DYNAMICS OF THE SPHERICAL CLOUD OF CHARGED PARTICLES

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Abstract

In work the behavior spherically of a symmetric dense cloud of the charged particles which are under the influence of own field is studied. The kinetic description based on "Meshchersky's integral" is used. The movements of particles from the center of symmetry and to the center of symmetry of system are studied.

INTRODUCTION

In this work the behavior spherically of a symmetric clot is considered. At the same time for the description of the radial movement Kapchinsky-Vladimirsky's integral as it is made in work [1] won't be used, and other approach connected with "Meshchersky's integral" will be offered (see [2,3]). Two types of the movement of particles - from the center of system and to the center of system of coordinates are considered.

THE MOVEMENT FROM THE CENTER OF SYSTEM

Hamilton-Jacobi's Equation with spherical symmetry has an appearance (see [4]):

$$\frac{\partial S}{\partial t} + \frac{1}{2m}\left(\frac{\partial S}{\partial r}\right)^2 + \frac{1}{2mr^2}\left(\frac{\partial S}{\partial \theta}\right)^2 + \frac{1}{2mr^2\sin^2\theta}\left(\frac{\partial S}{\partial \phi}\right)^2 + U(r,t) = 0 \quad (1)$$

Here $r, \theta, \ phi$- coordinates in spherical system, S- Hamilton function. We will look for the decision (15) in a look:

$$S = \pm\int\sqrt{2m\left(H - U(r',t) - \frac{L}{2r'^2}\right)}dr' + \psi(t) + M_\phi \phi \pm \int_{\arcsin|\frac{M_\phi}{\sqrt{L}}}^{\theta} d\theta' \sqrt{L - \frac{M_\phi^2}{\sin^2\theta'}},$$

where M_ϕ - a moment projection to an axis of z, $L = \frac{M_\phi^2}{\sin^2\theta} + m^2(r^2\dot{\theta})^2$ - a square of the full moment.

The integrals interfaced to energy H have an appearance:

$$J_H^{\pm} = \frac{\partial S}{\partial H} = \pm\int\frac{dr'}{\sqrt{\frac{2}{m}(H - U(r')) - \frac{L}{2m^2r'^2}}} - t$$

We will pass from H to an invariant of I, ("Meshchersky's integral") remaining at a certain dependence of potential function from $\vec{r}$ and t. In this case the Hamiltonian has an appearance:

$$H = \frac{p_r^2}{2m} + \frac{L}{2mr^2} + \frac{1}{\xi^2(t)}U\left(\frac{r}{\xi(t)}\right).$$

Here $p_r = m\dot{r}$. Using expression of a Hamiltonian, it is possible to receive expression for an invariant:

$$I = \frac{m}{2}(\dot{r}\xi - r\dot{\xi})^2 + U\left(\frac{r}{\xi(t)}\right) + \frac{\lambda m}{2}\frac{r^2}{\xi^2} + \frac{L}{2m}\frac{\xi^2}{r^2}, \quad (2)$$

Similar to integral of $J_H^{\pm}$ can construct integral of $J_I^{\pm}$, we will consider interfaced to I. In this section that there are only particles described by J_I^{+} integral. Density of particles is expressed by integral in phase space:

$$n = \int d\vec{q} f(I, J_I^{+}, L). \quad (3)$$

We will present an element of phase space in the form:

$$d\vec{q} = dq_r dq_\theta dq_\phi, \quad dq_\phi = \frac{dM_\phi}{r\sin\theta},$$

$$dq_r = \frac{dI}{\xi\sqrt{\frac{2}{m}(I - U) - \lambda\frac{r^2}{\xi^2} - \frac{L\xi^2}{m^2r^2}}},$$

$$dq_\theta = \frac{dL}{2r\sqrt{L - \frac{M_\phi^2}{\sin^2\theta}}}.$$

Averaging on M_ϕ leads to expression:

$$n = \frac{\pi}{2r^2}\int\frac{dIdLf(I, L, J_I^{+})}{\xi\sqrt{\frac{2}{m}(I - U) - \lambda\frac{r^2}{\xi^2} - \frac{L\xi^2}{m^2r^2}}}.$$

At the same time density of current of j_r has an appearance: ($\dot{r}$ can be expressed through I from (1.2)) $j_r = \frac{r\dot{\xi}}{\xi}n + \frac{\pi}{2r^2\xi^2}\int fdIdL$. We will make, further, replacement of variables: $r = \rho\xi, \tau = \int\frac{dt'}{\xi^2(t')}$. At the same time Poisson's equation takes a form:

$$\frac{1}{\xi^4(t)}\frac{1}{\rho^2}\frac{d}{d\rho}\rho^2\frac{dU}{d\rho} = -\frac{4\pi e^2}{\xi^3(t)\rho^2}\int\frac{dIdLf(I, L, J_I^{+})}{\sqrt{\frac{2}{m}(I - U(\rho)) - \lambda\rho^2 - \frac{L}{m^2\rho^2}}}. \quad (4)$$

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In order that both parts of equality (4) equally depended from ξ, function of distribution has to contain the multiplier which is exponential depending on J_I^+. We will put: $f = \kappa_* \delta(I - I_0)\delta(L - L_0)\exp\{\frac{1}{2\tau_0} J_I^+\}$. We will notice that in variables ρ, τ the invariant of J_I^+ interfaced to I has an appearance:

$$J_I^+ = -\tau + \int_0^\rho \frac{d\rho'}{\sqrt{\frac{2}{m}(I - U(\rho')) - \lambda\rho'^2 - \frac{L}{m^2\rho'^2}}}.$$

If condition is satisfied by $\xi \exp\left(-\frac{\tau}{2\tau_0}\right) \equiv \xi_0$, then as an independent variable enters Poisson's equation only ρ. Thus, $\xi(t) = \sqrt{\frac{t}{\tau_0} + \xi_0^2}$,
$\lambda = -\frac{1}{4\tau_0^2}$. We will designate, further.

$$v_0^2 = \frac{2I_0}{m}, s = \frac{\rho}{2\tau_0 v_0}, y = \frac{2U}{mv_0^2}, l = \frac{L_0}{4m^2\tau_0^2 v_0^4},$$

$$u(s) = \int_0^s \frac{ds'}{\sqrt{1 - y(s') + s'^2 - l/s'^2}}.$$

Then follows from Poisson's equation:

$$\frac{d}{ds} s^2 \frac{d}{ds} y(s) = -\theta u' e^{u(s)}, \quad (5)$$

$$u'(s) = \frac{1}{\sqrt{1 - y(s) + s^2 - l/s^2}}.$$

Constant θ is defined by task parameters - κ_*, m, v_0, τ_0 and a charge of e: $\theta = \frac{8\pi e^2 \kappa_*}{mv_0^3}\xi_0$. If to use equality of $y = -\frac{\theta}{s^2}\exp(u(s)) + \frac{C_0}{s^2}$, the system (5) can be written down in the form of one equation:

$$\frac{d}{ds} s^2 \frac{dy(s)}{ds} = -\frac{C_0 - s^2 y'(s)}{\sqrt{1 - y(s) + s^2 - l/s^2}}. \quad (6)$$

Density of particles can be written down in a look:

$$n = n_1 \frac{C_0/s^2 - y'}{\xi^4\sqrt{1 - y(s) + s^2 - l/s^2}}$$

$$= a(s) n_1/\xi^4,$$

and current density:

$$j_r = n_1 v_0 \Big(\frac{s}{\sqrt{1 - y(s) + s^2 - 1/s^2}} + 1\Big)\times$$

$$\frac{C_0/s^2 - y'}{\xi^5} = b(s)\frac{n_1 v_0}{\xi^5},$$

where $n_1 = \frac{m}{32\pi e^2 \tau_0^2 \xi_0}$. Density of particles and density of current can be also written down in a look:

$$n = \frac{\pi\kappa_*}{2v_0}\frac{u' e^u}{r^2\xi^2},$$

$$j_r = \frac{\pi\kappa_*}{4v_0\tau_0}\frac{r u' e^u}{r^2\xi^4} + \frac{\pi\kappa_*}{2}\frac{e^u}{r^2\xi^3}.$$

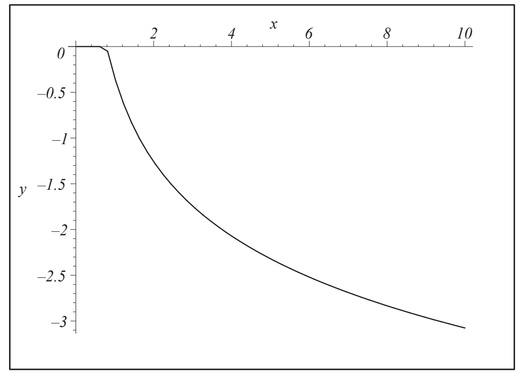

Figure 1: Dependens $y(s)$ at $y_0 = y_0' = 0, C_0 = 0$.

These expressions satisfy to the continuity equation.

Decisions for the potential of $y(s)$ and density of particles of $a(s)$ at $y(0) = y'(0) = 0$ are provided on the Fig.1. Because $L \neq 0$, density near the beginning of coordinates is equal to zero. At great values of coordinate density decreases, however the full number of particles grows in the area limited to some value of coordinate beyond all bounds with growth of this value. It is possible that the states studied here will be of interest to studying of influence of own fields on acceleration of particles The decision for $a(s)$, density of particles describing dependence on s is provided on the Fig. 2. The movement from the center of system was studied the same in work [3].

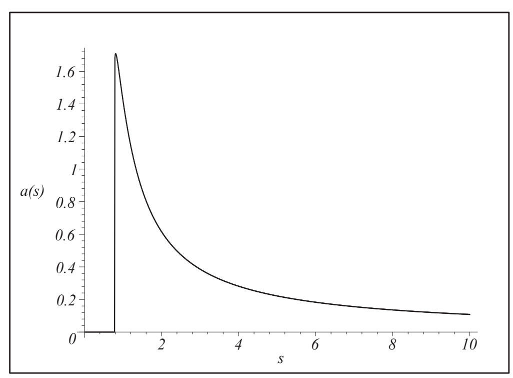

Figure 2: Dependens $a(s)$ at $y_0 = y_0' = 0, C_0 = 0$.

THE MOVEMENT OF PARTICLES TO THE CENTER OF SYSTEM

Also the description of system with the speeds directed towards decrease of radius is of interest. Instead of integral of J_I^+ it is necessary to use

$$J_I^- = -\int \frac{dr'}{\sqrt{\frac{2}{m}(I - U(r')) - \frac{L}{2m^2 r'^2}}} - t.$$

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At the same time density of particles and density of current look as follows:

$$n = \frac{\pi\kappa_*}{2v_0}\frac{u' e^{-u}}{r^2\xi^2},$$

$$j_r = \frac{\pi\kappa_*}{4v_0\tau_0}\frac{r u' e^{-u}}{r^2\xi^4} - \frac{\pi\kappa_*}{2}\frac{e^{-u}}{r^2\xi^3}.$$

It is possible to receive the system of the equations:

$$y'(s) = -\frac{\theta}{s^2}\exp(-u(s)) + C_0, \qquad (7)$$

$$u'(s) = \frac{1}{\sqrt{(1 - y(s) + s^2 - \frac{l}{s^2}}}.$$

It is characteristic that the type of dependence of density

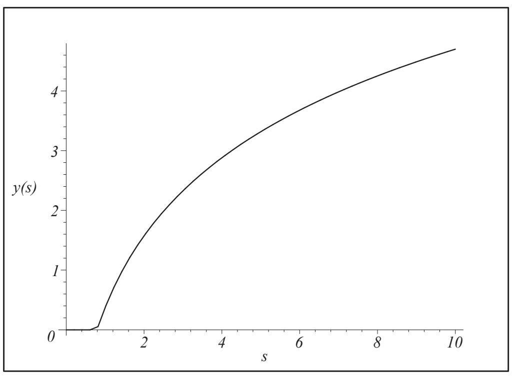

Figure 3: Dependens $y(s)$ at $y_0 = y_0' = 0, C_0 = 0$.

practically doesn't change from the size of C_0 and the direction of the movement of particles. However it should be noted that at the movement to the center full number of particles of N in the field of $s > S$ is final. It is possible to receive $N = \frac{c}{o} nst\xi(t)$, i.e. full number decreases over time. At $s = S$ there has to be a full absorption of the flying particles, and the radius of the absorbing sphere grows: $r(t) = 2\tau_0 v_0 \xi(t)$. The curve describing behavior of potential of $y(s)$ at the movement of particles to the center of spherical system of coordinates is given on the Fig. 3 On the Fig. 4 the behavior of density as functions of s, and on the Fig. 5 - dependence of full quantity of particles is presented at $s > S$, where $s = S$ - border of the area occupied with particles.

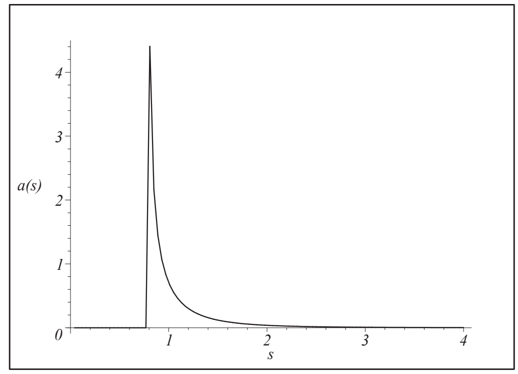

Figure 4: Dependens $a(s)$ at $y_0 = y_0' = 0, C_0 = 0$.

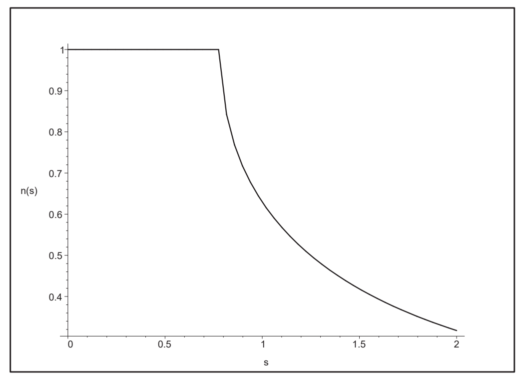

Figure 5: Dependens $N(s)$, $y_0 = y_0' = 0, C_0 = -3$.

CONCLUSION

In work self-consistent decisions for the potential of a spherical bunch of the particles interacting with own field are studied.

Results of numerical decisions for density and potential as functions of a self-similar variable of s are given.

REFERENCES

[1] Chikhachev A.S.. Sov. Phys-Tech. (USA) 1984, vol. 29, N.9. pp. 990-992.

[2] J.Mestschersky Astron.Nachr., 1893, v.132, p.129; Astron.Nachr.,1902, v.159, p.229.

[3] Chikhachev A.S. . Technical Physics, 2014, v.59,N.4, pp.489-493.

[4] G. Goldstein, Classical mechaniks, M.: 1957

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