STUDY OF THE NON-UNIFORM COUPLED SYSTEM MODEL OF CDS SECTION AND WAVEGUIDE SEGMENT BASED ON MULTIMODE APPROXIMATION

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Abstract

With the development of the new CDS cavity for the first four-section cavity of the main part of INR RAS linac replacement application of the bridge devices, similar to existing rectangular waveguide segments-based devices, is supposed. Numerical simulation of a complete cavity coupled with bridge devices requires unattainable computing resources. For this reason, the analytical representation for the coupled non-uniform system based on multimode approximation is considered. The analytical model for a system consisting of a short CDS section coupled with a rectangular waveguide segment is presented. The model is calibrated by direct numerical simulation. A generalization allowing parameters selection for the new CDS cavity sections and bridge devices coupling based on the presented model is proposed.

INTRODUCTION

The first cavity of the main part of INR linac works for proton acceleration in the range β=0.4313 – 0.4489 with acceleration gradient $E_0Tcos\varphi_s$ = 2.5 MV/m and the synchronous phase φ_s=-33°. The cavity has the aperture radius r_a= 17 mm, operating frequency f_a=991.0 MHz and the required operating regime is with RF pulse length τ=200 μs and Repetition Rate (RR) up to 100 Hz. The cavity consists of four DAW sections connected by three rectangular-cross bridge devices through slots [1]. The input of RF power is implemented through the bridge between sections 2 and 3. Each section of the cavity consists of 18 – 21 periods of the structure.

The new CDS structure was proposed for the first cavity replacement [2]. The bridge devices for the new cavity are proposed to be similar with existing system. A simplified schematic of the connection between two CDS sections is shown at Fig. 1.

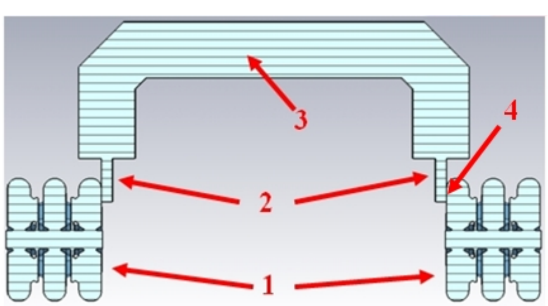

Figure 1: Schematic sketch of two CDS sections with a bridge device, 1 – cavity sections, 2 – transition waveguide, 3 – bridge device, 4 – coupling slot.

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In case of RF input through the bridge device the full cavity with bridge devices should be tuned as complete system [3]. In this case numerical simulations of the cavity with bridge devices require unattainable computing resources. Therefore the analytical model based on multimode approximation is considered [3].

ANALYTICAL MODEL

We consider the non-uniform system consisting of biperiodic structure cavity coupled with a segment of rectangular-cross waveguide (bridge) through the slot. The field distribution in the cavity could be represented as a sum of eigenmodes with frequencies $\omega^c{}_n$ [4]:

$$\overline{H}^c = \sum_n h_n^c \overline{H}_n^c, \mu_0 \int_{V_c} \overline{H}_n^c \overline{H}_n^{c*} dV = 2W_0, \quad (1)$$

where V_c is the cavity volume, W_0 is stored energy. The magnetic field in the waveguide segment could be presented similarly:

$$\overline{H}^b = \sum_k h_k^b \overline{H}_k^b, \mu_0 \int_{V_b} \overline{H}_k^b \overline{H}_k^{b*} dV = 2W_0, \quad (2)$$

$h_n{}^b$ and $h_n{}^c$ are the amplitude coefficients. The magnetic field in the bridge and cavity is excitated by the tangential electric field of the slot:

$$h_n^c = \frac{j\omega \int_{S^s} \left[\overline{E}^s \overline{H}_n^{c*}\right] d\overline{S}}{2W_0(\omega^2 - \omega_n^{c\,2})}, \quad h_k^b = \frac{j\omega \int_{S^s} \left[\overline{E}^s \overline{H}_k^{b*}\right] d\overline{S}}{2W_0(\omega^2 - \omega_k^{b\,2})} \quad (3)$$

where E^s is the slot field, dS is normal to the slot area S^s. Herewith the magnetic field on the slot radius r_s in the cavity and the bridge can be presented as:

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$$H^c(r_s) = \sum_n \frac{j\omega H_n^c(r_s) \int_{S^s} \left[\bar{E}^s \bar{H}_n^{c*}\right] dS}{\mu_0 \int_{V_c} \bar{H}_n^c \bar{H}_n^{c*} dV (\omega^2 - \omega_n^{c\,2})}$$

$$H^b(r_s) = \sum_k \frac{j\omega H_k^b(r_s) \int_{S^s} \left[\bar{E}^s \bar{H}_k^{b*}\right] dS}{\mu_0 \int_{V_b} \bar{H}_k^b \bar{H}_k^{b*} dV (\omega^2 - \omega_k^{b\,2})} \quad (4)$$

The slot is represented as a waveguide segment in cut-off mode with geometrical parameters l_s, h_s, t_s. Electric field in the slot could be presented as:

$$\bar{E}^s = \sum_m e_m^s \bar{E}_m^s, \varepsilon_0 \int_{V^s} \bar{E}_m^s \bar{E}_m^{s*} dV = 2W_0 , \quad (5)$$

where e_m^s is amplitude coefficient. The field in the slot is excited by tangential fields of the cavity and bridge:

$$e^s = \frac{-j\omega\left(\int_{S^s} \left[\bar{E}^s \bar{H}^{b*}\right] d\bar{S} + \int_{S^s} \left[\bar{E}^s \bar{H}^{c*}\right] d\bar{S}\right)}{\varepsilon_0 \int_{V^s} \bar{E}_m^s \bar{E}_m^{s*} dV (\omega^2 - \omega^{s\,2})} \quad (6)$$

The slot has smaller dimension and can be considered in single mode approximation. The first TE mode field distribution can be represented:

$$\bar{E}^s = E^s \sin\left(\frac{\pi x}{l_s}\right) \bar{y}_0, x = x(l_s) \quad (7)$$

where $x=x(l_s)$ is the coordinate along the slot. Therefore the integral of slot electric field excitation:

$$\int_{S^s} \left[\bar{E}^s \bar{H}^{c*}\right] d\bar{S} = E^s H^c(r_s) \frac{2h_s l_s}{\pi} \quad (8)$$

With the stored energy normalization $2W_0=(\varepsilon_0 E^{s2} l_s h_s t_s)/2$ one can get the electric field in the slot:

$$\bar{E}^s = \frac{-4j\omega(H^b(r_s) + H^c(r_s))}{\pi\varepsilon_0 t_s (\omega^2 - \omega_s^2)} \sin\left(\frac{\pi x}{l_s}\right) \bar{y}_0 \quad (9)$$

Introducing (9) into (3) with (4) one can get the system for h^c and h^b:

$$\begin{cases} h_n^c \left(\dfrac{\omega^2 - \omega_n^{c2}}{A_s \omega^2} - H_n^{c2} \Big|_{S^s} \right) - \left(H_n^c(r_s) \sum\limits_{i \ne n} h_i^b H_i^b(r_s) \Big|_{S^s} + H_n^c(r_s) \sum\limits_{i \ne n} h_i^c H_i^c(r_s) \Big|_{S^s} \right) = 0 \\ h_m^b \left(\dfrac{\omega^2 - \omega_m^{b2}}{A_s \omega^2} - H_m^{b2} \Big|_{S^s} \right) - \left(H_m^b(r_s) \sum\limits_{i \ne m} h_i^b H_i^b(r_s) \Big|_{S^s} + H_m^b(r_s) \sum\limits_{i \ne m} h_i^c H_i^c(r_s) \Big|_{S^s} \right) = 0 \end{cases}$$

(10), where A_s is:

$$A_s = -\frac{4 l_s h_s}{\pi^2 \varepsilon_0 t_s W_0 (\omega^2 - \omega_s^2)} \quad (11)$$

To simplify the system (10) one could introduce the coupling coefficients between i-th and j-th modes in the cavity and bridge:

$$\gamma_{(b,c)}^{(i,j)} = -A_s H_i^b(r_s) H_j^c(r_s) \Big|_{S^s} \quad (12)$$

And the coupling coefficients between modes inside the cavity and bridge separately:

$$\gamma_{(b,b)}^{(i,j)} = -A_s H_i^b(r_s) H_j^b(r_s) \Big|_{S^s} \quad (13)$$

$$\gamma_{(c,c)}^{(i,j)} = -A_s H_i^c(r_s) H_j^c(r_s) \Big|_{S^s} \quad (14)$$

Thus one could rewrite the system (10):

$$\begin{cases} h_n^c \dfrac{(\omega^2 + \gamma_{(c,c)}^{(n,n)} \omega^2 - \omega_n^{c\,2})}{\omega^2} + \sum\limits_{i \ne n} \gamma_{(b,c)}^{(n,i)} h_i^b + \sum\limits_{i \ne n} \gamma_{(c,c)}^{(n,i)} h_i^c = 0 \\ h_m^b \dfrac{(\omega^2 + \gamma_{(b,b)}^{(m,m)} \omega^2 - \omega_m^{b\,2})}{\omega^2} + \sum\limits_{i \ne m} \gamma_{(b,c)}^{(m,i)} h_i^c + \sum\limits_{i \ne m} \gamma_{(b,b)}^{(m,i)} h_i^b = 0 \end{cases}$$

(15).

Using the characteristics of the individual elements of the system calculated or determined analytically, the values of the eigenfrequencies and the magnetic field on the slots between the accelerating structure and the bridge, the resulting frequencies in the cavity and the field distributions over the elements of the cavity are calculated.

To calculate the system (15) numerically it should correspond to generalized eigenproblem:

$$Ax = \lambda Bx \quad (16)$$

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Where A and B are real square matrices. For this purpose the ω value in (11) could be set constant as operating frequency of the cavity.

CDS STRUCTURE SIMULATIONS

Calibration

To calibrate the analytical model a direct numerical simulation of 3 simplified CDS periods coupled with a short bridge device with length l_1+l_2 was provided. This test structure is shown at Fig. 2.

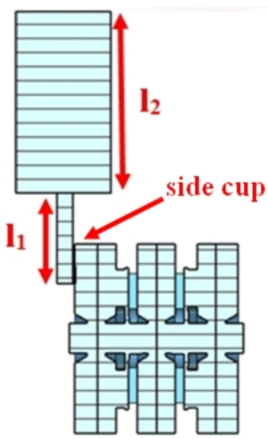

Figure 2: The testing CDS cavity model.

The length l_2 of bridge device was varied from 15 to 55 cm, each iteration provides resulting eigenmodes of the system. The resulting frequencies dependencies are shown at Fig. 3.

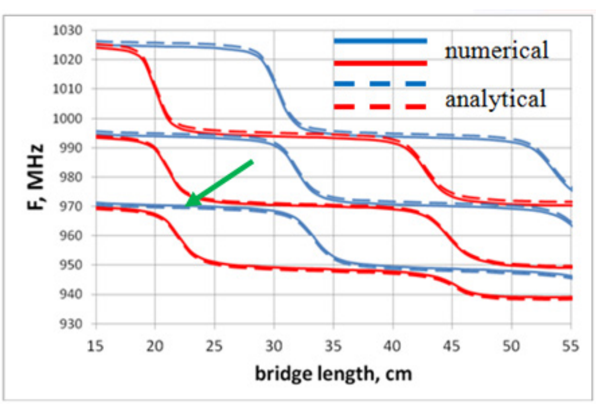

Figure 3: Results of numerical simulation and analytical calculation compared.

The results of analytical calculation and numerical simulation are consistent within a relative error δ=0.5%.

Thus the analytical model of non-uniform system could be applied to predict resulting eigenmodes frequencies with high accuracy [5].

Tuning of the Section

The technique of CDS cavity section with slots tuning assumes similar with described for DAW [5]. A plunger with electric wall installed in the bridge is used to regulate the l_2 length of the bridge. The frequency dependencies are taken to determine the point where the operating mode is on equal distance from side modes. Then the operating mode frequency is adjusted by the side cup (Fig. 3) to desired value.

Using the data from direct numerical simulations one can approximate values for side cup parameters influence on the resulting eigenfrequencies. Thus using the analytical multi-mode model the parameters for cavity frequency adjustment could be predicted. It is shown at Fig. 4.

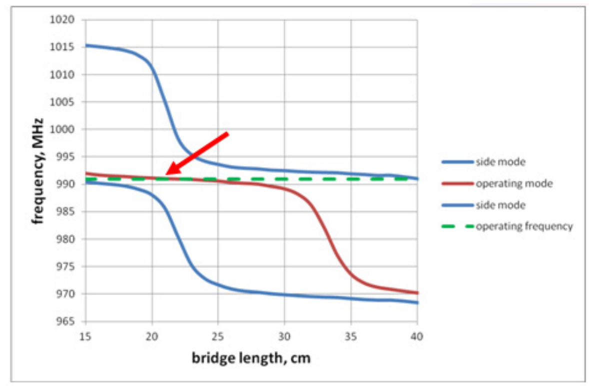

Figure 4: Operating mode adjustment.

With the calculated parameters of individual elements this procedure could be applied for the full four sections cavity with three bridge devices. Herewith the numerical simulation of full system is not required.

SUMMARY

The analytical model of non-uniform system of the cavity and bridge device is presented and substantiated. With the results of direct numerical simulation the high accuracy of the presented technique is shown. For the new CDS structure for the first cavity of INR linac replacement the possibility of cavity-bridge system adjustment is proposed. The main advantage of the technique presented is absence of full system numerical simulation requirement.

AKNOLEDGEMENTS

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