APPROXIMATION OF SPACE CHARGE EFFECT IN THE PRESENCE OF LONGITUDINAL MAGNETIC FIELDS

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Abstract

The space charge effect plays a significant role in the evolution of phase space during beam transport. Applying an external longitudinal magnetic field has been shown to effectively reduce beam expansion through the mechanism of beam rotation. In this article, we present a fast approximation algorithm for estimating the impact of an external magnetic field on beam expansion. The algorithm enables efficient computations and provides insights into controlling the phase space dynamics of the beam in the presence of longitudinal magnetic fields.

INTRODUCTION

In particle accelerators, space charge forces have a significant impact on the transmission process, causing the beam to expand in both the transversal and longitudinal dimensions. This expansion leads to an increase in emittance, the formation of beam halo, and even beam loss [1]. Notably, the space charge force is proportional to $1/\gamma^2$, where γ represents the Lorentz factor [2]. Therefore, the influence of the space charge force becomes more pronounced for low-energy high-intensity beams. As the transmission distance increases, the impact of the space charge force on high-energy beams gradually becomes significant. In the simplest scenario, using an infinitely long uniform beam model, the expression for the space charge force can be given as follows:

$$F_r = \frac{qIr}{2\pi\varepsilon_0\beta c a^2}\frac{1}{\gamma^2} \qquad (1)$$

As shown in Figure 1, a is the radius of the beam.

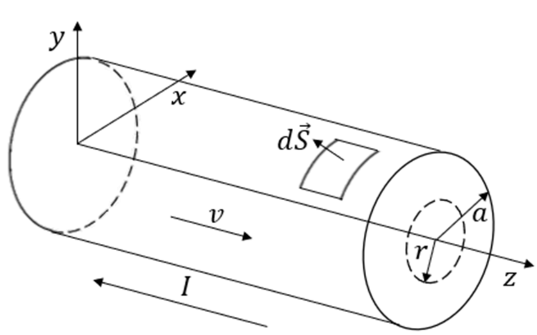

Figure 1: The infinitely long uniform beam model.

When a longitudinal magnetic field B is applied, it can help alleviate the space charge effect. As macroparticles experience space charge forces and gain radial velocity, the Lorentz force causes the beam to rotate in the angular direction. This rotation dissipates some of the total energy derived from the electromagnetic potential. Consequently, this process can effectively slow down the radial expansion of the beam.

Furthermore, the angular rotation of particles gives rise to an induced longitudinal magnetic field B_1, which acts to weaken the original magnetic field B. However, through simulation, it has been observed that B_1 is significantly smaller than B. Consequently, in the subsequent discussion, we will neglect the influence of B_1. Additionally, we assume that the transverse velocity is much smaller than the longitudinal velocity, which is close to the speed of light. In the Cartesian coordinate system, with the longitudinal direction defined as the z-axis, the equation of motion for the macroparticles can be expressed as follows:

$$\ddot{x} = A_1 x - A_2\dot{y};\ \ddot{y} = A_1 y + A_2\dot{x} \qquad (2)$$

Wherein $A_1 = qI/\gamma^3 m_0 2\pi\varepsilon_0\beta c a^2; A_2 = qB/\gamma m_0$. m_0 is the static mass of an electron. By utilizing the fourth-order Runge-Kutta method, we can update the position of a macroparticle based on an appropriate time step. This numerical technique allows us to accurately calculate the particle's trajectory and track its motion throughout the simulation.

From Eq. (1) and Eq. (2), we observe that in a uniform electron beam, the accelerations and velocities of particles are both proportional to their radial positions. As a result, after expansion, the ratio of radial coordinates for different particles remains constant, indicating that the beam remains uniform. In order to simplify the computation, a fast approximation algorithm can be employed, where only the motion of the outermost particle is calculated. This approximation allows for a more efficient calculation process while still capturing the overall behaviour of the beam.

PROGRESS

Verification with Astra

We have developed a fast approximation algorithm utilizing Eq. (1) and Eq. (2). In order to validate the accuracy of this approximation algorithm, we can compare its results with those obtained from Astra, a space charge tracking algorithm developed by DESY [3]. For this comparison, we can utilize the single-bunch model. If the length of the beam remains relatively constant after transport, its equivalent current can be considered constant as well. Under such circumstances, Astra's finite-cylinder model is equivalent to the infinitely long beam model. By comparing the results of our approximation algorithm with Astra's

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simulations under these conditions, we can assess the correctness of our approach.

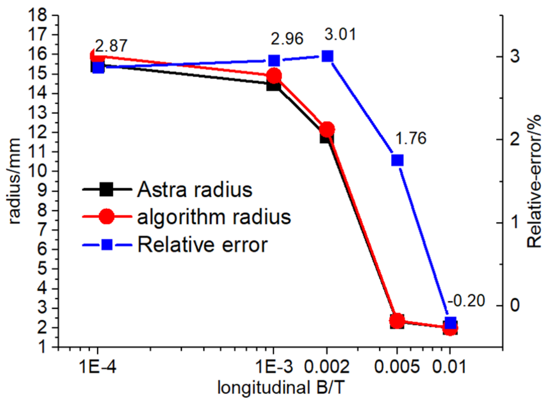

Figure 2: Comparison with Astra.

Based on the results depicted in Fig. 2, it is evident that the fast approximation algorithm accurately approximates the expansion of the beam under various longitudinal magnetic fields. The initial parameters of the electron beam used in the simulations are presented in Table 1. When compared to the results obtained from Astra, the algorithm exhibits a relative error of less than 3%. Moreover, it significantly reduces the computation time from several minutes to less than a second. These findings demonstrate the effectiveness and efficiency of the fast approximation algorithm in accurately modelling the beam's expansion behaviour.

Table 1: Initial Parameters of the Beam

Parameters	**Value**
Gamma	20.57
Initial radius/mm	2.0
Initial length/mm	50
Transport distance/m	50
Charge/nC	-0.5

Algorithm Results

Figure 2 clearly illustrates that as the longitudinal magnetic field strength increases, the mitigation of beam expansion becomes more pronounced. However, the presence of a longitudinal magnetic field introduces a non-monotonic behaviour in the beam's radius with respect to the transport distance. Instead of continuously increasing, the beam radius exhibits periodic oscillations, as demonstrated in Fig. 3. The parameters employed in these simulations are the same as those used in Fig. 2.

Indeed, in the scenario where the beam radius exhibits periodic oscillations due to the presence of a longitudinal magnetic field, focusing solely on the final radius at a fixed transport distance may not provide a comprehensive understanding. Instead, it becomes crucial to analyse the oscillation period and amplitude.

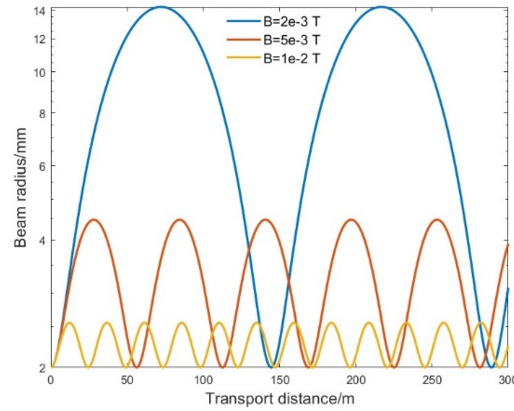

Figure 3: Oscillation of the beam radius.

As depicted in Fig. 3, we observe that with increasing magnetic field strength, the oscillation period becomes shorter, and the maximum radius decreases. This implies that by employing a solenoid to generate an appropriate magnetic field, we can effectively suppress the space charge effect of the beam to the desired extent. Stronger magnetic fields yield better results in terms of beam control. Developing an approximation method for such a process would significantly contribute to the phase space control of the beam, enabling precise manipulation of its behaviour.

Further Approximation

By simulating the beam described in Table 1 under various longitudinal magnetic fields, we can obtain the maximum radius R_m during oscillation and the corresponding distance D required to reach it, which is also half of the oscillation period. To enhance the visual representation, we use the reciprocal of the magnetic field strength $(1/B)$ as the x-axis in Figs. 4(a, b). The simulation results are depicted as a blue line. Notably, both curves appear to be smooth and nearly linear. Consequently, we can approximate these curves using polynomials to provide a simplified representation.

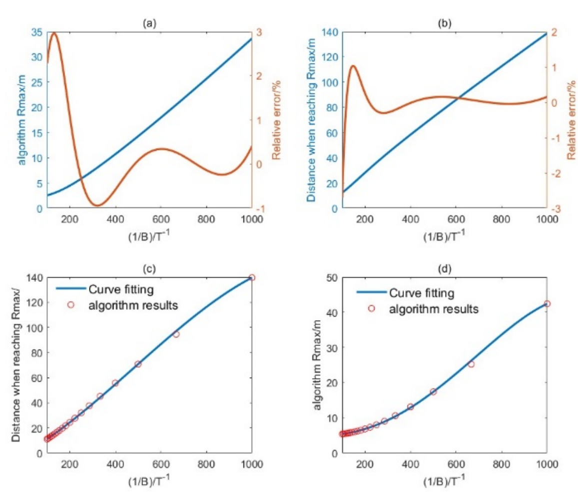

Figure 4: Fitting of the oscillation.

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It's obvious that when B is zero, the R_m will be ∞; when B is $+\infty$, the R_m will be R_0, which is the initial radius of the beam. Thus, we can get the fitting curve using the expression below:

$$R_m = \sum_{i=1}^{4} m_i B^{i-5} + R_0 \tag{3}$$

Considering that for large values of B, the corresponding distance D becomes negligible, we can simplify the approximation of D using a third-order polynomial, whose expression is:

$$D = \sum_{i=1}^{4} n_i B^{i-4} \tag{4}$$

The relative errors of the fitting process are illustrated as orange lines in Figs. 4(a, b), with all relative errors falling below 3%. This suggests that the approximation provided by Eq. (3) and Eq. (4) accurately captures the behaviour of the beam. In order to further validate the effectiveness of these equations, we select another set of initial parameters for a test beam, as presented in Table 2. By analysing the behaviour of the test beam and comparing it with the approximation derived from Eq. (3) and Eq. (4), we can assess the validity and generalizability of the proposed equations.

Table 2: Initial Parameters of the Test Beam

Parameters	**Value**
Gamma	20
Initial radius/mm	5.0
Initial length/mm	50
Transport distance/m	50
Charge/nC	-1.0

To validate the form of Eqs. (3, 4), we employ the algorithm with four different magnetic field strengths (B) to obtain four sets of results for the test beam. Subsequently, we solve for the parameters in Eqs. (3, 4) and compare the newly obtained curve fitting with the simulation results of the test beam. The comparison results are presented in Figs. 4(c, d). Remarkably, despite using only four data points for approximation, the curves fit closely with the algorithm results, thereby confirming the validity of the proposed equations.

CONCLUSION

In summary, when an external longitudinal magnetic field is introduced, the expansion of the beam under the influence of space charge is mitigated through beam rotation. Additionally, the beam's radius exhibits periodic oscillations as the transport distance increases. By employing a fast approximation algorithm, we are able to accurately fit the oscillation parameters using polynomials of the reciprocal of the magnetic field. These results provide valuable insights for controlling the phase space of the beam, enabling precise manipulation of its behaviour and characteristics.

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