FOUR-BEAM COMPENSATION WITH TWO BEAMS

N. A. Vinokurov
Budker Institute of Nuclear Physics SB RAS
Novosibirsk, Russia

Outline

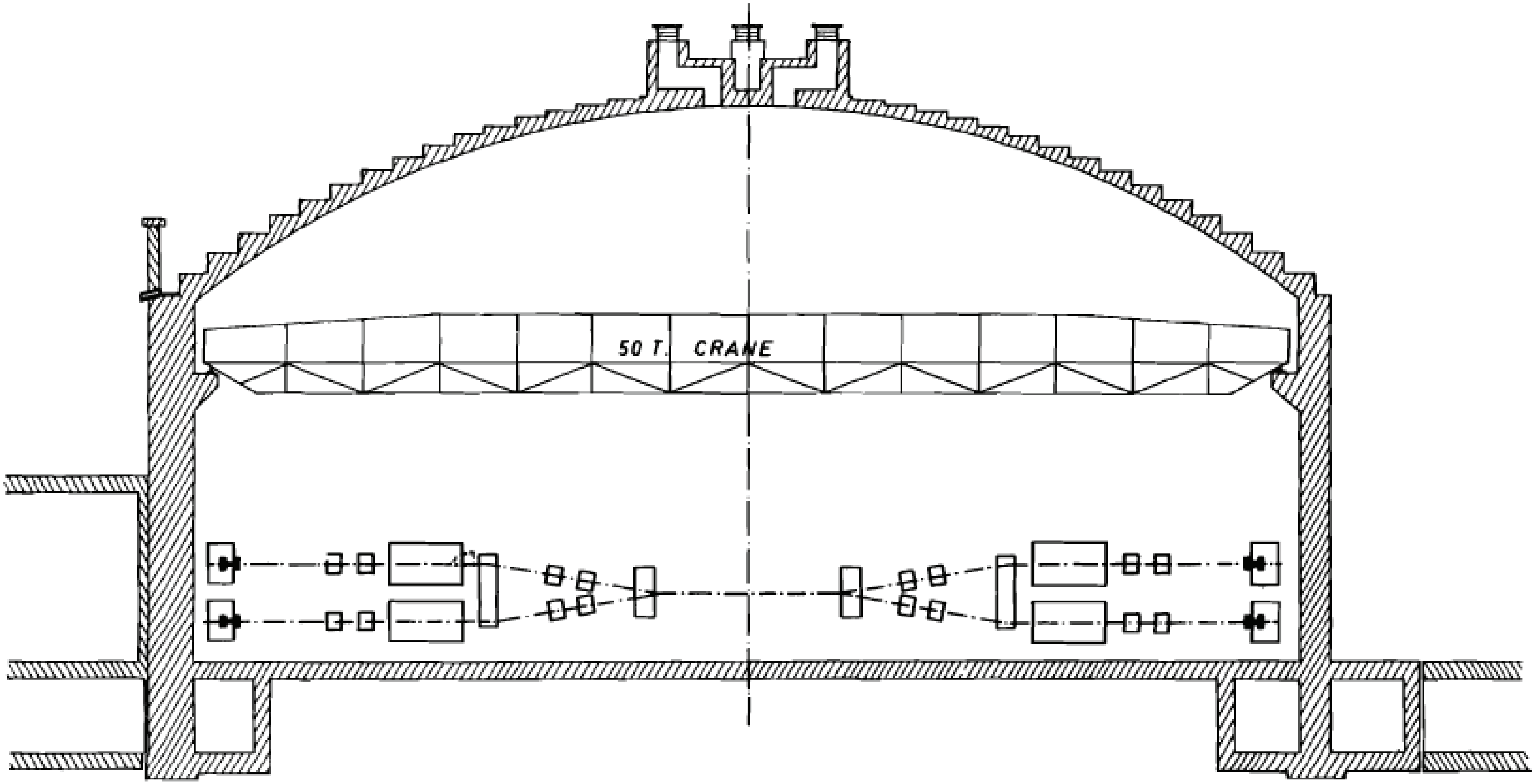
- Introduction
- Conventional scheme
- “Figure eight” collider
- Conclusion

The compensation of non-linear focusing in the storage ring colliders by the opposite-charge beam, circulating in other storage ring, was proposed and tested many years ago.

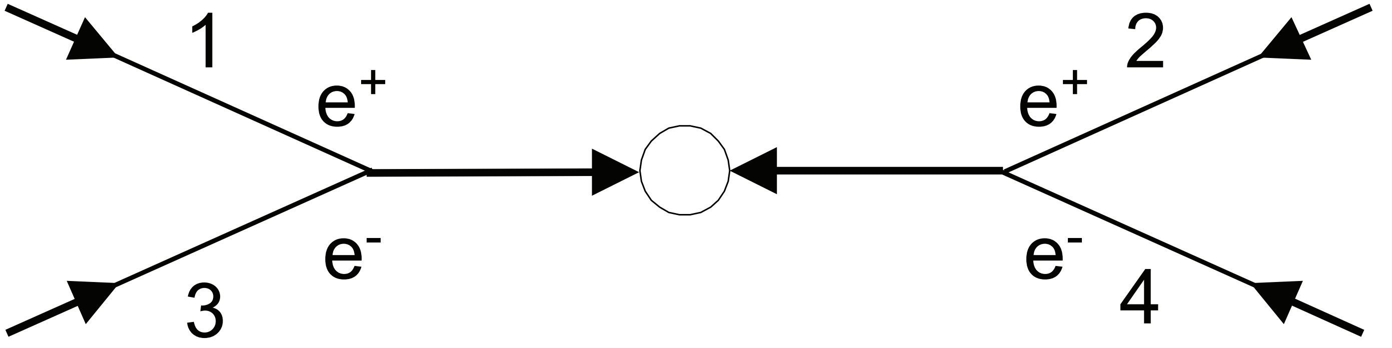
Ya. S. Derbenev had shown (1972) first that the scheme suffers from the tune shifts of coherent betatron oscillations, which shift betatron frequencies to nearest integer or half-integer resonance. In this paper we will revisit the stability condition at the simple model of rigid short bunches and discuss other configurations of colliders with beam compensation.

The Orasy compensated colliding beam ring D. C. I.

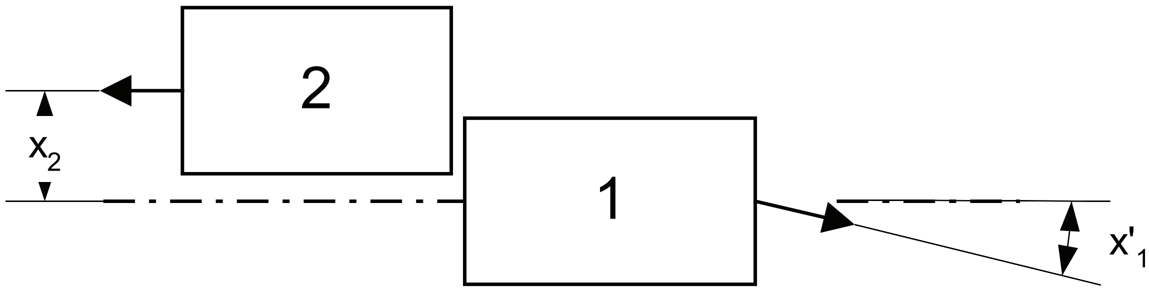
(France, 1972)



Scheme of the four-beam collision



Displaced bunch 2 kicks bunch 1.

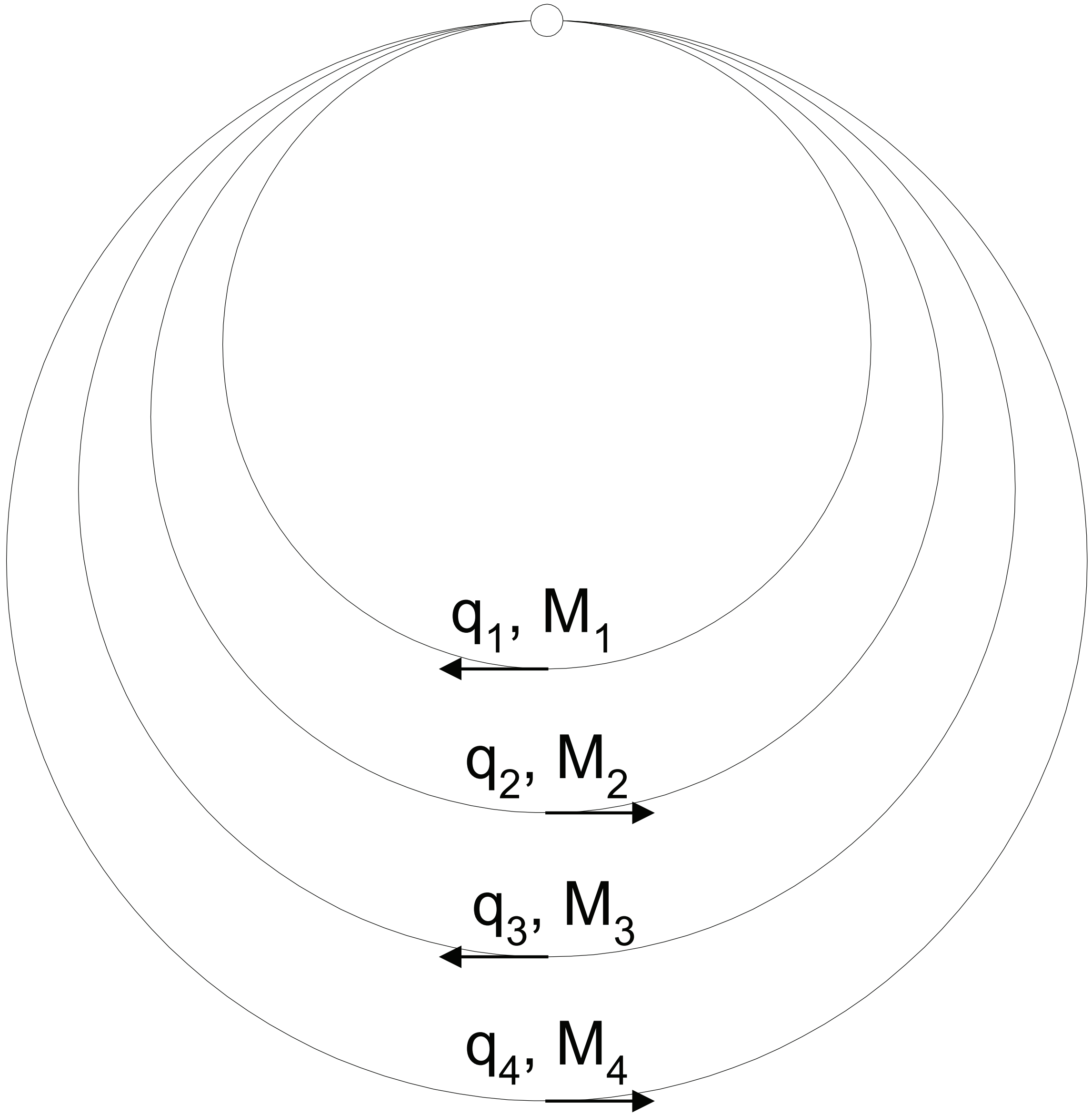


$$\beta\gamma x_1' = -d x_2$$

d is the interaction parameter (optical strength of the beam field focusing, multiplied by $\beta\gamma$).

CONVENTIONAL SCHEME

Scheme of the four-ring collider



$2(q_1 + q_2 + q_3 + q_4) \times 2(q_1 + q_2 + q_3 + q_4)$ matrix of collision

$$
S = \begin{pmatrix}
E & & & & C & & & & & & & & -C & & & \\
& E & & & & & & & & & & & & & & \\
& & \ddots & & & & & & & & & & & & & \\
& & & E & & & & & & & & & & & & \\
C & & & & E & & & & -C & & & & & & & \\
& & & & & E & & & & & & & & & & \\
& & & & & & \ddots & & & & & & & & & \\
& & & & & & & E & & & & & & & & \\
& & & & -C & & & & E & & & & C & & & \\
& & & & & & & & & E & & & & & & \\
& & & & & & & & & & \ddots & & & & & \\
& & & & & & & & & & & E & & & & \\
-C & & & & & & & & C & & & & E & & & \\
& & & & & & & & & & & & & E & & \\
& & & & & & & & & & & & & & \ddots & \\
& & & & & & & & & & & & & & & E
\end{pmatrix}
$$

$$
E = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}
$$

$$
C = \begin{pmatrix} 0 & 0 \\ -d & 0 \end{pmatrix}
$$

Matrix of the single turn of bunches after the collision

$$M = \begin{pmatrix} M_1 & & & & & & & & & & & & & & & \\ & E & & & & & & & & & & & & & & \\ & & \ddots & & & & & & & & & & & & & \\ & & & E & & & & & & & & & & & & \\ & & & & M_2 & & & & & & & & & & & \\ & & & & & E & & & & & & & & & & \\ & & & & & & \ddots & & & & & & & & & \\ & & & & & & & E & & & & & & & & \\ & & & & & & & & M_3 & & & & & & & \\ & & & & & & & & & E & & & & & & \\ & & & & & & & & & & \ddots & & & & & \\ & & & & & & & & & & & E & & & & \\ & & & & & & & & & & & & M_4 & & & \\ & & & & & & & & & & & & & E & & \\ & & & & & & & & & & & & & & \ddots & \\ & & & & & & & & & & & & & & & E \end{pmatrix}$$

Matrix of the bunches transposition in one RF period

$$
T = \left(\begin{array}{cccccccccccccccccccc}
0 & E \\
 & \ddots & \ddots \\
 & & \cdot & \cdot \\
 & & & 0 & E \\
E & & & & 0 \\
 & & & & & 0 & E \\
 & & & & & & \cdot & \cdot \\
 & & & & & & & \cdot & \cdot \\
 & & & & & & & & 0 & E \\
 & & & & & E & & & & 0 \\
 & & & & & & & & & & 0 & E \\
 & & & & & & & & & & & \cdot & \cdot \\
 & & & & & & & & & & & & \cdot & \cdot \\
 & & & & & & & & & & & & & 0 & E \\
 & & & & & & & & & & E & & & & 0 \\
 & & & & & & & & & & & & & & & 0 & E \\
 & & & & & & & & & & & & & & & & \cdot & \cdot \\
 & & & & & & & & & & & & & & & & & \cdot & \cdot \\
 & & & & & & & & & & & & & & & & & & 0 & E \\
 & & & & & & & & & & & & & & & E & & & & 0
\end{array}\right)
$$

Eigenvalues λ can be found from the characteristic equation

$$\left| TMS - \lambda E \right| = 0$$

$$0 = \left| TM - \lambda S^{-1} \right| =$$

$$\begin{vmatrix}
-\lambda E & E & & & \lambda C & & & & & & & & -\lambda C & & & \\
 & -\lambda E & E & & & & & & & & & & & & & \\
 & & \ddots & E & & & & & & & & & & & & \\
M_1 & & & -\lambda E & & & & & & & & & & & & \\
\lambda C & & & & -\lambda E & E & & & -\lambda C & & & & & & & \\
 & & & & & -\lambda E & E & & & & & & & & & \\
 & & & & & & \ddots & E & & & & & & & & \\
 & & & & M_2 & & & -\lambda E & & & & & & & & \\
 & & & & -\lambda C & & & & -\lambda E & E & & & \lambda C & & & \\
 & & & & & & & & & -\lambda E & E & & & & & \\
 & & & & & & & & & & \ddots & E & & & & \\
 & & & & & & & & M_3 & & & -\lambda E & & & & \\
-\lambda C & & & & & & & & \lambda C & & & & -\lambda E & E & & \\
 & & & & & & & & & & & & & -\lambda E & E & \\
 & & & & & & & & & & & & & & \ddots & E \\
 & & & & & & & & & & & & M_4 & & & -\lambda E
\end{vmatrix}$$

Simplifying the determinant, one can reduce this equation to

$$0 = \begin{vmatrix} M_1 - \lambda^{q_1} E & \lambda^{q_1} C & 0 & -\lambda^{q_1} C \\ \lambda^{q_2} C & M_2 - \lambda^{q_2} E & -\lambda^{q_2} C & 0 \\ 0 & -\lambda^{q_3} C & M_3 - \lambda^{q_3} E & \lambda^{q_3} C \\ -\lambda^{q_4} C & 0 & \lambda^{q_4} C & M_4 - \lambda^{q_4} E \end{vmatrix} =$$

$$\begin{vmatrix} M_1 - \lambda^{q_1} E & \lambda^{q_1} C & 0 & 0 \\ 0 & M_2 - \lambda^{q_2} E & -\lambda^{q_2} C & M_2 - \lambda^{q_2} E \\ M_3 - \lambda^{q_3} E & -\lambda^{q_3} C & M_3 - \lambda^{q_3} E & 0 \\ 0 & 0 & \lambda^{q_4} C & M_4 - \lambda^{q_4} E \end{vmatrix}$$

The matrix is symplectic, therefore, the characteristic polinom is reciprocal:

$$\frac{1}{d^2} =$$

$$\left[\frac{(M_1)_{12}}{\lambda^{q_1} + \lambda^{-q_1} - \mathrm{Sp}\, M_1} + \frac{(M_3)_{12}}{\lambda^{q_3} + \lambda^{-q_3} - \mathrm{Sp}\, M_3}\right]\left[\frac{(M_2)_{12}}{\lambda^{q_2} + \lambda^{-q_2} - \mathrm{Sp}\, M_2} + \frac{(M_4)_{12}}{\lambda^{q_4} + \lambda^{-q_4} - \mathrm{Sp}\, M_2}\right]$$

$$\lambda = e^{i\varphi} \qquad \mathrm{Sp}\, M_i = 2\cos\mu_i$$

$$\frac{4}{d^2} =$$

$$\left[\frac{(M_1)_{12}}{\cos(q_1\varphi) - \cos\mu_1} + \frac{(M_3)_{12}}{\cos(q_3\varphi) - \cos\mu_3}\right]\left[\frac{(M_2)_{12}}{\cos(q_2\varphi) - \cos\mu_2} + \frac{(M_4)_{12}}{\cos(q_4\varphi) - \cos\mu_4}\right]$$

In the simplest but not the best case of two equal rings $q_1 = q_2 = q_3 = q_4 = \mu/\varphi$ with minima of beta functions in the meeting point, one obtains a simple result

$$\cos\mu = \cos\mu_1 \pm d\beta \sin\mu_1$$

and corresponding stability condition $\left|\cos\mu\right| < 1$ **,**

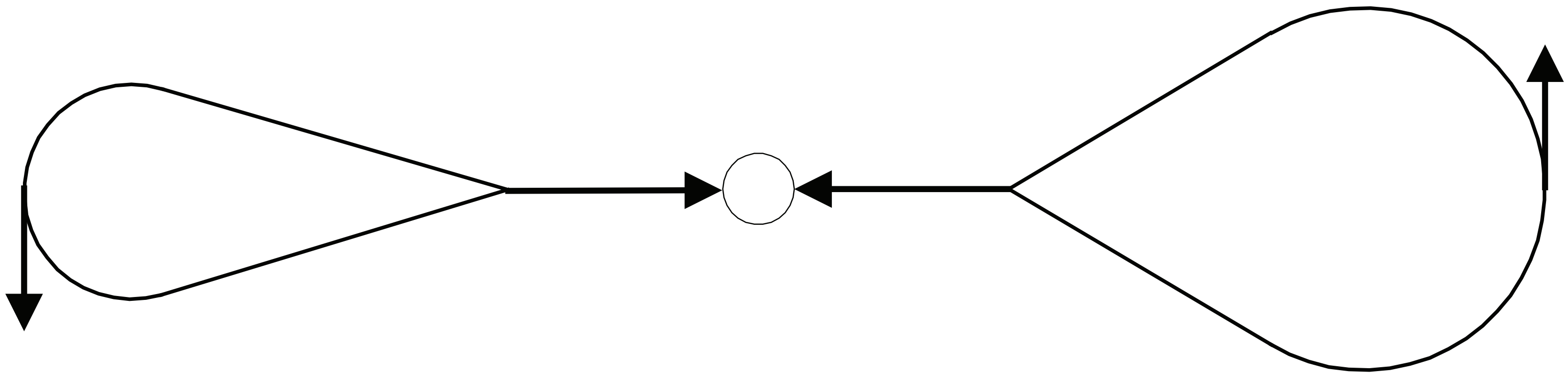
expressed as limitation for the incoherent tune shift ξ,

$$\left|\xi\right| = \frac{\left|d\beta\right|}{4\pi} < \frac{1-\left|\cos\mu_1\right|}{4\pi\left|\sin\mu_1\right|} =$$

$$\frac{1}{4\pi}\min\left(\left|\tan\frac{\mu_1}{2}\right|, \left|\tan\frac{\mu_1}{2}\right|^{-1}\right) \le \frac{1}{4\pi}$$

«FIGURE-8» COLLIDER

Consider first the scheme of electron-electron collider



All electrons are moving near the same equilibrium orbit in the same direction. Therefore, one can say that it is single-beam (but, certainly, multi-bunch) collider.

Injecting positrons in such a collider, one can obtain beam-beam compensation.

This scheme was considered in 1991 for the project of the Novosibirsk φ - factory.

STATUS OF THE NOVOSIBIRSK PHI-FACTORY PROJECT.

L.M.Barkov, S.A.Belomestnykh, V.V.Danilov, N.S.Dikansky, A.N.Filippov, B.I.Grishanov, P.M.Ivanov, I.A.Koop, O.B.Malyshev, B.L.Militsyn, S.S.Nagaitsev, I.N.Nesterenko, E.A.Perevedentsev, D.V.Pestrikov, L.M.Schegolev, I.K.Sedlyarov, Yu.M.Shatunov, E.A.Simonov, A.N.Skrinsky, I.B.Vasserman, V.G.Vescherevich, P.D.Vobly ,E.I. Zinin
Institute of Nuclear Physics, Novosibirsk, USSR

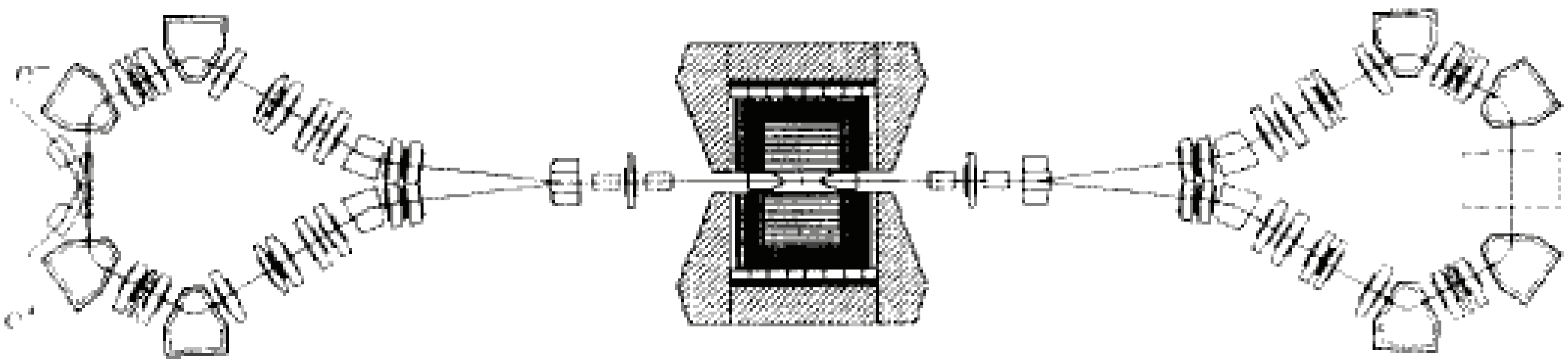
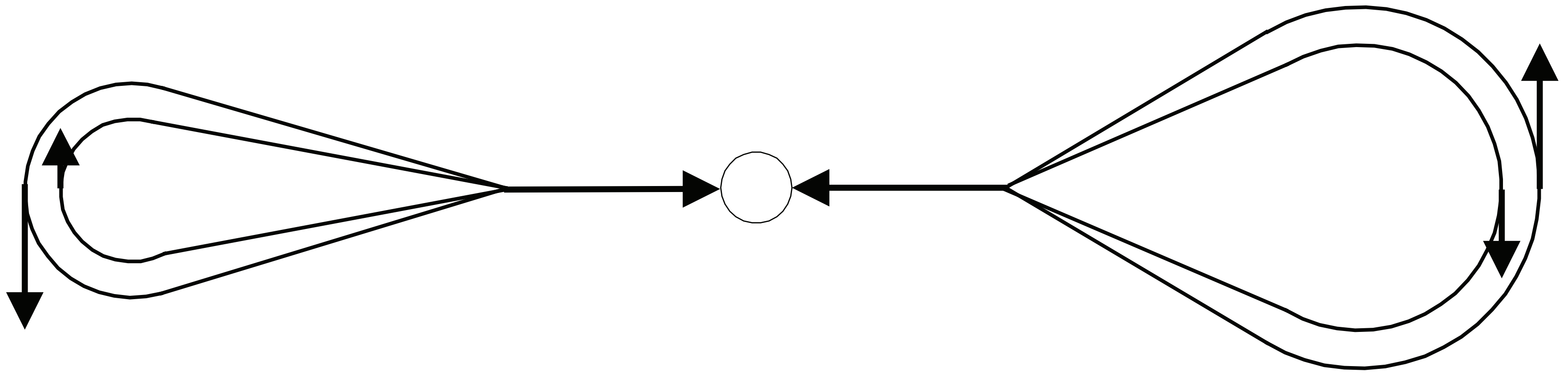


Fig.1. Layout of the Novosibirsk φ-factory.

In more general case of different energies of electrons and positrons, their orbits will be separated outside the collision straight section



Only the matrix of the bunches transposition in one RF period is different:

$$
T_2 = \begin{pmatrix}
0 & E & & & & & & & & \\
 & \ddots & \ddots & & & & & & & \\
 & & \ddots & \ddots & & & & & & \\
 & & & 0 & E & & & & & \\
E & & & & 0 & & & & & \\
 & & & & & 0 & E & & & \\
 & & & & & & \ddots & \ddots & & \\
 & & & & & & & \ddots & \ddots & \\
 & & & & & & & & 0 & E \\
 & & & & & E & & & & 0
\end{pmatrix}
$$

Eigenvalues λ can be found from the characteristic equation

$$0 = \begin{vmatrix} -\lambda^{q_1} E & M_2 + \lambda^{q_1} C & 0 & -\lambda^{q_1} C \\ M_1 + \lambda^{q_2} C & -\lambda^{q_2} E & -\lambda^{q_2} C & 0 \\ 0 & -\lambda^{q_3} C & -\lambda^{q_3} E & M_4 + \lambda^{q_3} C \\ -\lambda^{q_4} C & 0 & M_3 + \lambda^{q_4} C & -\lambda^{q_4} E \end{vmatrix}$$

$$\lambda = e^{i\varphi}$$

$$0 = 1 + 2d\left[\frac{(M_1)_{12}\cos(q_1\varphi) + (M_2)_{12}\cos(q_2\varphi)}{2\cos\left[(q_1+q_2)\varphi\right] - \mathrm{Sp}(M_1M_2)} + \frac{(M_3)_{12}\cos(q_3\varphi) + (M_4)_{12}\cos(q_4\varphi)}{2\cos\left[(q_3+q_4)\varphi\right] - \mathrm{Sp}(M_3M_4)}\right] +$$

$$d^2\left[\begin{array}{l}\dfrac{(M_1)_{12}(M_2)_{12}}{2\cos\left[(q_1+q_2)\varphi\right] - \mathrm{Sp}(M_1M_2)} + \dfrac{(M_3)_{12}(M_4)_{12}}{2\cos\left[(q_3+q_4)\varphi\right] - \mathrm{Sp}(M_3M_4)} - \\ \dfrac{(M_1M_2)_{12}(M_4M_3)_{12} + (M_2M_1)_{12}(M_3M_4)_{12}}{\left\{2\cos\left[(q_1+q_2)\varphi\right] - \mathrm{Sp}(M_1M_2)\right\}\left\{2\cos\left[(q_3+q_4)\varphi\right] - \mathrm{Sp}(M_3M_4)\right\}} + \\ 2\,\mathrm{Re}\dfrac{\left[e^{iq_1\varphi}(M_1)_{12} + e^{-iq_2\varphi}(M_2)_{12}\right]\left[e^{iq_4\varphi}(M_4)_{12} + e^{-iq_3\varphi}(M_3)_{12}\right]}{\left\{2\cos\left[(q_1+q_2)\varphi\right] - \mathrm{Sp}(M_1M_2)\right\}\left\{2\cos\left[(q_3+q_4)\varphi\right] - \mathrm{Sp}(M_3M_4)\right\}}\end{array}\right]$$

In the case of equal energies $q_1 = q_3$, $q_2 = q_4$, and minima of beta functions at collision point

$$M_1 = M_3 \qquad M_2 = M_4 \qquad \left(M_1\right)_{12} = \beta \sin \mu_1 \quad \left(M_2\right)_{12} = \beta \sin \mu_2$$

$$\mathrm{Sp}\left(M_1 M_2\right) = 2\cos \mu_0$$

and the characteristic equation is

$$\cos \mu - \cos \mu_0 + 2d\beta \left(\sin \mu_1 \cos \frac{q_1 \mu}{q_1 + q_2} + \sin \mu_2 \cos \frac{q_1 \mu}{q_1 + q_2} \right) +$$

$$2d^2 \beta^2 \sin \mu_1 \sin \mu_2 = 0$$

where $\mu = (q_1 + q_2)\varphi$.

The corresponding stability conditions is

$$\sqrt{(d\beta)^2\left(\frac{\sin\mu_1-\sin\mu_2}{2}\right)^2+\cos^2\frac{\mu_0}{2}}+d\beta\left|\frac{\sin\mu_1+\sin\mu_2}{2}\right|<1$$

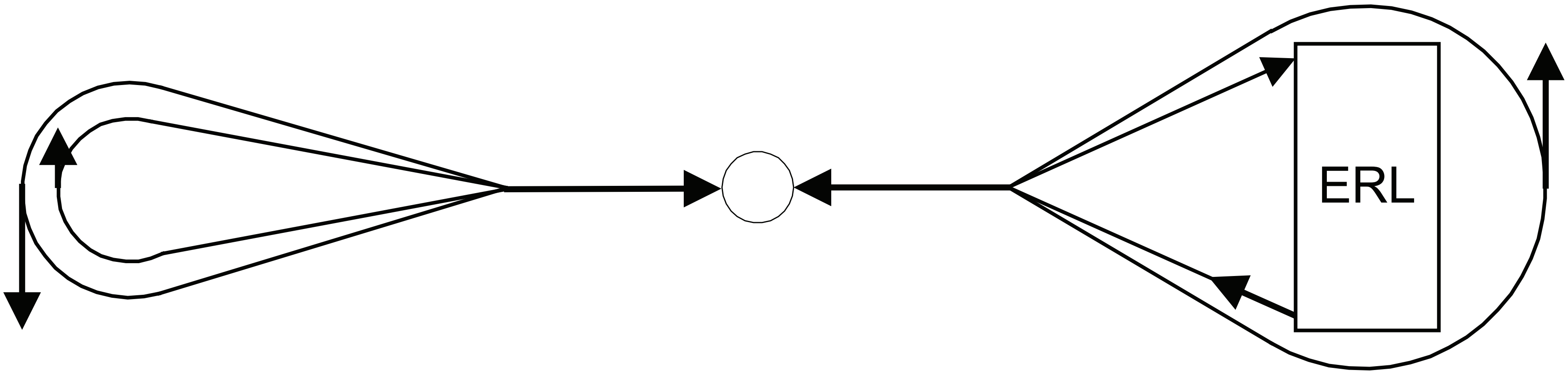
For equal loops $\mu_1 = \mu_2$,

$$\cos\frac{\mu}{2}=-d\beta\sin\frac{\mu_0}{2}\pm\cos\frac{\mu_0}{2}$$

and the stability condition is $\left|\cos\frac{\mu}{2}\right|<1$**, or**

$$\xi=\frac{d\beta}{4\pi}<\frac{1}{4\pi}\left|\tan\frac{\mu_0}{4}\right|,\frac{1}{4\pi\left|\tan\frac{\mu_0}{4}\right|}<\frac{1}{4\pi}$$

The scheme may be modified by replacing the electron storage ring with the energy recovery linac (ERL).



In this case, M_4 is the matrix of the left loop, but matrix M_3 describes the influence of decelerating beam to the accelerating one. In the simplest case, it may be zero. Then,

$$0 = 1 + 2d\,\frac{(M_1)_{12}\cos(q_1\varphi) + (M_2)_{12}\cos(q_2\varphi)}{2\cos\left[(q_1+q_2)\varphi\right] - \mathrm{Sp}(M_1M_2)} +$$

$$d^2\,\frac{(M_1)_{12}(M_2)_{12}}{2\cos\left[(q_1+q_2)\varphi\right] - \mathrm{Sp}(M_1M_2)} +$$

$$de^{-iq_3\varphi}(M_4)_{12}\left[1 + d\,\frac{e^{iq_1\varphi}(M_1)_{12} + e^{-iq_2\varphi}(M_2)_{12}}{2\cos\left[(q_1+q_2)\varphi\right] - \mathrm{Sp}(M_1M_2)}\right]$$

The last term, which is proportional to effective length $(M_4)_{12}$ of the ERL loop, describes the influence of ERL beam on stability of the beam in storage ring. This influence is similar to the one of the feedback system with beam position monitor, amplifier and kicker.
In more general case, one can use dedicated feedback system, which provides beam stability with proper M_3.

CONCLUSION

In this paper I used the simple model for instability for compensated beams to demonstrate that not all interesting schemes of compensated colliders are considered yet.

More detail analysis of solutions in different cases, including coupling, ERLs and additional feedback systems, is required.

Further investigation of such schemes will, hopefully, show their feasibility.

Thank you for your attention.