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BEAM-BEAM INTERACTION WITH LONGITUDINAL IMPEDANCE AND ITS APPLICATION IN TMCI STUDY

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Abstract

Simulations have showed a novel coherent head-tail instability induced by beam-beam interaction with a large Piwinski angle. The localized cross-wake force has been introduced to explain the instability. The longitudinal impedance would cause coherent and incoherent synchrotron tune shift and distort the particle's trajectories in longitudinal phase space. Further beam-beam simulation revealed that the longitudinal impedance has strong impacts on the beam stability, squeezing the horizontal stable tune area seriously. The instability has become an important issue during the designs of CEPC and FCC-ee. In this paper, we develop a transverse mode coupling analysis method that could be used to study beam-beam instability with and without longitudinal impedance. This method can also be applied in synchrotron light sources to study transverse mode coupling instability (TMCI) with longitudinal impedance and harmonic cavity. Some preliminary results at Shenzhen Innovation Light Source (SILF) are also shown.

INTRODUCTION

Beam-beam interaction with a crossing angle has been studied for many years. Usually, it is believed that the horizontal oscillation of colliding bunch would be very stable. However, during the study of FCC-ee, the simulations [1] showed that there exists a coherent head-tail instability (X-Z instability) in collision with a large Piwinski angle. The "cross-wake force" induced by beam-beam interaction has been introduced to successfully explain this newfound instability [2, 3].

The stability of horizontal motion is sensitive to the longitudinal dynamics. The longitudinal impedance would modify the beam distribution, distort the longitudinal phase space trajectory, and produce incoherent synchrotron tune shift. Strong-strong simulation [4] showed that the stable tune area would be shifted, and the width would be squeezed when the longitudinal impedance is included in the simulation. It is interesting to study how the longitudinal impedance influences the X-Z instability analytically.

The ordinary transverse mode coupling instability (TMCI) theory [5] is derived as a perturbed Vlasov equation. In this theory, the transverse impedance, a perturbation source, represents the averaged wake force around the circumference of the ring. The TMCI is based on the solution of Sacherer's integral equation, only a few analytic solutions are known for some specific beam distributions so far. Some transverse mode coupling analytical methods have been developed to treat the localized wake force [2, 3]. However, the distortion of longitudinal phase space trajectory and the incoherent synchrotron tune shift were not considered in these papers. In this paper, we will develop a new transverse mode coupling method where the effects of longitudinal phase space trajectory distortion and incoherent synchrotron tune shift induced by longitudinal impedance could be considered.

LONGITUDINAL MOTION WITH WAKEFIELD

We use $s = z + v_0 t$ with s the longitudinal Serret-Frenet coordinate, representing the arc length measured along the closed orbit from an initial point, $v_0 \approx c$ the synchronous velocity and t clock time. z is the longitudinal distance from the synchronous particle and $z > 0$ is the bunch head. In the following, we will use s as the timelike variable and z as the longitudinal coordinate.

As the particle moves along the beamline, the head of the bunch will act as a source of an electromagnetic field that kicks the tail. In one revolution, the relative longitudinal momentum kick $\Delta\delta(z)$ received by a particle at z can be expressed by a wake function [5],

$$\Delta\delta(z) = -\frac{N_0 r_e}{\gamma}\int_{-\infty}^{\infty} W_z(z-z')\,\rho(z')\,dz'. \tag{1}$$

$W_z(z)$ is the ordinary longitudinal wake function with the property $W_z(z) = 0\ (z > 0)$. N_0 represents the single bunch population, r_e is the classical radius of the electron, γ is the relativistic factor and $\rho(z)$ is normalized line density.

Including the longitudinal wakefield, the Hamiltonian of the particle then reads,

$$\begin{aligned} -H = &\frac{\eta_p}{2}\delta^2 + \frac{\mu_z^2}{2\eta_p L^2} z^2 \\ &- \frac{1}{L}\frac{N_0 r_e}{\gamma}\int_0^z dz''\int_{-\infty}^{\infty} dz' W_z(z''-z')\,\rho(z') \end{aligned} \tag{2}$$

where L represents the circumference of the ring, ν_s is the synchrotron tune, $\mu_z = 2\pi\nu_s$, η_p is the slippage factor.

For electron machine, due to the synchrotron radiation, the stationary distribution should have a Gaussian distribution with the RMS value σ_δ in δ,

$$\psi(z,\delta) = \frac{1}{\sqrt{2\pi}\sigma_\delta}\exp\left(-\frac{\delta^2}{2\sigma_\delta^2}\right)\rho(z). \tag{3}$$

$\rho(z)$ cam be obtained by solving Haissinski equation.

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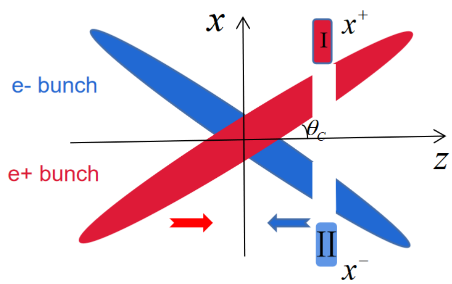

Figure 1: Beam-beam interaction with a crossing angle in Lorentz boost frame.

CROSS-WAKE FORCE INDUCED BY BEAM-BEAM INTERACTION

Considering electron (e^-) and positron (e^+) bunches colliding with half crossing angle θ_c in the $x-z$ plane, the two bunches head on each other tilted horizontally by θ_c in the Lorentz boost frame shown in Fig. 1. As the collision proceeds from the head of the bunch to the tail, the perturbed momentum kick due to horizontal betatron oscillation experienced by a $e^{\mp}$ particle at z is expressed as follows [3],

$$\Delta p_x^{(\mp)}(z) = -\int_{-\infty}^{\infty} W_x^{(\mp)}(z-z')\,\rho_x^{(\pm)}(z')\,dz' \tag{4}$$

where $\rho_x(z) = \rho(z)\cdot x(z)$ is the dipole moment of bunch, and $W_x^{(\mp)}(z)$ is the cross-wake function [2,3] for $e^{\mp}$ beams induced by beam-beam interaction.

For symmetric collider, we assume the colliding bunches have the same parameters: $N_0^+\gamma^+ = N_0^-\gamma^-$, $\sigma_{xz}^+ = \sigma_{xz}^-$, $\nu_{xz}^+ = \nu_{xz}^-$. The stability of colliding bunches can be studied separately for the σ mode $\rho_x^{(+)}(z) = \rho_x^{(-)}(z)$ and π mode $\rho_x^{(+)}(z) = -\rho_x^{(-)}(z)$. The momentum kick in Eq. (1) is reduced to a normal wake force for single bunch

$$\Delta p_x(z) = \mp\int_{-\infty}^{\infty} W_x(z-z')\,\rho_x(z')\,dz, \tag{5}$$

where the "−" and "+" signs represent σ and π modes, respectively.

TRANSVERSE MODE COUPLING THEORY WITH LONGITUDINAL IMPEDANCE

We use the normalized coordinates, where x and p_x are normalized by

$$x/\sqrt{\beta_x} \to x, \quad p_x\sqrt{\beta_x} \to p_x. \tag{6}$$

Since the dipole amplitudes $x(J,\phi), p_x(J,\phi)$ are periodic functions of ϕ with period 2π in the longitudinal phase space, we expand them as Fourier series,

$$x(J,\phi) = \sum_{l=-\infty}^{\infty} x_l(J)e^{il\phi}, \quad p_x(J,\phi) = \sum_{l=-\infty}^{\infty} p_l(J)e^{il\phi}. \tag{7}$$

In the arc section, the synchro-betatron motion for the vector $(x_l(J), p_l(J))$ is described by the matrix,

$$M_0 = e^{-2\pi il\nu_s(J)}\begin{pmatrix} \cos\mu_x & \sin\mu_x \\ -\sin\mu_x & \cos\mu_x \end{pmatrix}. \tag{8}$$

Note that the synchrotron tune $\nu_s(J)$ is a function of J.

At IP, the change of dipole moment, which is induced by cross-wake force, can be expressed as:

$$\Delta p_x(J,\phi) = \mp\beta_x\int W_x(z-z')\,x(J',\phi')\,\rho(z')\,dz'. \tag{9}$$

Using $\rho(z') = \int\psi(J',\phi')\,d\delta'$, we can rewrite the equation

$$\Delta p_x(J,\phi) = \mp\beta_x\int W_x(z-z')\,x(J',\phi')\,\psi(J')\,dJ'd\phi', \tag{10}$$

Substituting the expansions in Eq. (7) into Eq. (10), we obtain the momentum change for each azimuthal mode

$$\Delta p_l(J) = \mp\frac{\beta_x}{2\pi}\sum_{l'}\int dJ' W_{ll'}(J,J')\,\psi(J')\,x_{l'}(J'), \tag{11}$$

where

$$W_{ll'}(J,J') = \iint d\phi d\phi' e^{-il\phi+il'\phi'}W_x(z-z'). \tag{12}$$

Next we truncate l at $\pm l_{max}$, and discretize J at $J_1, J_2, ..., J_{n_J}$. The momentum kick of Eq. (11) is converted to

$$\begin{aligned}\Delta p_l(J_i) &= \mp\frac{\beta_x}{2\pi}\sum_{l'}\sum_{i'}\Delta J_{i'}W_{ll'}(J_i,J_{i'})\,\psi(J_{i'})\,x_{l'}(J_{i'}) \\ &\equiv \beta_x M_{lil'i'}x_{l'}(J_{i'}).\end{aligned} \tag{13}$$

The transformation at IP, therefore, can be written in a more condensed matrix form,

$$M_W = \begin{pmatrix} 1 & 0 \\ \beta_x M_{lil'i'} & 1 \end{pmatrix}. \tag{14}$$

Finally, the stability of the colliding beams is determined by the eigenvalues $(\lambda's)$ of the revolution matrix M_0M_W.

APPLICATIONS

We use the CEPC-Z mode parameters [6] to study beam-beam interaction with and without longitudinal impedance. Figure 2 shows growth rate versus horizontal tune without longitudinal impedance obtained by our action discretization method and the conventional raidal mode expansion method [5]. The two methods agrees well. We can see there are stable tune aeras for both σ and π mode. Figure 3 shows the growth rate for various horizontal tunes where the longitudinal impedance is included. Comparing to the results without impedance shown in Fig. 2, the gap $\Delta\nu$ between two neighboring peaks is reduced from 0.014 to 0.011. Besides the change of gap $\Delta\nu$, the once-stable working tune has turned unstable for both σ and π modes when we consider the influence of impedance.

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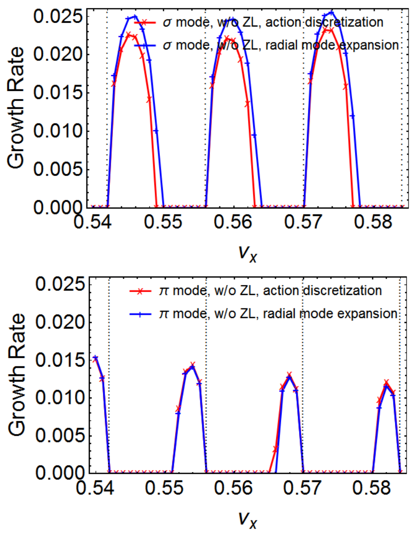

Figure 2: Growth rate vs. horizontal tune without longitudinal impedance (ZL). The upper and lower plots show the results of σ mode and π mode, respectively. The vertical lines are synchro-betatron sidebands $\nu_x = 0.5 + n\nu_s$, $\nu_s = 0.014$.

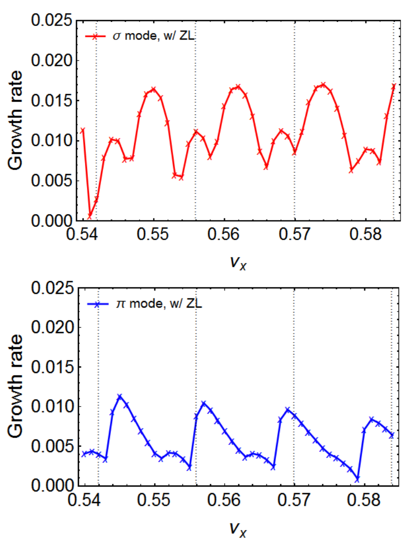

Figure 3: Growth rate vs. horizontal tune with longitudinal impedance (ZL). The red and blue points represent the σ and π modes, respectively.

The action discretization method can also be used to study TMCI with the replacement of cross wake force $W_x(z)$ in Eq. (5) by the ordinary beam-environment coupling wake function. Here we use SILF parameters [7] without harmonic cavity to study TMCI. As shown in Fig. 4, with the longitudinal impedance, the eigentunes of each azimuthal mode as a spread, and this spread becomes larger as the beam intensity increases. This spread makes the growth rate appear at very low beam intensity.

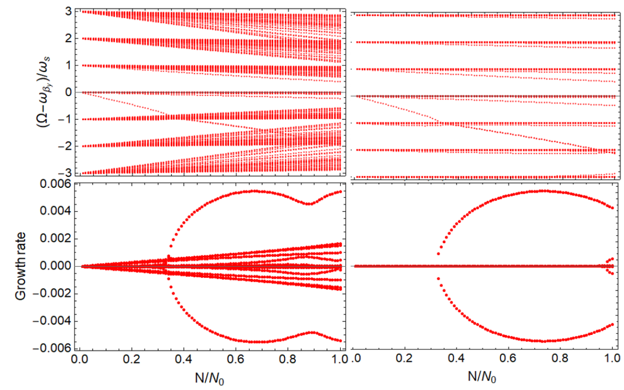

Figure 4: Eigentune and growth rate versus normalized beam intensity with (left) and without (right) longitudinal impedance.

CONCLUSIONS

The beam-beam coherent head-tail instability in collision with a large crossing angle is strongly dependent on the longitudinal beam dynamics. In the absence of longitudinal impedance, distinct stable regions with horizontal working tunes, separated by ν_s, can be observed. However, when considering the longitudinal impedance, these stable regions experience a considerable reduction. The conventional TMCI appears a growth rate at a relative low beam intensity when the longitudinal impedance is included. However we should keep in mind, in the above analysis, the nonlinear part of the force and other damping mechanisms are not included.

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