EM FIELDS IN A METAL IN AN EXTERNAL MAGNETIC FIELD AT LOW TEMPERATURS

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Abstract

Radio waves do not penetrate deep into the metal due to the high density of charge carriers in the metal. In the shorter-wave part of the spectrum, metals can be “transparent” only starting from ultraviolet, since the plasma frequencies in metals lie in the ultraviolet range. However, for a metal cooled to a low temperature and placed in an external magnetic field, the situation may change. Under these conditions, cyclotron orbits can be formed, and the relaxation time can significantly exceed the period of cyclotron motion of the charge carriers. For an electromagnetic wave with polarization normal to the induction vector of an external magnetic field, the exchange of energy with charge carriers in the metal turns out to be suppressed. Such a wave can propagate in the metal for relatively large distances (in comparison with the skin layer). In this connection, in electron storage rings at the azimuths, for example, of superconducting strong-field wigglers, with the magnetic field rise, the effect in the surface impedance change of the wiggler vacuum chamber metal can be manifested. This report is a brief review of some of the results known in plasma and solid state physics applicable to these conditions and of interest to accelerator technology.

INTRODUCTION

The electromagnetic waves can propagate in some conducting media in external magnetic fields at the frequencies much lower the plasma frequencies. This phenomenon is widely studied and used in plasma physics.

In metals under low temperatures, the relaxation time can significantly exceed the periods of cyclotron motion in external magnetic field and the cyclotron orbits can be formed. Under these conditions, the radiofrequency electromagnetic waves can propagate in metal in external magnetic field.

In this report, two theoretical approaches are used for the numerical analysis. One of them is the theoretical model of wave propagation in plasma [1] and the other one is the theoretical model of wave propagation in metal [2], [3]. The practical object of this report is the beam dynamics investigation in storage ring Siberia – 2.

The electron storage ring Siberia - 2 is 124 m in length with electron beam energies from 75 Mev up to 2.5 Gev. Beam life time is about 20 -30 hours in regular mode at the electron beam currents above 100 mA. Siberia-2 storage ring is equipped with a superconducting wiggler with magnetic field up to 7.5 Tesla.

ELECTROMAGNETIC FIELDS IN A PLASMA IN AN EXTERNAL MAGNETIC FIELDS

To describe the behaviour of electrons in metal, the simplest approximation is the model of free valent electrons. The energy spectrum of these electrons is discrete because of their motion is finite in metal volume. At zero temperature, electrons occupy in sequence lower energy levels up to the highest level – Fermi energy.

Two-component cold gas plasma can successfully illustrate some of the features and quantitative parameters of electromagnetic wave propagation in metals.

In external magnetic field, for plasma plane wave with time dependence

$$e^{i\left(\left(\vec{k}\vec{r}\right)-\omega t\right)},$$

within the linear approximation on the wave amplitudes, the equation for wave current density $\vec{j}$ and electrical field $\vec{E}$ can be written as [1]

$$\omega(\omega+i\nu)\vec{j} =$$
$$= i\frac{\omega_0^2\omega}{4\pi}\vec{E}+\omega_i\omega_e\left(\vec{j}-\vec{h}\left(\vec{j}\vec{h}\right)\right)-i\omega\vec{\omega}_e\left[\vec{j}h\right] \qquad (1)$$

In these relations, $\vec{k}$ is the wave vector, ω is the field frequency, ω_i and ω_e are the cyclotron frequencies of the ions and of the electrons, respectively, in external magnetic field.

The external magnetic field is introduced in the equation (1) by the field direction unit vector $\vec{h}$ and by the ion and electron cyclotron frequencies ω_i and ω_e determining the magnitude of the external magnetic field.

The own plasma wave magnetic field is neglected in the equation (1) because of the production of the current density $\vec{j}$ on the own magnetic field is the value of the second order of smallness.

In equation (1), energy dissipation is taken into account by the scattering effective frequency - ν.

The equation (1) describes the dynamics of charge carriers in electromagnetic fields and therefore must be completed by the Maxwell’s equation determining the field dependence on currents:

$$k^2\vec{E}-\vec{k}\left(\vec{k}\vec{E}\right)=\frac{\omega^2}{c^2}\vec{E}+i\omega\mu_0\vec{j} \qquad (2)$$

The complete set of equations (1) - (2) gives the self-consistent solution for the wave current density $\vec{j}$ and the wave electrical field $\vec{E}$.

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For plane wave $\left(\vec{k}\vec{E}\right)=0$ and for $\left|\frac{\omega^2}{c^2}\vec{E}\right|\langle\langle\left|i\omega\mu_0\vec{j}\right|$, the equation (2) can be simplified to

$$\vec{E}=i\omega\mu_0\vec{j}\,. \tag{3}$$

Substituting (3) into equation (1) gives the closed equation for current density $\vec{j}$

$$\omega(\omega+i\nu)\vec{j}=\left(\omega_i\omega_e-\frac{\omega_0^2\omega^2}{k^2c^2}\right)\vec{j}-i\omega\omega_e\left[\vec{j}\vec{h}\right]. \tag{4}$$

This equation has nonzero solution only for frequency ω and wave number k satisfying the dispersion relation

$$\frac{k^2c^2}{\omega^2}=\frac{\omega_0^2}{\omega_i\omega_e-\omega(\omega+i\nu)\mp\omega\omega_e}\,. \tag{5}$$

For massive compared to electrons ions $\omega_i\langle\langle\omega_e$ and in frequency range $\omega_i\langle\langle\omega\langle\omega_e$, in the limit $\frac{\omega_i}{\omega}\Rightarrow 0$, the dispersion equation (5) transforms into equation

$$\frac{k^2c^2}{\omega^2}=\frac{\omega_0^2}{\omega(\omega_e-\omega-i\nu)}\,. \tag{6}$$

The solution of the equation (6) can be found in the form

$$k=r+i\delta\,. \tag{7}$$

It is useful to introduce the dimensionless parameter

$$s=\frac{\omega_e-\omega}{\nu},\quad p=\sqrt{s^2+1}\,.$$

Substitution of form (7) into equation (6) gives the solutions

$$r=\sqrt{\frac{\omega\omega_0^2(p+s)}{2c^2\nu p^2}}\,, \tag{8}$$

$$\delta=\sqrt{\frac{\omega\omega_0^2}{2c^2\nu p^2(p+s)}} \tag{9}$$

Known r and δ make it possible to determine the wave length $\lambda=\frac{2\pi}{r}$ in the plasma, the depth $l=\frac{1}{\delta}$ of the field penetration into the plasma, the number $N=\frac{l}{\lambda}=\frac{r}{2\pi\delta}$ of wavelength at the attenuation length in the plasma.

At the wave frequency $\omega=2\pi\cdot10^9\,Hz$, in plasma with plasma frequency $\omega_0=2\pi\cdot8{,}56\cdot10^{14}\,Hz$ and the effective electron scattering frequency $\nu=5\cdot10^8\,\frac{1}{c}$, in the external magnetic field providing the electron cyclotron frequency $\omega_e=2\pi\cdot2{,}5\cdot10^{10}\,Hz$, the dispersion relation solutions (8)-(9) determine numerically the wave length $\lambda=1{,}72\cdot10^{-6}\,m$ in plasma, the depth $l=164{,}8\cdot10^{-6}\,m$ of the field penetration into the plasma, the number $N=\frac{l}{\lambda}=95{,}8$ of wavelength at the attenuation length in the plasma.

The considered transverse plane wave propagates in plasma along the external magnetic field. The polarization of this wave is circular.

ELECTROMAGNETIC FIELDS IN A METAL IN AN EXTERNAL MAGNETIC FIELDS

In metal, the weak influence of the crystal lattice on the particle dynamics can be taken into consideration by the introduction of the effective mass of the particle [2]. This effective mass may differ significantly from the mass of the particle in the free space.

The presence of the crystal lattice is appeared also by the appearance of the positive charge carriers – holes.

In this approximation, the electromagnetic wave penetration can be illustrated by the simple model [3] using the expression for the current density

$$\vec{j}=e\left(n_h\vec{v}_h-n_e\vec{v}_e\right) \tag{10}$$

and the particle dynamic equations

$$-i\omega\vec{v}_e\mu_e=-e\vec{E}-e[\vec{v}_e\vec{B}]\,, \tag{11}$$

$$-i\omega\vec{v}_h\mu_h=e\vec{E}+e[\vec{v}_h\vec{B}]\,. \tag{12}$$

In expressions (10)-(12), n_e and n_h are the concentrations, μ_e and μ_h are the effective masses, $\vec{v}_e$ and $\vec{v}_h$ are the velocities of the electrons and holes, respectively. The right sides of equations (11)-(12) contain the wave electrical field $\vec{E}$ and the external magnetic field $\vec{B}$. The wave magnetic field is neglected in these equations because of the equations (11)-(12) are linear in the wave amplitudes.

In a rectangular Cartesian coordinate system with the z axis along the external magnetic field $\vec{B}$, substitution of the equation set (11)-(12) solution in the expression (10) gives

$$j_x=\frac{-i\omega E_x\left(n_h\mu_h+n_e\mu_e\right)-eBE_y\left(n_h-n_e\right)}{B^2}\,, \tag{13}$$

$$j_y=\frac{-i\omega E_y\left(n_h\mu_h+n_e\mu_e\right)+eBE_x\left(n_h-n_e\right)}{B^2}\,, \tag{14}$$

$$j_z=i\frac{e^2}{\omega}\left(\frac{n_h}{\mu_h}+\frac{n_e}{\mu_e}\right)E_z\,. \tag{15}$$

In the derivation of relations (13)-(14), the situation was considered when the cyclotron frequencies ω_e and ω_h

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of charge carriers in the external magnetic field $\vec{B}$ are much higher than the wave frequency ω.

In the typical case for normal metals when the electrons concentration and the holes concentration are equal to each other, the equations (13)-(14) are simplified and reduced to the form

$$j_x = \frac{-i(\omega + i\nu)(n_h\mu_h + n_e\mu_e)}{B^2} E_x, \quad (16)$$

$$j_y = \frac{-i(\omega + i\nu)(n_h\mu_h + n_e\mu_e)}{B^2} E_y. \quad (17)$$

The metal conductivity tensor becomes diagonal.

The energy dissipation is taken into account in the equations (16)-(17). That is done by the insertion of relaxation time τ,

$$\nu = \frac{1}{\tau}.$$

For plane wave $(\vec{k}\vec{E}) = 0$ and for $\left|\omega^2 c^{-2}\vec{E}\right| \langle\langle \left|i\omega\mu_o\vec{j}\right|$,

The Maxwell's equations give

$$\vec{E} = \frac{i\omega\mu_0}{k^2}\vec{j}. \quad (18)$$

For wave penetrating along the external magnetic field, substitution of equation (18) into the equations (16)-(17) determines the dispersion relation

$$k^2 = \omega(\omega + i\nu)\frac{\omega_0^2}{c^2\omega_e^2}\frac{n_h\mu_h + n_e\mu_e}{nm}. \quad (19)$$

The dispersion relation solution can be found in the form

$$k = r + i\delta,$$

$$r = \frac{\omega}{c}\frac{\omega_0}{\omega_e}\sqrt{u^{-1}(w+u)A}, \quad (20)$$

$$\delta = \frac{\omega}{c}\frac{\omega_0}{\omega_e}\frac{\sqrt{A}}{\sqrt{u(w+u)}}, \quad (21)$$

where the dimensionless parameters

$$u = \frac{\omega}{\nu},\ w = \sqrt{u^2+1} \text{ and } A = \frac{n_h\mu_h + n_e\mu_e}{2nm}.$$

The dimensionless parameter A depends on effective masses of particles in metal.

At the wave frequency $\omega = 2\pi\cdot 10^9\,Hz$, in metal with plasma frequency $\omega_0 = 2\pi\cdot 8{,}56\cdot 10^{14}\,Hz$ and the effective electron scattering frequency $\nu = 5\cdot 10^8\frac{1}{c}$, in the external magnetic field providing the electron cyclotron frequency $\omega_e = 2\pi\cdot 2{,}5\cdot 10^{10}\,Hz$, the dispersion relation solutions (8)-(9) determine numerically the wave length $\lambda = 6{,}19\cdot 10^{-6}\frac{1}{\sqrt{A}}$ m in metal, the depth $l = 25{,}38\cdot 10^{-6}\frac{1}{\sqrt{A}}$ m of the field penetration into the metal, the number $N = \frac{l}{\lambda} = 4{,}1$ of wavelength at the attenuation length in the metal.

The considered transverse plane wave propagates in metal along the external magnetic field. The polarization of this wave is linear.

CONCLUSION

This report is a brief review of the theory background for phenomena of radio wave penetration in metals at low temperatures in external magnetic field. Two theoretical models are used for the numerical analysis. One of them is the theoretical model of wave propagation in plasma and the other one is the theoretical model of wave propagation in metal. The qualitative and numerical results obtained in these two models are similar.

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