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DEVELOPMENT OF A VLASOV SOLVER FOR ARBITRARY SUB-OPTIMAL LENGTHENING CONDITIONS IN DOUBLE-RF SYSTEM

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Abstract

Solving Vlasov equation is a classic method for analyzing collective beam instabilities. Considering longitudinal impedance and the nonlinear longitudinal potential well, we developed a new Vlasov solver which can be used to study the transverse mode-coupling instability under the arbitrary sub-optimal lengthening and the optimal lengthening conditions in a double-RF system. Several different techniques to deal with the radial direction of longitudinal phase space have been tested. Numerical discretization method is selected in this paper. The development of the solver is presented in details here. Benchmarks and crosscheck of the solver have been made and presented as well.

INTRODUCTION

Most (semi-)analytical Vlasov solvers are based on a single RF cavity or do not contain synchrotron tune spread [1,2]. In 2014, A. Burov proposed the NHTVS (Nested Head-Tail Vlasov Solver), which can contain small synchrotron tune spread but it is based on Gaussian bunches [3]. In 2018, Venturini proposed radial discretization which contains large synchrotron tune spread and flat-top distribution, it is only applicable for optimal lengthening. [4]. They may result in significant errors under conditions where the longitudinal distribution is completely different from Gaussian or the synchrotron tune spread cannot be ignored. There is currently no general Vlasov solver for sub-optimal lengthening conditions.

In this paper, we proposed a method to deal with arbitrary sub-optimal lengthening bunch by discretization of Vlasov equation. It is a general method to dominate Gaussian, sub-optimal and optimal lengthening bunches.

At first, we will derivate Vlasov equation similar to Ref. [5]. Then we choosed two typical cases and compared our method with Chuntao Lin's transfer matrix method [6]. The specific sampling process will be described in this section.

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FORMULA DERIVATION

The single-particle equations of motion are:

$$\begin{cases} \dot{y}(s) = p_y \\ \dot{p_y}(s) = -\left(\frac{\omega_\beta}{c}\right)^2 y + \frac{1}{E}F_y(z,s) \\ \dot{z}(s) = -\eta_p\delta \\ \dot{\delta}(s) = \frac{eV_1}{EC}\mathscr{V}(z) \end{cases} \tag{1}$$

Here F_y is transverse wake force, E is particle energy, ω_β is betatron frequency, η_p is slippage factor, C is circumference of the storage ring, and $V_1\mathscr{V}(z)$ is the voltage of double RF system, where

$$\begin{aligned} \mathscr{V}(z) &= \sin\left(\phi_s - \frac{2h_1\pi}{C}z\right) - \sin\phi_s \\ &+ r\sin\left[\phi_{2s} - \frac{2h_1 h\pi}{C}z\right] - r\sin\phi_{2s}. \end{aligned} \tag{2}$$

We use action-angle variables J, ϕ in the longitudinal phase space and polar coordinates q, θ in the transverse. So the perturbation formalism of density distribution $\psi(J,\phi,q,\theta;s)$ can be written as

$$\psi = f_0(q)g_0(J) + f_1(q,\theta)g_1(J,\phi)e^{-i\Omega s/c}.$$

Substitute ψ, Eqs. (6.168) and (6.173) in Ref. [5] into the Vlasov equation,

$$\begin{aligned} i(\Omega-\omega_\beta)g_1 &= \frac{ce^2}{2E\omega_\beta T_0^2}g_0\sum_p \tilde{\rho}_1(\omega')Z_1^\perp(\omega')e^{i\omega' z/c} \\ &+ B(J,\phi)\left[\frac{\partial g_1}{\partial J} - \frac{1}{D}\frac{f_0}{f_0'}\frac{\partial g_0}{\partial J}e^{i\Omega s/c - i\theta}\right] + C(J,\phi)\frac{\partial g_1}{\partial \phi}, \end{aligned} \tag{3}$$

here

$$\begin{aligned} B(J,\phi) &= \dot{\delta}(z)\frac{\partial J}{\partial \delta}\bigg|_z + \dot{z}(t)\frac{\partial J}{\partial z}\bigg|_\delta = \frac{d\vec{r}(t)}{dt}\cdot\nabla J = \frac{dJ}{dt} = 0, \\ C(J,\phi) &= \dot{\delta}(z)\frac{\partial \phi}{\partial \delta}\bigg|_z + \dot{z}(t)\frac{\partial \phi}{\partial z}\bigg|_\delta = \frac{d\vec{r}(t)}{dt}\cdot\nabla\phi = \omega_s(J). \end{aligned} \tag{4}$$

Here $\vec{r}$ is the vector from the original point to the particle coordinate (z,δ) in the longitudinal phase space. By the same manipulation of Fourier expansion of g_1 in Ref. [5], we finally get

$$\begin{aligned}&(\Omega^{(l)}-\omega_\beta)R_l(J)\\ =&g_0(J)\sum_{l'}\int_0^\infty dJ'\,R_{l'}(J')G_{l,l'}(J,J')\\ &+\sum_{l'}l'R_{l'}(J)\delta_{l,l'}\omega_s(J).\end{aligned}\tag{5}$$

Here $g_0(J)$ is the normlized radial distribution, $R_l(J)$ is radial distribution of angular mode l, $\omega_s(J)$ is synchrotron tune, $G_{l,l'}(J,J')$ defines as

$$G_{l,l'}(J,J')=-\frac{iN\pi ce^2}{E\omega_\beta T_0^2}\int_{-\infty}^{\infty}d\omega Z_1^\perp(\omega)S_l^*(\omega,J)S_{l'}(\omega,J')\tag{6}$$

which corresponds to Eq. (6.195) in Ref. [5], but considering angular mode coupling. Here $S_l(\omega,J)$ defines as

$$S_l(\omega,J)=\frac{1}{2\pi}\int_0^{2\pi}d\phi e^{il\phi-i\frac{z(J,\phi)}{c}\left(\omega-\frac{\xi\omega_\beta}{\eta}\right)},\tag{7}$$

which degenerates to

$$i^{-l}J_l\left(\frac{r}{c}\left(\omega-\xi\omega_\beta/\eta\right)\right)$$

in Eqs. (6.177-6.178)in Ref. [5]. Eq. (7) is a compromise on asymmetric potential. Almost all existing models, such as water-bag model, hollow bunch, parabolic bunch, small amplitude approximation of Gaussian and ideal-lengthening bunch, are symmetric. Because of asymmetry, function $S_l(\omega,J)$ will be numerically calculated combined with discretization that not only in radial direction as Venturini did but also angular direction.

By discretization of Eq. (5) in radial direction, it becomes an eigen value problem

$$\begin{aligned}(\Omega^{(l)}-\omega_\beta)\vec{R}_{lj}&=M_{lj,l'j'}\vec{R}_{l'j'}\\ M_{lj,l'j'}&=g_0(J_j)G_{l,l'}(J_j,J_{j'})\Delta J_{j'}+l'\delta_{ll'}\omega_s(J_j)\end{aligned}.\tag{8}$$

Notice that at vanishing beam intensity $N=0$, $M_{lj,l'j'}$ will be a diagonal matrix with elements $\{l\omega_s(J_j)|j=1,...,n_J\}$. That's the reason of tune spread of each azimuthal mode, except $l=0$.

COMPARE WITH TRANSFER MATRIX METHOD

Settings

The toy model of HEPS is shown in Table 1 and settings of two cases are shown in Table 2, Case #1 represents a set of sub-optimal lengthening parameters, while Case #2 represents optimal lengthening parameters.

Sampling Method

Assuming maximum z of sampling points reaches $n\sigma_z$, the number of radial, angular discretization meshes is n_J, n_ϕ, where n_ϕ is an even number. Then we can sample as described below:

Table 1: Toy Model of HEPS

Parameter	**Value**
Beam energy, E, GeV	9
Circumference, C, m	1360.4
Primary harmonic number, h_1	756
Ratio of harmonic number, $h=h_2/h_1$	3
Energy spread, σ_δ	1.06×10^{-3}
Momentum compaction factor, α_c	1.56×10^{-5}
Vertical betatron tune, ν_y	106.23
Bunch charge, N, nC	2

Table 2: Settings of Two Cases

Parameter	**#1**	**#2**
Primary RF Voltage, E, MV	3.6395	3.6395
Voltage ratio, $r=V_2/V_1$	0.1819	0.1802
Primary RF phase, ϕ_s, rad	1.8928	2.0390
Harmonic RF phase, ϕ_{2s}, rad	5.2598	5.7005
Bunch length, σ_z, cm	2.5763	2.9078
Average synchrotron tune, $\langle\nu_s\rangle$, 10^{-4}	1.6145	0.9912
Number of radial samples, n_J	60	80
Number of angular samples, n_ϕ	300	400
Maximum angular mode, l_m	3	3
$n=z_{\max}/\sigma_z$	4	4

- Calculate longitudinal density $\rho(z)$;
- Uniformly sample n_J points within the range of 0 to $n\sigma_z$, which is n_J $z_{\max}$'s of Hamiltonian tori;
- Calculate n_J $z_{\min}$'s, Hamiltonian H's, radial unperturbated density $g_0(J)=\rho(z_{\max})$ of these Hamiltonian tori.
- Calculate action variable J of each torus according to

$$J=\frac{1}{\pi}\int_{z_{\min}}^{z_{\max}}\delta dz;$$

- Calculate synchrotron tune spread $\nu_s(J)$ and average synchrotron tune $\langle\nu_s\rangle$ by:

$$\begin{aligned}\nu_s(J)&=\frac{C}{2\pi}\frac{dH}{dJ},\\ \langle\nu_s\rangle&=2\pi\int_0^\infty g_0(J)\nu_s(J)dJ.\end{aligned}$$

- Take angular sample of each torus between $z_{\min}$ to $z_{\max}$ by

$$z_i=z_p+r\cdot\cos\frac{i\cdot2\pi}{n_\phi},(0\le i\le n_\phi)$$

here $z_p=0.5(z_{\min}+z_{\max})$, $r=0.5(z_{\max}-z_{\min})$.
- Caculate angle variable ϕ at each point according to

$$\begin{aligned}F_2(z,J)&=\int_z^{z_{\max}}\delta(z,J)dz,\\ \phi&=\frac{\partial F_2}{\partial J}.\end{aligned}$$

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Results

We compared our method with Chuntao Lin's transfer matrix method [6]. We compared two typical settings, sub-optimal and optimal lengthening, and the corresponding longitudinal distribution $\rho(z)$ and synchrotron tune spread $\nu_s(z)$ was shown in Fig. 1. As shown in Fig. 1, the synchrotron tune spread cannot be ignored, or the density distribution is asymmetric.

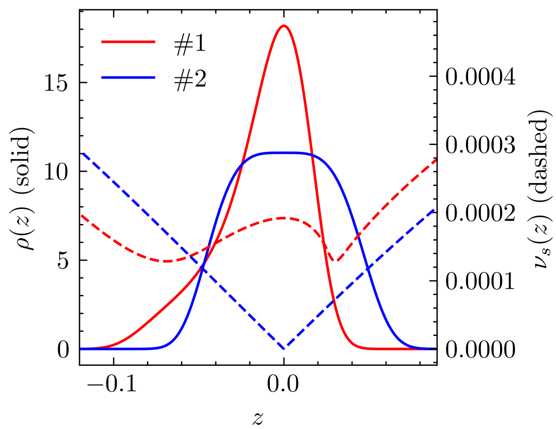

Figure 1: Distribution(solid) and synchrotron tune spread(dashed) of Case #1(red) and Case #2(blue) lengthening.

Under sub-optimal and optimal lengthening conditions, our results are consistent. As shown in Fig. 2, we can clearly see when the $l = 0$ mode and the $l = -1$ mode are coupled, the instability occurs. In Fig. 3, we cannot find the threshold of instability clearly as Venturini said [4]. In fact, the tune shift points at $N = 0$ nC are dominated by radial sampling points, and the maximum tune shift of $l = -1$

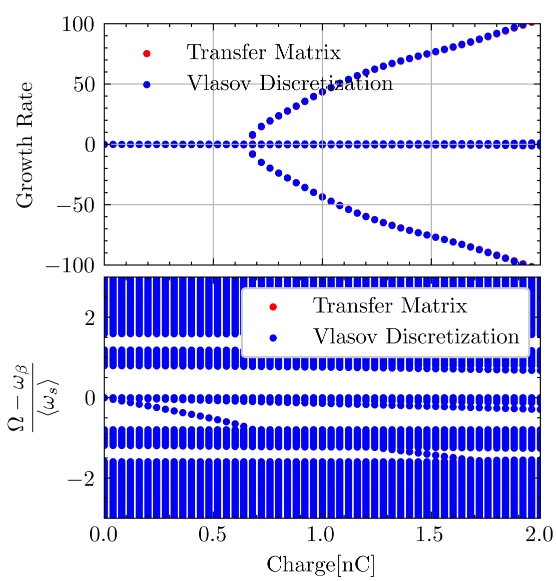

Figure 2: Growth rate per second and tune shift of Case #1(sub-optimal).

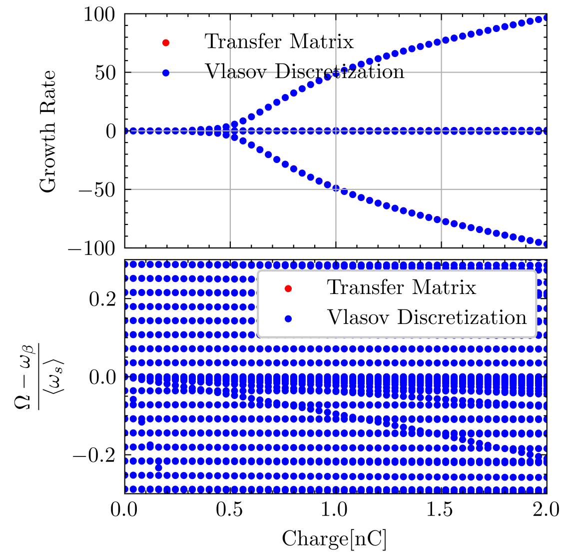

Figure 3: Growth rate per second and tune shift of Case #2(optimal).

mode is broadened to $l = 0$ mode under optimal lengthening condition.

CONCLUSION

In this paper, we proposed a general Vlasov discretization method that can calculate growth rate and mode coupling for arbitrary sub-optimal and optimal lengthening bunches.

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