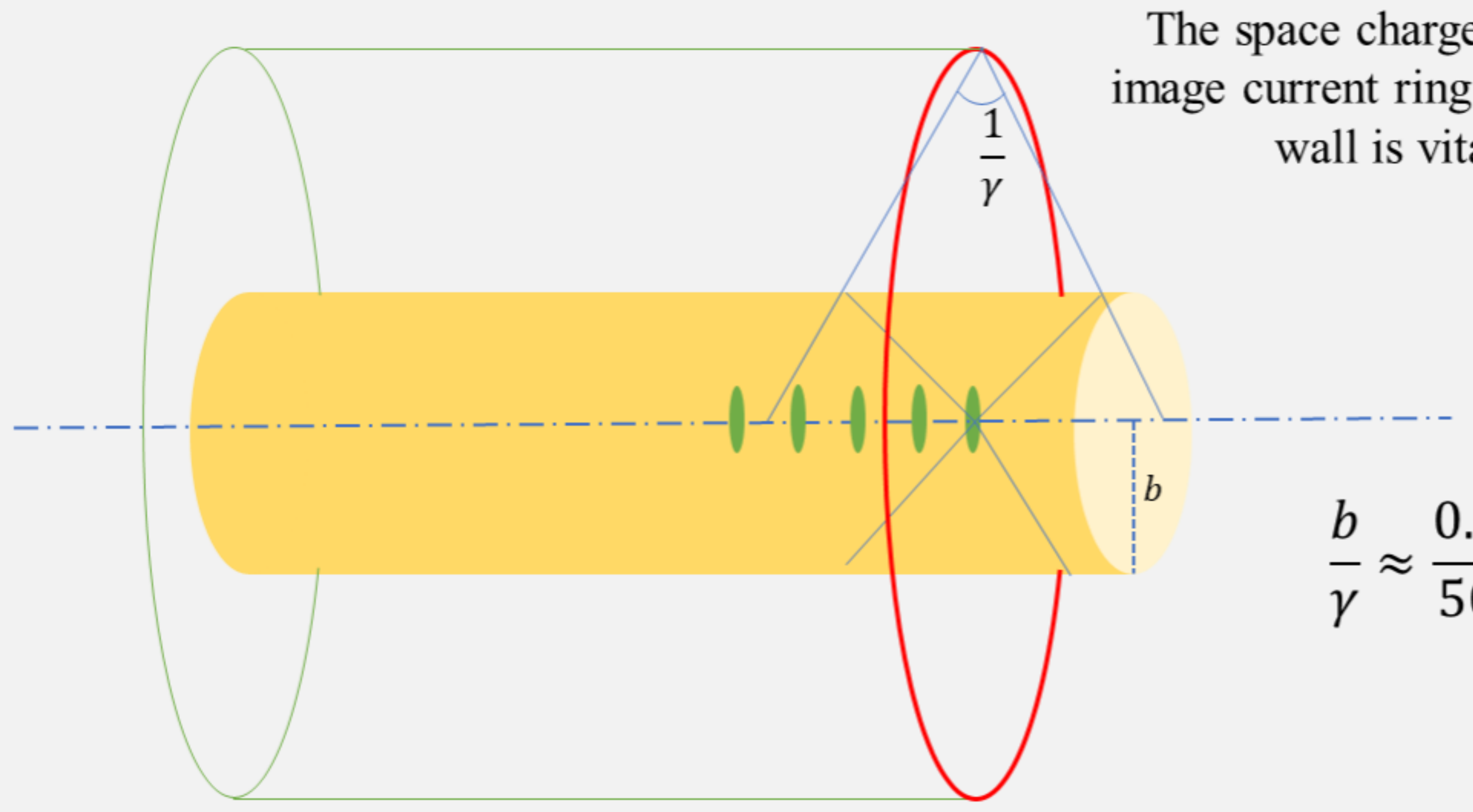
Abstract

With the development of the steady state micro bunching (SSMB) storage ring, its parameters reveal that the ultrarelativistic assumption which is wildly used is not valid for the electron beam bunch train, which has length in the 100nm range, spacing of 1um and energy in hundreds MeV range. The strength of the interaction between such bunches and the potential instability may need careful evaluation. At the same time, the effect of the space charge inside a single bunch due to space charge effect also needs to be considered. We reorganized the lowest-order longitudinal wakefield under non-ultra relativistic conditions, and modified the inconsistent part in the theoretical derivation in some essays of the lowest-order transverse wakefield. We present the modified theoretical results and analysis. The action area are then divided into three parts. It lays foundation in future research.

Sketch of the Physical model



The space charge component caused by image current ring induced on the resistive wall is vital in SSMB case

$$\frac{b}{\gamma} \approx \frac{0.02}{500} \approx 2\times 10^{-5} m$$

SSMB Parameter

Parameter	Value	Remarks
Length	$10\ nm$	For High Longitudinal Coherent
Transverse size	$10-100\mu m$	
Spacing	$1\mu m$	For High Average Power
Energy	$250MeV$	$\gamma \approx 490.2$

Source Term

Monopole Source

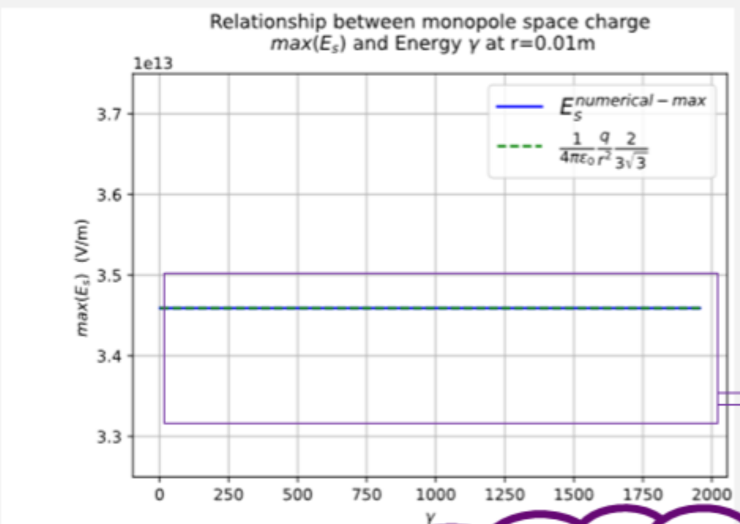
$$\rho_0 = q\delta(s-vt)$$
$$\boldsymbol{j_0} = v\rho_0\hat{\boldsymbol{s}}$$

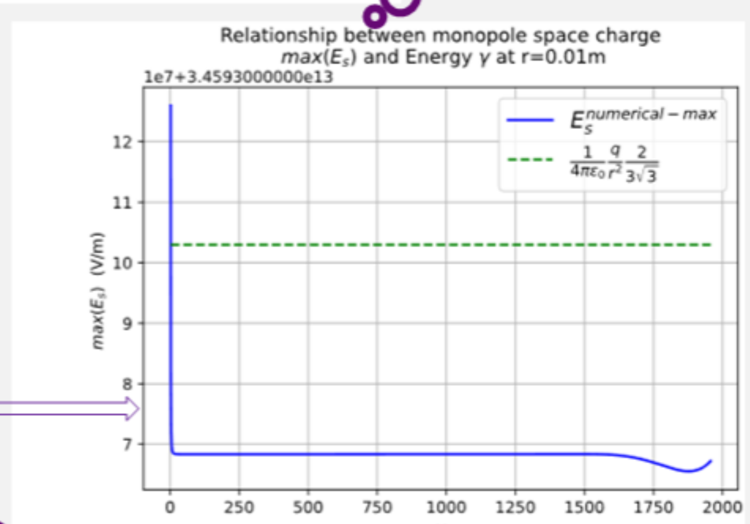
Dipole Source

$$\rho_1 = \frac{q}{\pi a}\delta(s-vt)\delta(r-a)\cos\theta$$
$$\boldsymbol{j_1} = v\rho_1\hat{\boldsymbol{s}}$$

- *Max Longitudinal Electric Field*

$$E_z(z) = \frac{1}{4\pi\varepsilon_0}\frac{2q}{3\sqrt{3}r^2}$$



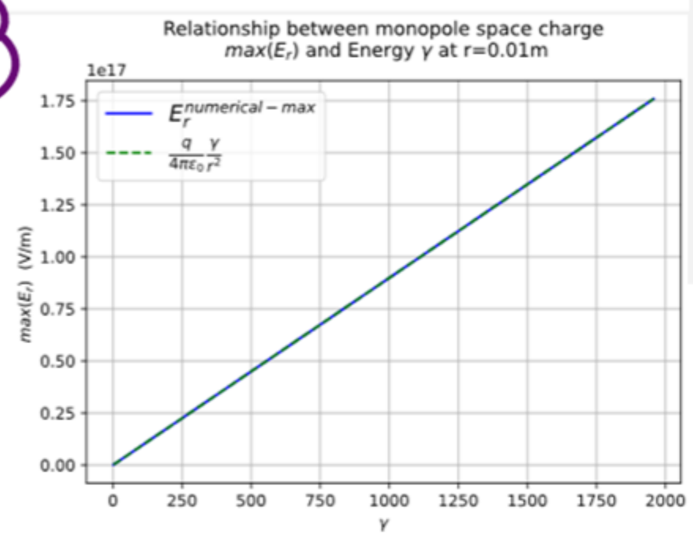


σ → 0 Limit Longitudinal Space Charge

- *Max Transverse Electric Field*

$$E_z(z) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\gamma$$

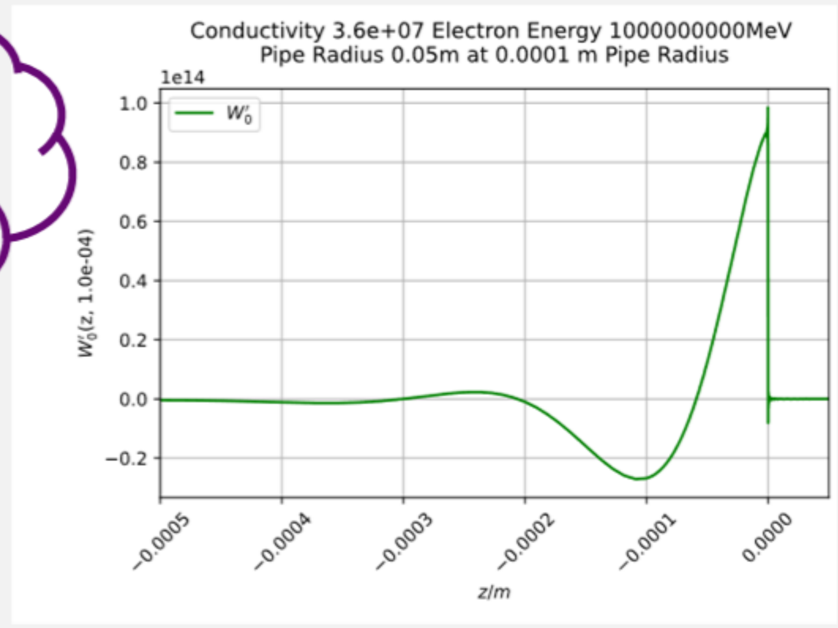
σ → 0 Limit Transverse Space Charge



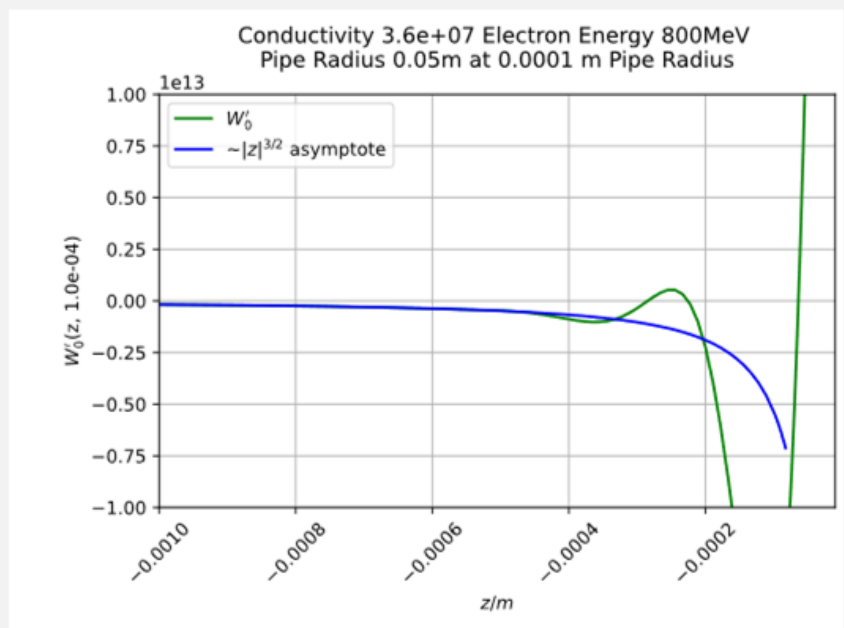
$max(E_s)$by formula	$max(E_s)$by numerical calculation
$3.4593068\times 10^{13}\ V/m$	$3.4593102\times 10^{13}\ V/m$

E	$max(E_r)$by formula	$max(E_r)$by numerical calculation
$500\ MeV$	$8.79408198\times 10^{13}\ V/m$	$8.79408704\times 10^{13}\ V/m$

Benchmark



Ultrarelativistic Limit

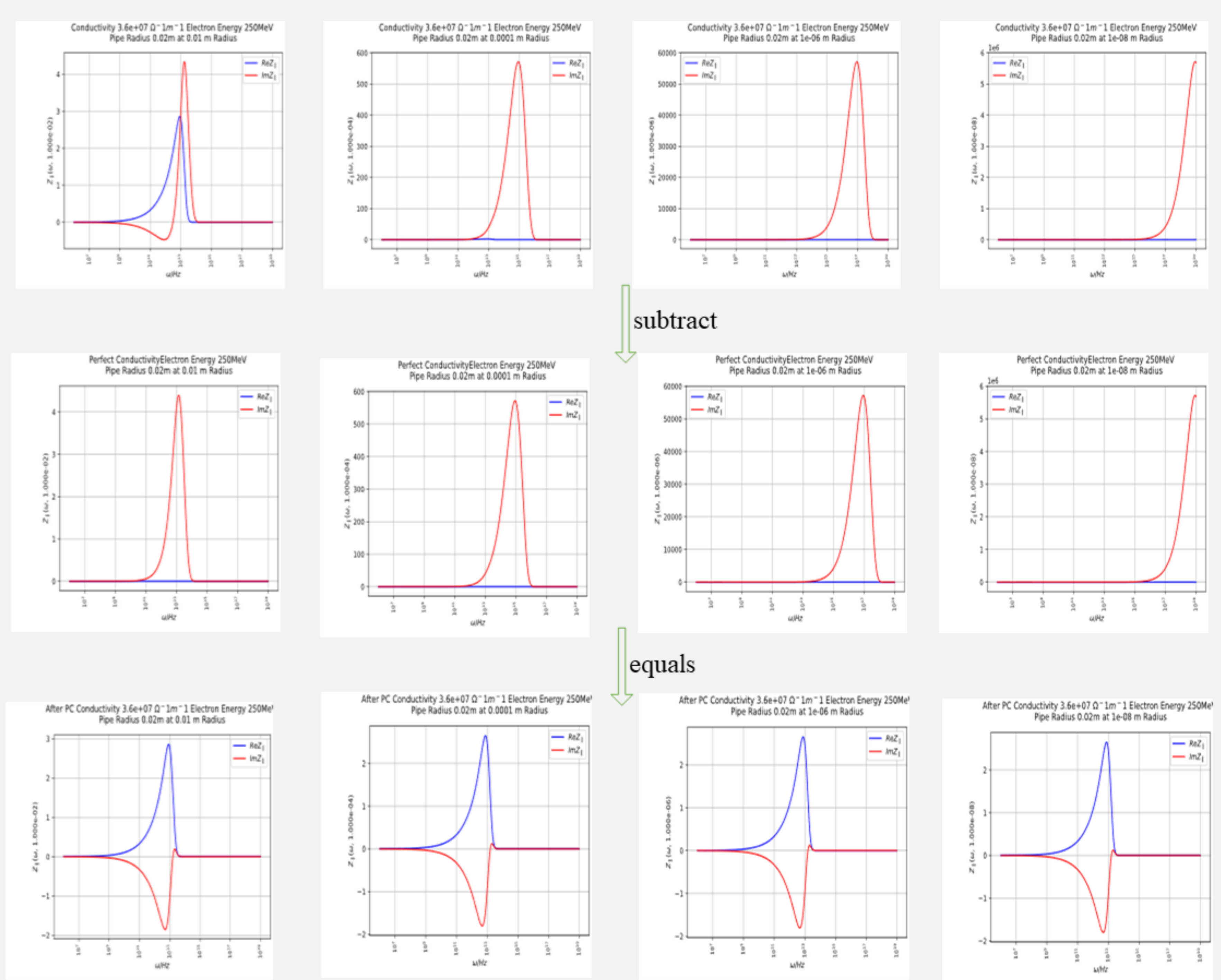


Long Range Limit

- Asymptote function

$$W_0'(z) = -\frac{c}{4\pi b}\sqrt{\frac{Z_0}{\pi\sigma_c}}\frac{L}{|z|^{\frac{3}{2}}}, \qquad z \ll -\chi^{\frac{1}{3}}b$$

z/m	$\chi^{\frac{1}{3}}b/m$	$W_0'(z)$ by formula	$W_0'(z)$ by numerical calculation
-0.01	5.7×10^{-5}	-5.47×10^{9}	-5.35×10^{9}



Reduce Resistivity Method

Monopole Case

$$Z_{\parallel,sc}(\omega,r) = \lim_{\sigma\to\infty}\frac{iZ_0ck_r^2}{2\pi\omega}\left[K_0(k_rr)+I_0(k_rr)\frac{\omega^2\lambda K_1(bk_r)K_0(b\lambda)+k_rc^2(\lambda^2-k^2)K_0(bk_r)K_1(\lambda b)}{\omega^2\lambda I_1(bk_r)K_0(b\lambda)-k_rc^2(\lambda^2-k^2)I_0(bk_r)K_1(\lambda b)}\right]$$

$$= \lim_{\sigma\to\infty}\frac{iZ_0ck_r^2}{2\pi\omega}\left[K_0(k_rr)+I_0(k_rr)\frac{\omega^2\lambda K_1(bk_r)+k_rc^2(\lambda^2-k^2)K_0(bk_r)}{\omega^2\lambda I_1(bk_r)-k_rc^2(\lambda^2-k^2)I_0(bk_r)}\right] = \frac{iZ_0ck_r^2}{2\pi\omega}\left[K_0(k_rr)-I_0(k_rr)\frac{K_0(bk_r)}{I_0(bk_r)}\right]$$

Conclusion

Resistive Wall WakeField + *Space Charge Effect* → *Source Space Charge* + *Image Space Charge* + *Resistive Wall*

Todo…

- **Analytical Formula** for dipole current go through perfect conductor wall

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