WAKEFIELD STUDIES FOR THE STEP STRUCTURE AND THE SKIN DEPTH OF COATED DIELECTRIC TUBES

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Abstract

Wakefield issues are always important in free electron laser (FEL) facilities. Since the wakefield in free electron laser facilities usually leads to a decrease of FEL performance, the research of the wakefield impacts is of great significance. Step structures are almost ubiquitous in the overall undulator section of an FEL facility, which always generate critical wakefields. In this paper, we systematically analyse and summarize the wakefield characteristics of step structures including the step-in and the step-out. In addition, the skin depth issue of the wakefield is still controversial. We study the skin depth of the wakefield field in the vacuum chamber of the kicker in the Shanghai high-repetition-rate XFEL and extreme light facility (SHINE), which is made of the dielectric tube. We proposed the conception of "effective skin depth" from two different perspectives and wrote simulation codes to calculate the "effective skin depth". We hope these methods could provide new mentalities for related research in the future.

INTRODUCTION

FEL lasing performance largely depends on the quality of electron bunches [1]. While the wakefield generated by the electron bunch always destroys its phase space, which leads to a decrease of lasing performance [2-4]. Since there are many different types of devices the inner apertures of chambers in different devices are always different. The connection of chambers with different apertures is called step structure. In our research before [5, 6], we found that the wakefield generated in step structures is an important component of the total wakefield of an FEL facility. The step structure wakefield even contributes half of the total wakefield. Therefore, the study of the step structure wakefield is important for FEL. There are currently some studies on the step structure wakefield, but there is still a lack of systematic summary of the characteristics of this kind of wakefield. In this paper, we calculate and analyse the step structure wakefield in different situation and initially list its characteristics comprehensively.

In our study of the wakefield in the kicker of the SHINE, we found that the skin depth issue in wakefield calculations is still unsolved. Different teams presented different perspectives. In this paper, we proposed the conception of "effective skin depth" from the perspective of the accumulation of free charges and the attenuation of the surface current. We calculate the effective skin depth using our own simulation code. The results demonstrate the feasibility of the development of this conception in the future.

WAKEFIELDS OF THE STEP STRUCTURE

The step structure includes two types of structures, the step-in and the step-out. The former means the structure whose aperture varies from large to small while the latter is the opposite structure to step-in. There are two connection types of step structures, the saltatory type is called the "hard connection" and the "soft connection" means a gradual change of aperture. Although there are some research about the step structure wakefield, some questions are still unsolved. In this section, we will answer these questions based on our simulation results (simulation code: ECHO2D [7]).

Short Bunch Situation

In our research before, we always ignore the effect of step-in wakefield since its value is much smaller than that of step-out wakefield. Taking the situation in the SHINE as an example, we calculated wakefields of a step-in and a step-out with soft (taper) and hard connection schemes. The parameters are listed in Table 1 and the simulation results are shown in Figure 1.

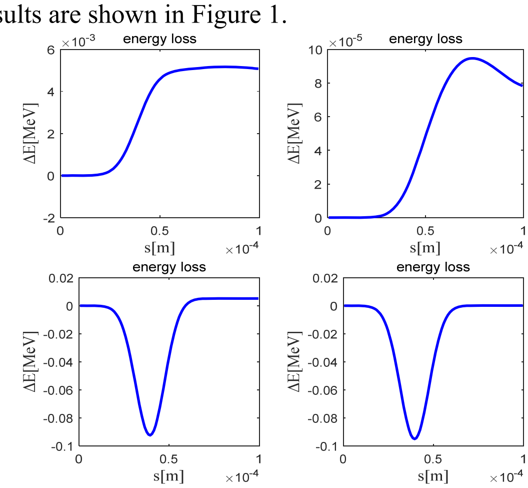

Figure 1: Simulation results of step structure wakefields in short bunch situation.

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Table 1: Parameters of Wakefield Simulation

Aperture 1 [mm]	Aperture 2 [mm]	Bunch length [GeV]	Charge [pC]
14	24	8	100

Figures in the first row are step-in wakefields and the second row contains step-out wakefields. The first column is soft connection, and the second column is hard connection. As shown in Figure 1, the step-out wakefield is almost unchanged in different connection schemes, while the step-in wakefield shows a significant growth in soft connection scheme. However, step-in wakefields are consistently inappreciable compared with step-out wakefields. In most FEL facilities, the bunch length is usually short enough that the step-in wakefields could be ignored.

To understand the characteristics of step structure wakefields, we referred to L. Palumbo's theory about step structures [8].

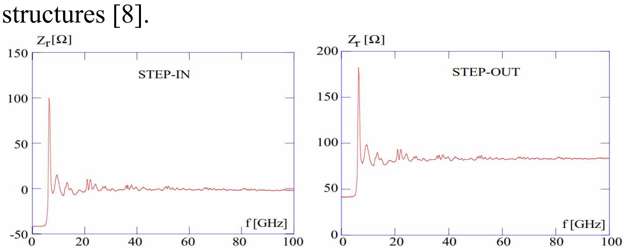

Figure 2: Impedance spectrum of step-in and step-out [8].

As we all know, the wakefield is the inverse Fourier transform of the impedance, so the larger impedance means the stronger wakefield and they have opposite signs. According to Figure 2, we could analyse the reason of step structure wakefield variation. In short bunch situation, the main part of the impedance comes from the high frequency part in the spectrum. Thus, the step-out wakefield is a negative number just like the most wakefields, while the step-in wakefield is positive due to the negative impedance. In soft connection scheme, the low frequency of impedance is growing with the slope decreasing. Then, the step-in wakefield is getting stronger and the step-out wakefield is getting weaker. The value of this variation is very small, so it is obvious for the step-in wakefield but not for the step-out wakefield.

Long Bunch Situation

When the bunch length becomes extremely long, the conclusion could be quite different from the short bunch situation. We calculated wakefields in the same structure but generated by a long bunch (rms=260mm, Q=670pC). The results are shown in Figure 3.

The correspondence between the wakefield and structure is the same as Figure 2. It is obvious that in long bunch situation, the value of the step-in wakefield and the step-out wakefield in soft connection scheme is almost the same. This is expectable according to Figure 2. In long bunch situation, the low frequency part becomes the key part of the impedance. When the frequency is approaching to zero, the impedances of step-in and step-out become almost opposite numbers. Thus, when the slope of connection part is getting smaller, the sum of their wakefields is getting closer to zero.

Note that the step-in wakefield is always positive in all simulation results. It seems that the electron bunch has gained energy at this position. This does not conform to the law of conservation of energy. In Palumbo's theory, the energy loss of an ultra-short bunch caused by step-out and step-in is given by the following equation:

$$q^2k^{out}\sim 2U(a<r<b),$$
$$q^2k^{in}\sim 0, \quad (1)$$

while the energy loss of an extremely long bunch caused by a long tapering step structure is given as:

$$q^2k^{out}\sim U(a<r<b),$$
$$q^2k^{in}\sim -U(a<r<b). \quad (2)$$

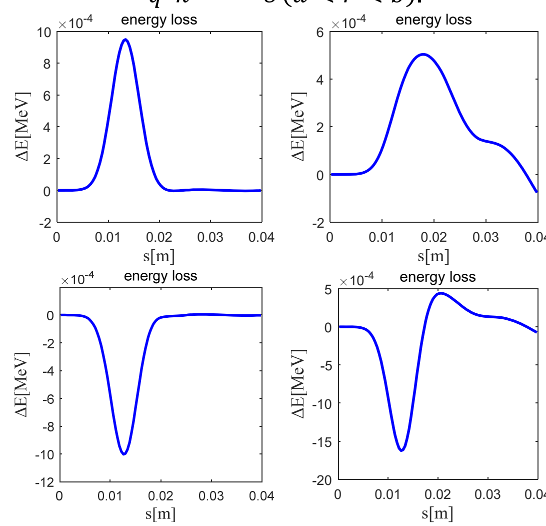

Figure 3: Simulation results of step structure wakefields in long bunch situation.

According to Eq. (1), when the high frequency part dominates, the wakefield of step-in could be ignored. While if the low frequency part dominates, as shown in Eq. (2), the wakefield value of step-in is equivalent to that of step-out. Under this circumstance, the effect of step-in wakefield should not be ignored anymore and its physical implication needs to be explained. Actually, the energy lost by the electron bunch at the position of step-out will not immediately dissipated but diffuses into the cavity space. We believe that the positive wakefield of the step-in structure means that the energy loss at the step-out structure will be partially returned through the step-in structure. We summarize the wakefield characteristics of the step structure as follows:

- The wakefield of step-in is positive, which means an energy compensation. This part of energy comes from the energy lost by the electron bunch itself at the position of step-out.
- In the case of ultra-short bunch and hard connections, the wakefield of step-in is extremely small. It is several orders of magnitude smaller than that of step-out.
- As the electron bunch becomes longer and the slope of the step structure becomes smaller, the low-frequency part of the impedance spectrum increases,

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resulting in a larger step-in wakefield and a smaller step-out wakefield. Ultimately, the absolute values of the two gradually approach. For a infinitely long bunch and an infinitely flat slopes, the sum of the wakefields of step-in and step-out is 0.

- The length of the electron bunch is the main factor that dominates the high and low frequency proportion of the impedance spectrum of the step structure.
- The value of the wakefield discussed above refers to the total wakefield from the beginning to the end of the step structure. In the case of ultra-short beam clusters, although the total wakefield cannot be obviously changed by a tapering connection, it can be distributed across the entire slope, which is helpful in weakening the thermal effect of the wakefield.

SKIN DEPTH ISSUES OF THE WAKEFIELD

The kicker in the SHINE employs a metal coated dielectric tube as its vacuum chamber. In order to better transmit the field, the coating thickness should be as thin as possible, while to avoid the wakefield of the dielectric tube, the coating thickness should be thick enough. Under the common requirements of both aspects, we recommend that setting the coating thickness as the skin depth is the best scheme. For a single frequency electromagnetic field, solving skin depth is never a problem. But in the wakefield issue, the skin depth of a multifrequency mixed field is hard to define and calculate. To find a suitable coating solution, we proposed the conception of “effective skin depth” from the perspectives of the accumulation of free charges and the attenuation of the surface current.

Effective Skin Depth 1: Accumulation of Free Charges

Assuming that the electromagnetic field enters the conductor perpendicular to the surface of the conductor. If only under the action of the electric field force and the Lorentz force, the electrons in the conductor will move deeply. At this time remainder residual positive charges will be generated on the surface, resulting in a charge force that attracts electrons towards the surface. As more and more electrons move towards the depth, this amount of charge gradually increases. Finally, the electrons achieve equilibrium under the combined action of the electromagnetic field and charge force, as illustrated in Figure 4.

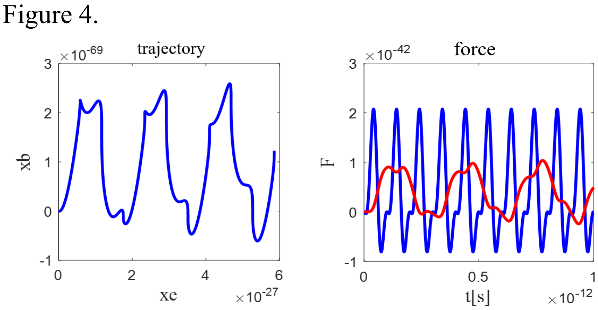

Figure 4: The trajectory and force of electron motion reaching equilibrium state

We calculated the attenuation total charge density at the equilibrium state as shown in Figure 5. Take the depth where the charge density attenuates to 1/e as the effective skin depth δ_{eff}, we got $\delta_{eff} = 162\ mm$.

Effective Skin Depth 2: Attenuation of The Surface Current

Actually, charge accumulation may not occur at all frequencies of electromagnetic fields, as evidenced by the decay function of free charge density within a conductor:

$$\rho(t) = \rho_0 e^{-\frac{\sigma}{\epsilon}t}. \quad (3)$$

When the attenuation characteristic time $\tau \ll \omega^{-1}$, charges will attenuate before accumulating. In the impedance spectrum of kicker's quasi coating material titanium, the curve almost comes to zero when the frequency exceeds 10THz [9,10] while $\tau \approx 10^{-17}s$. Thus, we have to calculate the effective skin depth from a new perspective. Considering the transvers motion of electrons along the conductor surface, we could calculate the surface current and figure out effective skin depth. We wrote a code and simulated electron motion under multi frequency superposition field. The result of surface current distribution is shown in Figure 5. Equally, we found the position where surface current attenuates to 1/e and the effective skin depth, in this situation, is 272mm.

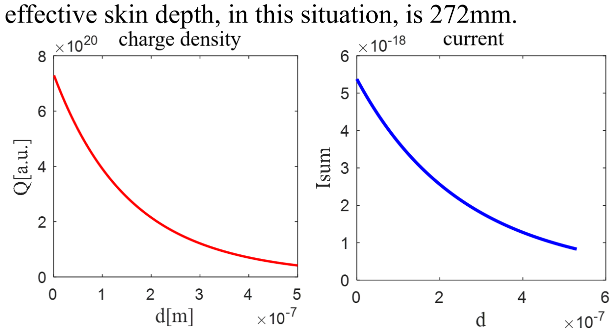

Figure 5: Attenuation curve of total charge density (left) and surface current distribution under multi frequency superposition field (right)

CONCLUSION

We systematically summarized the wakefield characteristics of the step structure. The conclusion could be helpful for related research. In addition, we proposed the conception of “effective skin depth” from two different perspectives. Although the simulation code is rough, we believe these methods could provide new mentalities for related research in the future.

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