# ABOUT FOCUSING BY GRIDS

A.S.Chikhachev,
Federal State Unitary Enterprise " RFYaTs VNIITF im. akad. E. I. Zababakhina" -VEI-branch
Krasnokazarmennaya, 12, 111250, Moscow, Russia

*Abstract*

The possibility of steady acceleration of particles in the presence in a tube of drift of a foil which plane is parallel to an axis is studied. For calculation of the field the method of conformal transformations is used. Conditions of simultaneous acceleration with stability of the cross and longitudinal movement are found out.

## INTRODUCTION

For focusing of bunches of heavy charged particles focusing by grids - the folga located along an axis which plane is parallel to an axis of the accelerating system is offered (see [1]). For studying of properties in the real work the simplest option of such system is considered - the foil settles down on a drift tube axis, and influence of a foil will be studied in flat geometry that gives the chance for calculation of the field to apply a method of conformal transformations.

## THE FIELD IN A STRIP WITH A CUT

Method of conformal transformations. Let $z = x + iy, w = u + iv$. Transformation

$$w + ia = \frac{a}{\pi}\ln(1 - e^{\frac{\pi z}{a}}) \tag{1}$$

transfers a strip with a cut to $z$ planes in a strip without cut in $w$ plane. At this transformation

$$u = \frac{a}{2\pi}\ln\left(1 - 2e^{\frac{\pi x}{a}}\cos(\frac{\pi y}{a}) + e^{\frac{2\pi x}{a}}\right), \tag{2}$$

$$v = -\frac{a}{\pi}\arctan\frac{\sin\frac{\pi y}{a}}{e^{-\frac{\pi y}{a}} - \cos\frac{\pi y}{a}} - \frac{a}{\pi} \tag{3}$$

The straight line of $y = +a, -\infty < x < +\infty$ passes in half line $v = -a, 0 < u < +\infty$, top coast of drift $y = 0, x < 0$ meets to half line $v = -a, -\infty < u < 0$. Borders of the accelerating interval $[0, b]$ meets to $[\alpha, \beta]$ where $\alpha = \frac{a}{\pi}\ln 2, \beta = \frac{a}{\pi}\ln(1 + e^{\frac{\pi}{b}a})$. Particle moves along axes $x$ in negative direction. Length of tube drift $d$ are $d >> a$, where $a$-an aperture. Actual width of a foil as it will be visible from expression for the field on an axis, can be about $a/\pi$ because of exponential decrease of the field. As boundary conditions we will put: $E_x(x, a) = f(x)$, $f = E_0\sigma x\sigma(b - x)$. Here $\sigma(x)$ - the Heaviside function. Components of the potential field $\vec{E}$ upon transition from $w$ to $z$ will be transformed by in a complaining way:

$$E_x = AE_u + BE_v, E_y = -BE_u + AE_v, \tag{4}$$

where

$$A = \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = \frac{\exp(\frac{2\pi x}{a}) - \exp(\frac{\pi x}{a})\cos(\frac{\pi y}{a})}{1 - 2\exp(\frac{\pi x}{a})\cos(\frac{\pi x}{a}) + \exp(\frac{2\pi x}{a})} \tag{5}$$

$$B = -\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x} = -\frac{\exp(\frac{\pi x}{a})\sin(\frac{\pi y}{a})}{1 - 2\exp(\frac{\pi x}{a})\cos(\frac{\pi x}{a}) + \exp(\frac{2\pi x}{a})} \tag{6}$$

We will pass, further, from boundary conditions for $E_x$ to boundary conditions for $E_u$.. Using ratios 1.4-1.6 for $E_x(x, a)$ we will find :

$$E_u(u, a) = g(u) = \frac{E_0}{(1 - \exp(\frac{-\pi u}{2})}\sigma(u - \alpha)\sigma(\beta - u) \tag{7}$$

We will find, further, field components in $w$ plane. From

$$E_u = \frac{1}{2\pi}\int\int dk du' g(u')\exp\{ik(u - u')\}\frac{\cosh(kv)}{\cosh(ka)}, \tag{8}$$

and for $E_v$ we have:

$$E_v = \frac{1}{2\pi i}\int\int dk du' g(u')\exp\{ik(u - u')\}\frac{\sinh(kv)}{\cosh(ka)}, \tag{9}$$

Integrals in (1.8) and (1.9) can be expressed in elementary functions. Using (1.4) it is possible to calculate $E_x(x, 0)$ and $E_y(x, 0)$. $E_x(x, 0) \neq 0$ only at $x > 0$, whereas $E_y \neq 0$ only at $x < 0$. When transforming (1.1) this half shaft passes into $v = 0$. From (1.5) for definition of $E_x(x, 0)$ to know $E_u(u, 0)$. We will finally receive :

$$E_x(x, 0) = -\frac{1}{\pi}\Big[\frac{1}{\sqrt{e^{\frac{\pi x}{a}} - 1}}\ln\left(\frac{\sqrt{1 + e^{\frac{\pi b}{a}}} - 1}{\sqrt{1 + e^{\frac{\pi b}{a}}} - 1}\frac{\sqrt{2} + 1}{\sqrt{2} - 1}\right) + 2\left(\arctan\sqrt{\frac{1 + e^{\frac{\pi x}{a}}}{e^{\frac{\pi b}{a}} - 1}} - \arctan\sqrt{\frac{2}{e^{\frac{\pi x}{a}} - 1}}\right)\Big] \tag{10}$$

For definition of $E_y(x, 0)$ needs to find $E_v(u, -a)$. Can be received:

$$E_y(x, 0) = \frac{1}{\pi}\frac{1}{\sqrt{1 - e^{\frac{\pi x}{a}}}}\ln\left(\frac{\sqrt{1 + e^{\frac{\pi b}{a}}} - 1}{\sqrt{1 + e^{\frac{\pi b}{a}}} - 1}\frac{\sqrt{2} + 1}{\sqrt{2} - 1}\right) - \frac{1}{\pi}\ln\left(\frac{\sqrt{1 + e^{\frac{\pi b}{a}}} - \sqrt{1 - e^{\frac{\pi x}{a}}}}{\sqrt{1 + e^{\frac{\pi b}{a}}} + \sqrt{1 - e^{\frac{\pi x}{a}}}}\frac{\sqrt{2} + \sqrt{1 - e^{\frac{\pi x}{a}}}}{\sqrt{2} + \sqrt{1 - e^{\frac{\pi x}{a}}}}\right) \tag{11}$$

ISBN 978-3-95450-197-7

## THE FOCUSING ACTION OF A GRID

Let charged particle fly from a point with $x = \infty$ to a point with $x = -\infty$ at $a >> y > 0$. Further, we will consider that the external field changes under the sinusoidal law, i.e. force operating on a particle is defined by a vector $\vec{E}(x(t), y(t))\cos(\omega t + \phi)$. The movement of the nonrelativistic particle are described by two equations:

$$\frac{d}{dt}m\dot{x} = E_x(x(t), y(t))\cos(\omega t + \phi), \qquad (1)$$

$$\frac{d}{dt}m\dot{y} = E_y(x(t), y(t))\cos(\omega t + \phi) \qquad (2)$$

The longitudinal movement is steady if steady solution of the equation

$$\ddot{q} = \frac{e}{m}\frac{\partial E_x}{\partial x}\cos(\omega t + \phi)q, \qquad (3)$$

where $q = x - x_s$ ($x_s$ - coordinate a basis of a particle, $x$ - the coordinate of the displaced particle). The equation (2.3) is steady if

$$\int_{-\infty}^{\infty}\frac{\partial E_x}{\partial x}\cos(\omega t + \phi) > 0. \qquad (4)$$

The increment of a longitudinal impulse when passing accelerating interval a has to be positive, i.e.

$$m\dot{x} = e\int_{-\infty}^{\infty}E_x(x(t), y(t))\cos(\omega t + \phi) > 0. \qquad (5)$$

We will demand, further, that $\dot{y} < 0$, if particle flies upper axis and $\dot{y} > 0$, if the particle flies below an axis. Because of antisymmetry of $E_y$ for $y$ these requirements are compatible Execution them results in stability in the cross direction.

$$m\dot{y} = e\int_{-\infty}^{\infty}E_y(x.y)\cos(\omega t + \phi) < 0. \qquad (6)$$

Below will be found area of phases $\phi$, where conditions (2.4), (2.5), (2.6) are executed at the same time. We will enter new variables:

$$\xi = \frac{\pi x}{a}, \eta = \frac{\pi y}{a},$$

$$E_\eta(\xi, \eta) = E_y(\frac{a\xi}{\pi}, \frac{a\eta}{\pi}), E_\xi(\xi, \eta) = E_x(\frac{a\xi}{\pi}, \frac{a\eta}{\pi})$$

also we will designate $p = \frac{\omega a}{\pi v}$. Conditions (2.4), (2.5), (2.6) will take the following form (taking into account that $t = -\frac{x}{v}$:

$$\int_{-\infty}^{\infty}\frac{\partial E_\xi(\xi, \eta)}{\partial \xi}\cos(p\xi - \phi)d\xi \qquad (7)$$

$$= p\int_{-\infty}^{\infty}E_\xi(\xi, \eta)\sin(p\xi - \phi)d\xi < 0,$$

$$m\dot{\xi} = \frac{e}{v}\int_{-\infty}^{\infty}E_\xi(\xi, \eta)\cos(p\xi - \phi)d\xi > 0, \qquad (8)$$

$$m\dot{\eta} = \frac{e}{v}\int_{-\infty}^{\infty}E_\eta(\xi, \eta)\cos(p\xi - \phi)d\xi > 0. \qquad (9)$$

For calculation of integrals we will use analyticity of function of $E(\xi + i\eta) = E_\xi + iE_\eta$, which follows from equalities $\nabla\vec{E} = 0$, $nabla \times \vec{E} = 0$. We will consider a contour $C$, formed by a trajectory of a particle, axis $\eta = 0$ and closed on $\pm\infty$.

From ratios $\int_C (E_\xi + iE_\eta e^{\pm i(p]xi+\phi)}d\zeta)$ =0 can be received

$$\int_{-\infty}^{\infty}E_\xi(\xi, \eta)\cos(p\xi - \phi)d\xi = \qquad (10)$$
$$\cosh(p\eta)\int_{-\infty}^{\infty}E_\xi(\xi, 0)\cos(p\xi - \phi)d\xi -$$
$$\sinh(p\eta)\int_{-\infty}^{\infty}E_\eta(\xi, 0)\sin(p\xi - \phi)d\xi$$

$$\int_{-\infty}^{\infty}E_\eta(\xi, \eta)\cos(p\xi - \phi)d\xi = \qquad (11)$$
$$\cosh(p\eta)\int_{-\infty}^{\infty}E_\eta(\xi, 0)\cos(p\xi - \phi)d\xi +$$
$$\sinh(p\eta)\int_{-\infty}^{\infty}E_\xi(\xi, 0)\sin(p\xi - \phi)d\xi$$

We will consider further that the parameter of $p = \frac{\omega a}{\pi v} <<$ 1. From a type of expressions (1.10) and (1.11) follows what can be put:

$$\int_0^{\infty}E_\xi(\xi, 0)\sin(p\xi)d\xi = -pK,$$

$$\int_0^{\infty}E_\xi(\xi, 0)\cos(p\xi)d\xi = -L$$

$$\int_{-\infty}^{0}E_\eta(\xi, 0)\sin(p\xi)d\xi = -pM,$$

$$\int_{-\infty}^{0}E_\eta(\xi, 0)\cos(p\xi)d\xi = N,$$

where the sizes K,L,M,N are positive. Simultaneous performance of conditions (2.7), (2.8), (2.9) determines the following area of values $\phi$:

$$\frac{3\pi}{2} - \arctan(\frac{pM}{N}) > \phi > \pi + \arctan(\frac{pK}{L}). \qquad (12)$$

At $p \to 0$ this area passes in $\frac{3\pi}{2} > \phi > \pi$. The area of rather great values of $p$ is characterized by existence of the impulse directed from an axis. I.e. steady acceleration is possible at rather big speeds of particles.

## CONCLUSION

In work the field in a difficult configuration is calculated and the area of steady acceleration of charged particles is defined.

ISBN 978-3-95450-197-7

## ACNOLIGEMENT

The author is grateful to V.A. Teplyakov who has offered a subject of the real work.

## REFERENCES

[1] D.V.Karetnikov, I.N. Slivkov, V.A.Teplyakov, B.K.Shembel. Linear particle accelerators of ions. M, Gosatomizdat, 1962



ISBN 978-3-95450-197-7