STABILITY OF CHARGED PARTICLE MOTION IN A STORAGE RING WITH FOCUSING BY LONGITUDINAL MAGNETIC DIELD.

S.A. Melnikov, I.N. Meshkov, Joint Institute for Nuclear Research (JINR), 141980, Moscow Region, Dubna, Russian Federation.

Abstract

Analytically calculated matrices of the optical conversion of the elements of the focusing system of the Low Energy Particle Toroidal Accumulator (LEPTA) and the longitudinal magnetic field perturbations appearing in the technical connections of the ring elements are presented. Based on the matrix data, a program was written in the Wolfram Mathematica environment, which allows simulating the multiturn dynamics of particles in a ring and investigating the stability of their motion.

THE LEPTA SET UP

The LEPTA (Low Energy Particle Toroidal Accumulator) setup (see Fig. 1) is a storage ring with a perimeter of 17.2 m with a circulating positron beam in the energy range 1-10 keV

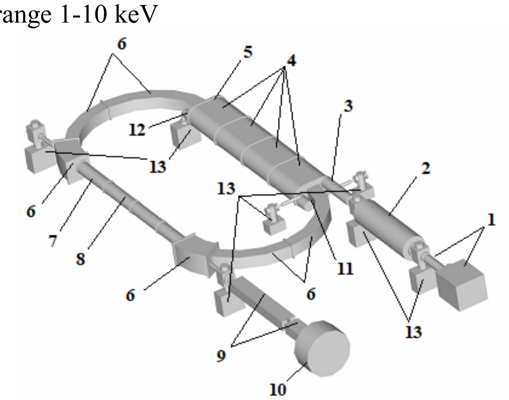

Figure 1: LEPTA set up diagram: 1 - injector; 2 - positron trap; 3 - section for injection of positrons; 4 - septum solenoids; 5 - kicker (located inside the septum solenoid); 6 - toroidal solenoids; 7 - solenoid and quadrupole coil; 8 - section of electron cooling, straight solenoid; 9-dipole analyzing magnet; 10 - coordinate-sensitive detector; 11 - electron gun; 12 - collector of electrons; 13 - the vacuum pump.

The LEPTA set up uses a section structure, which makes it possible to introduce additional straight sections where the injection and extraction sections of the beam and the diagnostic device are placed.

To maintain charged particle motion in the LEPTA, a longitudinal magnetic field is used that accompanies the particles from the source and along the entire orbit of the circulating beam. Thus, the particles in the LEPTA are "magnetized". The stability of the beam circulation is provided by introducing, in addition to the longitudinal magnetic field, the field of the helix quadrupole lens (further - quadrupole) in the section 7.

Between the individual focusing elements of the ring there are technical joints in which adiabatic perturbations of the magnetic field are formed (see Fig. 2). These gaps have a significant influence on the beam dynamics in the ring that also directly affects on the lifetime of circulating particles (see Fig. 3).

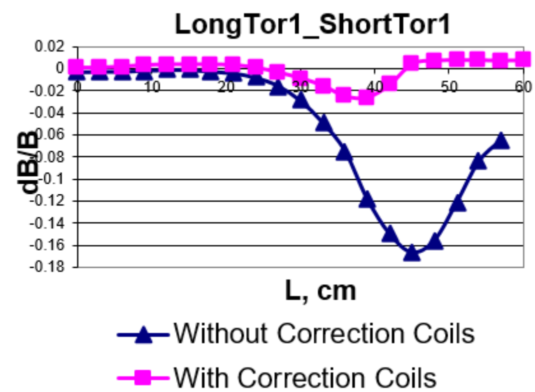

Figure 2: Experimentally measured perturbation of the longitudinal magnetic field before and after correction.

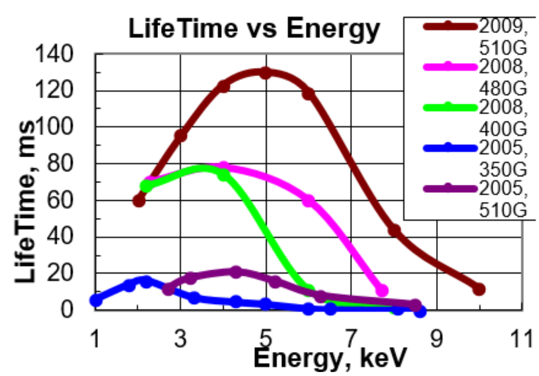

Figure 3: Experimentally measured dependences of the lifetime of a charged particle on their energy.

PARTICLE DYNAMICS IN THE LEPTA RING

The particle trajectory in an electromagnetic field is described by a differential equation [1]:

$$\frac{d\vec{p}}{dt} = e\vec{E} + \frac{e}{c} * [\vec{v} \times \vec{B}] \qquad (1)$$

For a region with a quadrupole and straight solenoid, equation (1) has the form:

Copyright © 2018 CC-BY-3.0 and by the respective authors

ISBN 978-3-95450-197-7

$$\begin{cases} x''(s) - \frac{1}{\rho_L} y'(s) + \frac{1}{\rho_L}\frac{B_y(s)}{B_0} = 0 \\ y''(s) + \frac{1}{\rho_L} x'(s) - \frac{1}{\rho_L}\frac{B_x(s)}{B_0} = 0 \end{cases}. \quad (2)$$

where $\rho_L = \frac{pc}{eB}$ is the radius of the Larmor rotation; B_0 is the longitudinal magnetic field. The transverse components of the quadrupole magnetic field are described by the formulas [2]:

$$\begin{cases} B_x(s) = -G(xSin[2ks] - yCos[2ks]) \\ B_y(s) = G(xCos[2ks] - ySin[2ks]) \end{cases}.$$

Performing the transformation $U(s) = x(s) + iy(s)$, we seek a solution in the Wentzel-Kramers-Brillouin approximation (WKB) [3] in the form $U(s) = A(s)e^{\chi(s)ds}$. In turn, $A(s)$ is sought in the form $A(s) = a(s) + ib(s) = C_a e^{\int \kappa ds} + iC_b e^{\int \kappa ds}$. As a result, we obtain a 4 × 4 transport matrix for the quadrupole. Below, because of the awkwardness of the results obtained, only a single element of the matrix is presented:

$$m_{Q11} = \left(1 + \frac{k\rho_L - T1A1}{T1A1 - T2A2}\right)(cos[T1z]cos[\chi] + A1sin[T1z]sin[\chi]) - \left(\frac{k\rho_L - T1A1}{T1A1 - T2A2}\right)(cos[T2z]cos[\chi] + A2sin[T2z]sin[\chi])$$

where $k = \frac{2\pi}{h}$, h – is the winding period of the quadrupole; $\chi = k\rho_L z$; $\begin{cases} A1 = \frac{\sqrt{2}\sqrt{A_D+B_D}C_D}{(A_D+B_D)+2D_D} \\ A2 = \frac{\sqrt{2}\sqrt{A_D-B_D}C_D}{(A_D-B_D)+2D_D} \end{cases}$, in its turn A_D, B_D, C_D and D_D − are polynomials of unit dimension containing quadrupole parameters k and $g = \frac{G}{B_0}$, as well as ρ_L and gradient of the magnetic field in the quadrupole G.

A detailed record of all elements of the optical transition matrix of the spiral quadrupole as well as the matrix elements described below will be given in [4].

$$\kappa = \kappa_{Q1} = \frac{T1}{\rho_L} = \pm\frac{1}{\rho_L}\frac{1}{\sqrt{2}}\sqrt{A_D + B_D} \longrightarrow \frac{1}{\rho_L} \quad (3)$$

$$\kappa = \kappa_{Q2} = \frac{T2}{\rho_L} = \pm\frac{1}{\rho_L}\frac{1}{\sqrt{2}}\sqrt{A_D - B_D} \longrightarrow \sqrt{k^2 - g^2} \quad (4)$$

The parameter κ is obtained from the equality to zero of the determinant of the matrix, constructing of the coefficients at x and y in equations, describing the particle motion.

(3) is the frequency of fast Larmor rotation of the particle around the line of force, (4) is the frequency of slow betatron oscillations made by the particle across the magnetic field line.

Limit values in (3) and (4) are reached when one of the the infinitesimal parameters $g\rho_L, k\rho_L \ll 1$

Here and below, all results are given in dimensionless coordinates $z = \frac{s}{\rho_L}$.

If we take $B_x = 0$, $B_y = 0$, in the system of equations (2), then, solving the system, we obtain the transition matrix for a section of a homogeneous longitudinal magnetic field:

$$M_S = \begin{pmatrix} 1 & sinI_1 & 0 & 1 - cosI_1 \\ 0 & cosI_1 & 0 & sinI_1 \\ 0 & -(1 - cosI_1) & 1 & sinI_1 \\ 0 & -sinI_1 & 0 & cosI_1 \end{pmatrix}. \quad (6)$$

For the case of an homogenous field, system (2) retains its form, but now the transverse components of the field will be written as $B_x(s) = \frac{-r}{2}\frac{dB_s}{ds}\Big|_{r=0}\frac{x}{r}$, $B_y(s) = \frac{-r}{2}\frac{dB_s}{ds}\Big|_{r=0}\frac{y}{r}$.

The solution is also sought by the WKB method. We find that for the region of the perturbed field the values of parameter κ is equal to

$$\kappa = \kappa_G \equiv \pm\begin{cases} i/\rho_L \\ g(s) \end{cases} (5)$$

that has the same physical meaning as κ_Q in (3) and (4), respectively.

The transport matrix for a section of the perturbed field has the form:

$$m_{G11} = \left[\frac{1 + g_\rho}{1 + {g_\rho}^2} g_\rho cos[I_1] + \frac{1 - g_\rho}{1 + {g_\rho}^2} Cosh[I_2]\right]$$

where $g_\rho = g_0\rho_L$, g_0 is the amplitude of the field perturbation, $I_1 = \frac{s}{\rho_L} = \frac{z\rho_L}{\rho_L} = z$, $I_2 = \int g(s)ds = \frac{-1}{2}\frac{1}{B_0}\rho_L \int \frac{dB_z}{dz}\Big|_{r=0} dz$.

In toroidal sections, an additional transverse magnetic field is applied to compensate for the centrifugal-gradient drift. The equations of particle motion in the toroidal section can be obtained from (1.9) - (1.11) [1, page 7]:

$$\begin{cases} x''(s) - \frac{1}{\rho_L} y'(s) + \frac{x(s)}{R_0^2} = \frac{1}{R_0} - \frac{1}{\rho_L}\frac{B_y(s)}{B_0} \\ y''(s) + \frac{1}{\rho_L} x'(s) = \frac{1}{\rho_L}\frac{B_x(s)}{B_0} \end{cases},$$

$$B_x = 0, B_y = \frac{\rho_L B_0}{R_0}$$

The solution for x(s) is sought in the WKB approximation in the following form $x(s) = A_0 + A_s s + A_x e^{i\kappa_T s}$. We obtain $\kappa_T = \pm\sqrt{\frac{1}{R_0^2} + \frac{1}{\rho_L^2}} = \pm T$, and the matrix itself has the form:

$$m_{T11} = (1 - 1/T^2)cos[Tz] + 1/T^2$$

The matrix of the ring can be obtained by multiplying the matrices of the individual elements:

$$M_{Lepta} = M_{Sept}M_{G2}M_{TL1}M_{G3}M_{TL2}M_{G4}M_{Str1}M_Q M_{Str2}M_{G5}M_{TS2}M_{G6} M_{TL2}M_{G1}$$

Multiplying the matrix of the ring by the initial conditions vector, and raising M_{LEPTA} to the power of N, we obtain the coordinates of the particle x, x', y, y' after the N turns in the ring.

Copyright © 2018 CC-BY-3.0 and by the respective authors

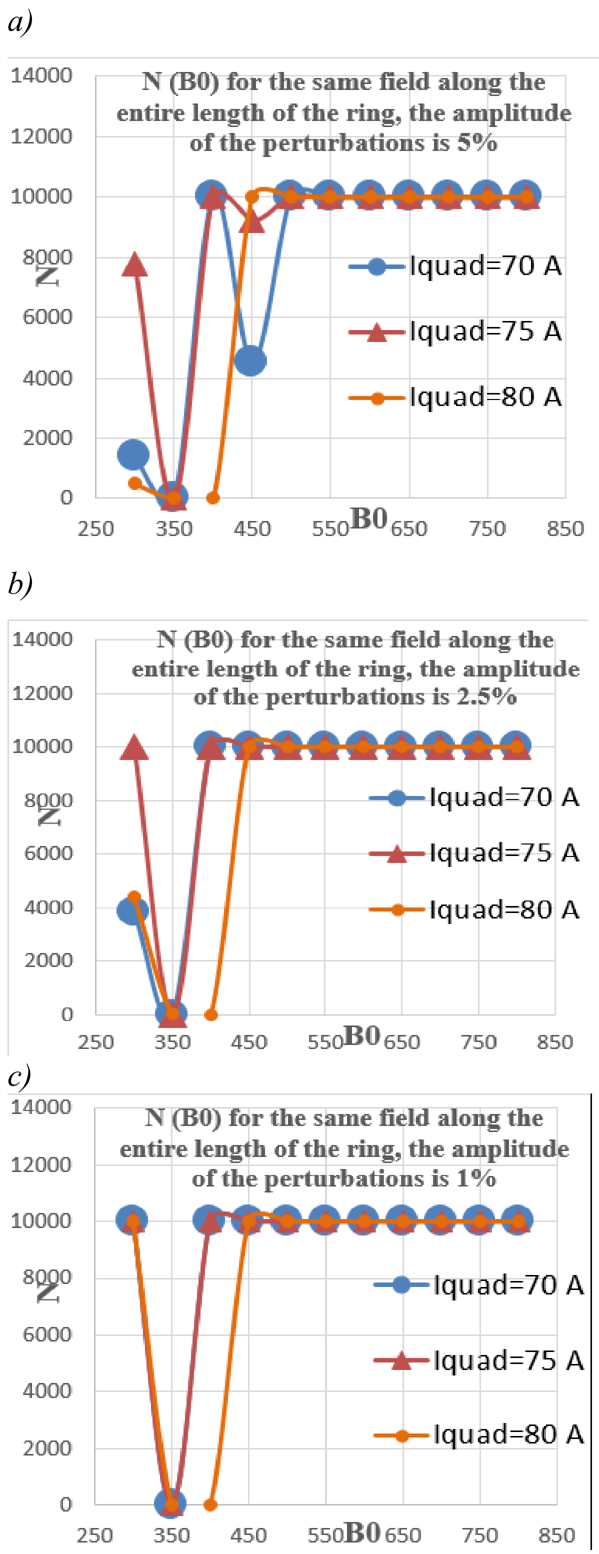

Figure 4: The dependence of the number of revolutions of the steady motion of a charged particle in the ring on the magnitude of the longitudinal magnetic field for different values of the quadrupole current I_GαG For cases with the perturbation amplitude g_0 a) 5%, b) 2.5%, c) 1%.

MAIN RESULTS

The results of the simulation (see Fig. 4) show that with a decrease in the amplitude of field perturbations, the number of revolutions of the stable motion of particles in the ring increases. In this case, there are areas of a sharp decrease, independent of the magnitude of the field perturbation. It was found that in these sections half of the spur of the quadrupole matrix $\left|\frac{1}{2}Tr[M_Q]\right| \geq 1$ (see Fig. 5), while similar values for the remaining matrices, including for the matrix of the ring, do not correlate with the number of particle turns in the ring.

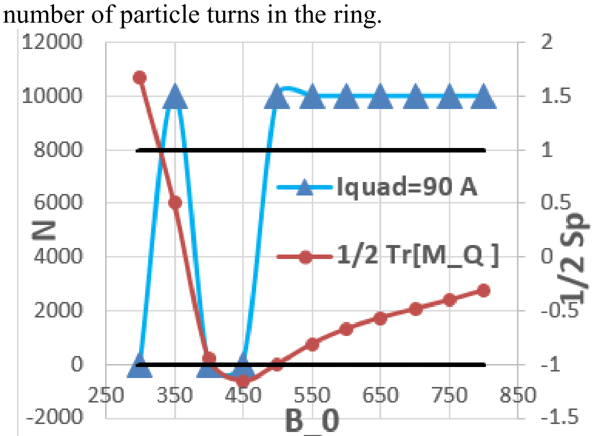

Figure 5: Correlation of the particle rotation speed in the ring with the value 1/2 Tr $[M\_Q]$.

Thus, the value of parameter $\frac{1}{2}Tr[M_Q]$ determines the stability of particle motion in the ring, which corresponds to the general theory of particle motion in focusing systems.

REFERENCES

[1] Meshkov IN, Transportation of beams of charged particles (Novosibirsk: Nauka, Sib., 1991).

[2] G.V. Trubnikov. The thesis abstract for the degree of candidate of physical and mathematical sciences (JINR, Dubna, 2005)

[3] A.B. Migdal, V.P. Krainov, Approximate methods of quantum mechanics (Moscow, Nauka, 1966).

[4] S.A. Melnikov, I.N. Meshkov, Investigation of the stability of the motion charged particles in the storage ring LEPTA (Preprint of the Joint Institute for Nuclear Research, Dubna, 2018, sent to the press).

Copyright © 2018 CC-BY-3.0 and by the respective authors

ISBN 978-3-95450-197-7