SOLITARY AND SHOCK WAVES IN FREE AND MAGNETIZED QUASI-NEUTRAL LASER INDUCED PLASMAS*

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Abstract

Starting from the Vlasov-Maxwell equations, an exact relativistic hydrodynamic closure for a special type of water-bag distributions satisfying the Vlasov equation has been derived. In the case of magnetized quasi-neutral plasma, the hydrodynamic substitution has been used to derive the hydrodynamic equations for the plasma density and current velocity, coupled to the wave equations for the self-consistent electromagnetic fields. Based on the method of multiple scales, a system comprising a vector nonlinear Schrodinger equation for the transverse envelopes of the self-consistent plasma wakefield, coupled to a scalar nonlinear Schrodinger equation for the electron current velocity envelope for free plasma, has been derived. In the case of magnetized plasma, it has been shown that the whistler wave envelopes of the three basic modes satisfy a system of three coupled nonlinear Schrodinger equations. Numerical examples for typical plasma parameters have been presented. It has been shown that in the case of magnetized plasma, the whistler waves facilitate the transverse confinement considerably.

INTRODUCTION

In the mid 1950s, Budker and Veksler [1, 2] proposed utilizing plasma collective fields to accelerate charged particles more compactly. Twenty years later this idea was further developed by the late John Dawson and his collaborators [3, 4]. Several mechanisms to generate large amplitude electron plasma waves are presently put into practice. The first one dates back to 1956, when V.I. Veksler's [2] suggestion of acceleration by means of collective fields was further elaborated by G.I. Budker [1], who proposed the concept of a self-stabilized ring beam. In recent years the beat wave mechanism [5], the laser-driven wakefield generation, and last but not least excitation of plasma wave structures by charged particle beams propagating in the plasma medium have attracted much attention. More recent numerical simulations and experimental investigations show that ultra-intense laser-matter interactions and laser-induced plasma wakefields can be used for new type of laser-driven positron sources [6] and acceleration of ion beams [7] with narrow energy spread.

Whistler waves are one of the first plasma waves observed and studied for more than a century. The first analytical approach to the linear dispersion properties of whistler waves is the one suggested by Appleton [8] and Hartree [9], who proposed the famous Appleton-Hartree dispersion equation. While the linear stability properties of the electromagnetic waves in the whistler mode are relatively well studied [10, 11], there is a serious gap in the understanding of their nonlinear behaviour. The general theory of nonlinear waves in a cold, collisionless relativistic plasma was initiated by Akhiezer and Polovin [12]. The study of the nonlinear behaviour of whistler waves has been initiated by Taniuti and Washimi [13], who obtained a nonlinear Schrodinger equation for the slowly varying wave amplitude.

We first review and summarize some basic properties of laser driven plasmas. Then, following Refs. 14 and 15, we reduce the Vlasov-Maxwell system to an *exact closure* of relativistic warm fluid dynamic equations for the plasma species, which are coupled to the wave equations for the radiation field. Using the method of multiple scales, we outline how a vector nonlinear Schrodinger equation describing the evolution of the slowly varying amplitude of the transverse plasma wakefield, coupled to a scalar nonlinear Schrodinger equation for the amplitude of the electron current velocity can be derived. Next, the derivation of the cold hydrodynamic picture by using the so-called hydrodynamic substitution has been outlined. We then obtain a system of coupled nonlinear Schrodinger equation describing the evolution of the slowly varying amplitudes of the three basic whistler modes. The analysis of an approximate traveling wave solution to the coupled nonlinear Schrodinger equations in both the relativistic and the non relativistic case concludes Section 4. Finally, we draw some conclusions in the last Section.

BASIC PROPERTIES OF LASER PLASMAS

To gain insight into the basic properties of laser induced plasmas, we consider a simple non-relativistic model

$$\partial_t n + \partial_x (n v_x) = 0, \tag{1}$$

$$\partial_t \mathbf{v} + v_x \partial_x \mathbf{v} = -\frac{e}{m}[\mathbf{E} + \mathbf{e}_x(\mathbf{v} \cdot \partial_x \mathbf{A}) - v_x \partial_x \mathbf{A}], \tag{2}$$

$$\partial_x E_x = -\frac{e}{\epsilon_0}\Big(n - \overline{Z} n_i\Big). \tag{3}$$

This model describes the plasma response to an external perturbation propagating longitudinally along the x-axis with a unit direction vector $\mathbf{e}_x$ and specified by the electromagnetic vector potential $\mathbf{A}$. Here, n and $\mathbf{v}$ are the electron number density and the current velocity, respectively, while $\mathbf{E}$ is the self-consistent electric field. In addition, m and e are the rest mass and the charge of the electron, respectively, ϵ_0 is the permittivity of free space and finally, $\overline{Z}$ and n_i are the

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mean charge number, and the number density of the background ions in the plasma. Neglecting the time variation of the transverse components $\mathbf{v}_\perp$ of the current velocity in Eq. (2), it follows that

$$\mathbf{v}_\perp = \frac{e\mathbf{A}_\perp}{m}. \quad (4)$$

This allows us to automatically rewrite Eq. (2) as

$$\partial_t v_x + \frac{1}{2}\partial_x v_x^2 = -\frac{eE_x}{m} - \frac{e^2}{2m^2}\partial_x A^2, \quad (5)$$

where $A^2 = A_y^2 + A_z^2$. The second term on the right-hand-side of Eq. (5) represents the so-called ponderomotive force. Let us now apply the traveling wave approximation to Eqs. (1), (3) and (5). Integrating once the resultant Eqs. (1) and (5) with due account of the initial conditions $n(0) = n_0$, $v_x(0) = 0$ and $\varphi(0) = 0$, we obtain

$$v_x = u\left(1 - \frac{n_0}{n}\right), \qquad v_x^2 - 2uv_x - \frac{2e\varphi}{m} + \frac{e^2}{m^2}\left(A^2 - A_0^2\right) = 0, \quad (6)$$

where $A_0 = \frac{mca_0}{e}$ (a_0 being the so-called the laser parameter), and φ is the scalar electromagnetic potential $\left(E_x = -\partial_\xi \varphi\right)$. Solving the second of Eqs. (6) for v_x and substituting the result into the first one, we obtain an expression for n/n_0. The latter substituted into Eq. (3) yields a single equation for the scalar potential

$$\partial_\xi^2 \varphi = \frac{en_0}{\epsilon_0}\left\{\left[1 + \frac{2e\varphi}{mu^2} - \frac{e^2}{m^2u^2}\left(A^2 - A_0^2\right)\right]^{-1/2} - 1\right\}. \quad (7)$$

Expanding the square root on the right-hand-side of Eq. (7), we obtain an equation for the electric field

$$\left(\partial_\xi^2 + k_e^2\right)E_x = -\frac{ek_e^2}{2m}\partial_\xi A^2, \qquad k_e = \frac{\omega_e}{u}, \quad (8)$$

where $\omega_e^2 = e^2 n_0/(m\epsilon_0)$ is the electron plasma frequency and $\xi = x - ut$ is a new traveling wave variable. For a sufficiently short driving laser pulse of the form $A^2 = \left(\frac{mca_0}{e}\right)^2 \exp\left(-\frac{\xi^2}{\sigma_l^2}\right)$, the driver [the source term in Eq. (8)] is resonant for $k_e\sigma_l = \sqrt{2}$. The evolution of the longitudinal electric field E_x for the resonant case is shown in Figure 1. For typical plasma number densities of the order of $10^{21}\ m^{-3}$ and intensity of the driving the laser pulse $a_0 = 1.3$, quite impressive acceleration gradients of the order of several gigavolts per meter can be achieved.

NONLINEAR WAVES AND COHERENT STRUCTURES IN QUASI-NEUTRAL PLASMAS

Theoretical Model and Basic Equations

We consider a quasi-neutral plasma in an external electromagnetic field depending on the dimensionless coordinates $\mathbf{x} = (x, y, s)$ and the dimensionless time t. The nonlinear

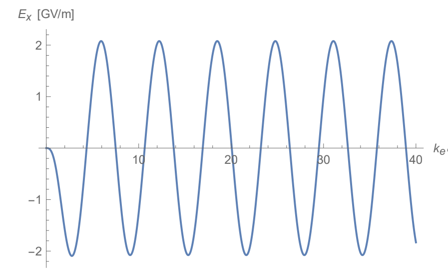

Figure 1: Evolution of the longitudinal electric field E_x in the resonant case $\left(k_e\sigma_l = \sqrt{2}\right)$. The amplitude of the driving laser pulse is $a_0 = 1.3$, while the electron plasma frequency is taken to be $\omega_e \sim 1.8$ THz.

Vlasov equation for the distribution function $f_a(\mathbf{x}, \mathbf{p}; t)$ of particle species a (electrons and ions) can be written as [14]

$$\partial_t f_a + \frac{\mathbf{p}_\perp - Z_a\mathbf{A}_\perp}{\mu_a\gamma_a}\cdot\nabla_\perp f_a + \frac{p_s}{\mu_a\gamma_a}\partial_s f_a + (Z_a\mathcal{F} - \mu_a\partial_s\gamma_a)\partial_{p_s} f_a = 0. \quad (9)$$

Here $\mu_a = m_a/m$ is the mass aspect ratio with respect to the electron mass and Z_a is the charge state of species a ($q_a = eZ_a$). Since the transverse canonical momenta are integrals of motion (no dependence on the transverse coordinates by assumption), Eq. (9) reduces to

$$\partial_t F_a + \frac{p_s}{\mu_a\gamma_a}\partial_s F_a + (Z_a\mathcal{F} - \mu_a\partial_s\gamma_a)\partial_{p_s} F_a = 0. \quad (10)$$

Consider now a class of water bag distributions solving exactly the one-dimensional Vlasov equation (10), which is given by the expression [14–17]

$$F_a(s, p_s; t) = C_a\left\{\Theta\left[p_s - p_a^{(-)}(s,t)\right] - \Theta\left[p_s - p_a^{(+)}(s,t)\right]\right\}, \quad (11)$$

where $\Theta(z)$ is the well known Heaviside step function. Following Ref. 15, we introduce the hydrodynamic variables n_a, V_a and Γ_a as certain combinations of the boundary curves $p_a^{(\pm)}(s,t)$. The important quantity Γ_a can be written as

$$\Gamma_a = \sqrt{\frac{1 + \frac{Z_a^2}{\mu_a^2}A^2}{\left(1 - V_a^2\right)\left(1 - 2v_{aT}^2 n_a^2\right)}}, \quad (12)$$

where $v_{aT}^2 = 1/\left(8C_a^2\right)$ is the thermal speed squared of the plasma species of the type a. The completion of the macroscopic fluid description can be performed in a similar to Ref. 15 manner by expressing the source terms entering the corresponding wave equations for the electromagnetic potentials as functions of n_a, V_a and Γ_a. Thus, the hydrodynamic equations for each plasma species a can be written as

$$\partial_t(n_a\Gamma_a) + \partial_s(n_a\Gamma_a V_a) = 0, \quad (13)$$

$$\partial_t(V_a\Gamma_a) + \partial_s\Gamma_a = \mathcal{F}_a = -\frac{Z_a}{\mu_a}(\partial_s\Phi + \partial_t A_s), \quad (14)$$

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$$\Box\Phi = -\frac{1}{n_{e0}} \sum_a Z_a n_a \Gamma_a, \tag{15}$$

$$\Box A_s = -\frac{1}{n_{e0}} \sum_a Z_a n_a \Gamma_a V_a, \tag{16}$$

$$\Box \mathbf{A}_\perp = \frac{\mathbf{A}_\perp}{n_{e0}} \sum_a Z_a^2 n_a \left(1 + \frac{2}{3} v_{aT}^2 n_a^2\right) + \Box \mathbf{A}_e. \tag{17}$$

For the case of constant phase-space density distribution specified by Eq. (11), the macroscopic fluid description consisting of Eqs. (13) – (17), is fully equivalent to the nonlinear Vlasov equation (10) supplemented by the corresponding wave equations for the self fields.

Nonlinear Waves in Laser Plasmas

Ions comprise a heavy plasma background, so that their effect on the dynamics of the plasma wakefield, triggered by the external pumping electromagnetic field can be neglected. The system of hydrodynamic and wave equations to be analyzed in the sequel can be written as follows [14]

$$\partial_t(n\Gamma) + \partial_s(n\Gamma V) = 0, \tag{18}$$

$$\Box\left[\partial_t^2(\Gamma V) + \partial_t\partial_s\Gamma\right] = -\Box(n\Gamma V), \tag{19}$$

$$\Box \mathbf{A}_\perp = n\left(1 + \frac{2}{3} v_T^2 n^2\right)\mathbf{A}_\perp. \tag{20}$$

According to the standard procedure of the multiple scales method [14, 16] applied to the system of equations (18) – (20), all variables n, V and $\mathbf{A}_\perp$ are expanded in a formal small parameter, which at the end of all calculations is set back to unity. Then, the corresponding perturbation equations are solved, such that secular terms are eliminated order by order. As a result, the evolution dynamics of the hydrodynamic and the field variables is being split on different spatial and time scales – fast ones involving rapid wave oscillations and slow scales on which coherent motion of certain wave amplitudes occurs. Omitting details [14], we can write

$$V(s;t) = \mathcal{B}(s;t)\mathrm{e}^{i\varphi(s;t)} + \mathcal{B}^*(s;t)\mathrm{e}^{-i\varphi(s;t)}, \tag{21}$$

$$\mathbf{A}_\perp(s;t) = \boldsymbol{\mathcal{A}}(s;t)\mathrm{e}^{i\psi(s;t)} + \boldsymbol{\mathcal{A}}^*(s;t)\mathrm{e}^{-i\psi(s;t)}, \tag{22}$$

where

$$\varphi = ks - \Omega t, \qquad \psi = ks - \omega t, \tag{23}$$

are the phases of the two basic waves propagating in the longitudinal direction. For a generic wave number k, the wave frequencies are given by

$$\Omega = \sqrt{1 + 2k^2 v_T^2} \qquad \omega = \sqrt{1 + k^2 + \frac{2}{3} v_T^2}. \tag{24}$$

The key result [14] is that the wave amplitudes $\mathcal{B}$ and $\boldsymbol{\mathcal{A}} = \mathcal{A}_x \mathbf{e}_x + \mathcal{A}_y \mathbf{e}_y$ satisfy the equations

$$i\partial_t \boldsymbol{\mathcal{A}} + i v_\omega \partial_s \boldsymbol{\mathcal{A}} = -\frac{1}{2}\frac{\mathrm{d}v_\omega}{\mathrm{d}k}\partial_s^2 \boldsymbol{\mathcal{A}} + \Gamma_{aa}\mathcal{A}^2 \boldsymbol{\mathcal{A}}^* + \Gamma_{ab}|\mathcal{B}|^2 \boldsymbol{\mathcal{A}}, \tag{25}$$

$$i\partial_t \mathcal{B} + i v_\Omega \partial_s \mathcal{B} = -\frac{1}{2}\frac{\mathrm{d}v_\Omega}{\mathrm{d}k}\partial_s^2 \mathcal{B} + \Gamma_{ba}|\mathcal{A}|^2 \mathcal{B} + \Gamma_{bb}|\mathcal{B}|^2 \mathcal{B}. \tag{26}$$

The explicit expressions for the dispersion and coupling coefficients can be found in Ref. 14. Equations (25) and (26) comprise a system of a nonlinear vector Schrodinger equations for $\boldsymbol{\mathcal{A}}$ coupled to a scalar nonlinear Schrodinger equation for $\mathcal{B}$. They describe the evolution of the slowly varying amplitudes of the generated transverse plasma wakefield and the current velocity of the plasma electrons.

It can be shown [14] that

$$\mathcal{A}_y = \mathcal{A}_x C, \tag{27}$$

where in the simplest case C is a real constant or a purely imaginary ($C = \pm i$) one. The subcase $C = 0$ corresponds to linear wave polarization, while $C = \pm i$ corresponds to circular wave polarization.

Numerical Results and Discussion

We analyze here circularly polarized plasma waves, in which case Eqs. (25) and (26) can be written as

$$i\partial_t \mathcal{A}_x + i v_\omega \partial_s \mathcal{A}_x = -\frac{1}{2}\frac{\mathrm{d}v_\omega}{\mathrm{d}k}\partial_s^2 \mathcal{A}_x + \Gamma_{ab}|\mathcal{B}|^2 \mathcal{A}_x, \tag{28}$$

$$i\partial_t \mathcal{B} + i v_\Omega \partial_s \mathcal{B} = -\frac{1}{2}\frac{\mathrm{d}v_\Omega}{\mathrm{d}k}\partial_s^2 \mathcal{B} + 2\Gamma_{ba}|\mathcal{A}_x|^2 \mathcal{B} + \Gamma_{bb}|\mathcal{B}|^2 \mathcal{B}. \tag{29}$$

The case of linear wave polarization can be treated in a similar manner. We shall describe now traveling wave solutions to Eqs. (28) and (29), which are generally sought through the standard ansatz

$$\mathcal{A}_x(\xi,\eta) = \mathrm{e}^{i(\mu\xi + \nu_1\eta)}\mathcal{P}(z), \qquad \mathcal{B}(\xi,\eta) = \mathrm{e}^{i(\mu\xi + \nu_2\eta)}\mathcal{Q}(z), \tag{30}$$

where ξ and η are new variables, $z = \eta - u\xi$ is the traveling wave independent variable, while $\mathcal{P}$ and $\mathcal{Q}$ are yet unknown complex (in general) traveling wave amplitudes [14]. The quantities μ, $\nu_{1,2}$ and the traveling wave velocity u are constants to be determined additionally.

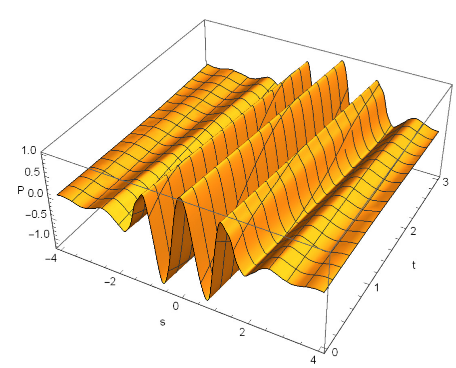

Figure 2: Evolution of the traveling wave amplitude $\mathcal{P}$ for the case $k = 1.543613$, $v_T^2 = 0.1$ and $\mu = -1.0$.

The resulting equations for $\mathcal{P}$ and $\mathcal{Q}$ have been solved by the method of formal series of Dubois-Violette [14, 16]. The solution of the corresponding equation for $\mathcal{P}$ (coupled to

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that for Q) is visualized in Figure 2. A careful inspection of the results presented in Figure 2 shows that the traveling wave solution of the coupled nonlinear Schrodinger equations represents a damping quasi-periodic wave [14]. The damping rate of this wave is proportional to $1/\eta$, and on a scale of $3 \sim 4 \; c/\omega_e$ it can be considered as practically completely subdued.

NONLINEAR WAVES AND COHERENT STRUCTURES IN MAGNETIZED QUASI-NEUTRAL PLASMAS

Kinetic and Hydrodynamic Picture

We analyze now the properties of quasi-neutral plasma immersed in an external constant magnetic field $\mathbf{B}_0 = B_0 \mathbf{e}_x$. The dimensionless Vlasov-Maxwell system is written as

$$\partial_t f_a + \mathbf{v} \cdot \boldsymbol{\nabla} f_a - \nu_a \mathbf{e}_x \times \mathbf{v} \cdot \boldsymbol{\nabla}_p f_a + Z_a (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \boldsymbol{\nabla}_p f_a = 0, \tag{31}$$

$$\Box \mathbf{A} = - \sum_a \lambda_a \int \mathrm{d}^3 \mathbf{p} \mathbf{v} f_a (\mathbf{x}, \mathbf{p}; t), \tag{32}$$

$$\Box \varphi = - \sum_a \lambda_a \int \mathrm{d}^3 \mathbf{p} f_a (\mathbf{x}, \mathbf{p}; t), \tag{33}$$

$$\partial_t \mathbf{E} = \boldsymbol{\nabla} (\boldsymbol{\nabla} \cdot \mathbf{A}) - \partial_t^2 \mathbf{A}, \qquad \mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}. \tag{34}$$

The quantity λ_a in Eqs. (32) and (33) is defined as

$$\lambda_a = \frac{Z_a n_a}{n_e}, \tag{35}$$

In addition, $\nu_a = \mu_a \omega_a / \omega_e$, $\omega_a = q_a B_0 / m_a$, where ω_a is the cyclotron frequency of particles of type a, and ν_a is the corresponding scaled cyclotron frequency with respect to the electron plasma frequency.

Consider the so-called hydrodynamic substitution [18, 19]

$$f_a (\mathbf{x}, \mathbf{p}; t) = \varrho_a (\mathbf{x}; t) \delta^3 \left[\mathbf{p} - \frac{1}{\mu_a} \gamma_a (\mathbf{x}; t) \mathbf{v}_a (\mathbf{x}; t) \right]. \tag{36}$$

Substituting the above expression (36) into the Vlasov-Maxwell system (31) – (33), we obtain the cold hydrodynamic equations

$$\partial_t \varrho_a + \boldsymbol{\nabla} \cdot (\varrho_a \mathbf{v}_a) = 0, \tag{37}$$

$$\partial_t (\gamma_a \mathbf{v}_a) + \mathbf{v}_a \cdot \boldsymbol{\nabla} (\gamma_a \mathbf{v}_a) + \bar{\omega}_a \mathbf{e}_x \times \mathbf{v}_a = \mu_a Z_a (\mathbf{E} + \mathbf{v}_a \times \mathbf{B}), \tag{38}$$

supplemented with the equations for the self-fields

$$\Box \mathbf{A} = - \sum_a \lambda_a \varrho_a \mathbf{v}_a, \qquad \Box \varphi = - \sum_a \lambda_a \varrho_a, \tag{39}$$

where the notation $\bar{\omega}_a = \omega_a / \omega_e$ has been introduced.

The frequency of the electromagnetic wave excited in the magnetized plasma is as a rule much higher than the (relative) ion-cyclotron frequency $\bar{\omega}_i$. Since $\mu_a \gg 1$ for $a \neq e$ (formally, $\mu_e = 1$), we can neglect the ion motion and take into account the contribution coming from the much lighter electrons only. We shall consider a special case of plasma wave anisotropy, implying that the longitudinal and the transverse plasma waves depend on the longitudinal (in the direction of the applied external magnetic field $\mathbf{B}_0$) x coordinate only. Although such assumption is not essential (for details see e.g. Ref. 20), it simplifies the analytic treatment considerably. It is convenient to complexify the transverse variables by introducing new notations $V = v_y + i v_z$ and $\mathcal{A} = A_y + i A_z$ and rewrite Eqs. (37) – (39) accordingly [19].

Nonlinear Waves in Magnetized Plasmas

It can be verified [19] that for typical values of the electron-cyclotron frequency $\bar{\omega}_e$ the dispersion equation for the whistler waves possesses three distinct real roots $\omega_n(k)$, where $n = 1, 2, 3$. Thus, for the transverse part of the self-consistent electromagnetic vector potential and the current velocity, we obtain

$$\mathcal{A} = \sum_{n=1}^{3} C_n \mathrm{e}^{i \psi_n}, \qquad V = \sum_{n=1}^{3} \left(\omega_n^2 - k^2 \right) C_n \mathrm{e}^{i \psi_n}, \tag{40}$$

where $\psi_n = kx - \omega_n t$. Separating real from imaginary part in Eq. (40), it is straightforward to verify that whistler waves are circularly polarized and this property expands on all other transverse field quantities and hydrodynamic variables. It can be shown [19] that the amplitudes C_n satisfy a system of three coupled nonlinear Schrodinger equations

$$i \partial_t C_n + i v_{gn} \partial_x C_n = - \frac{1}{2} \frac{\mathrm{d} v_{gn}}{\mathrm{d} k} \partial_x^2 C_n + \sum_m \Pi_{mn} C_n |C_m|^2 + \sum_{m \neq n} \Gamma_{mn} C_n |C_m|^2, \tag{41}$$

where the explicit form of the dispersion and the coupling coefficients can be found in Ref. 19. Note that terms with $m = n$ are excluded from the second sum on the right-hand-side of Eq. (41). This implies that the matrix of coupling coefficients Γ_{mn} represents a sort of a selection rule, according to which a generic mode n cannot couple with itself. The first term (not present in the non relativistic case) involving the coupling matrix Π_{mn} allows self-coupling and is entirely due to the relativistic character of the motion.

Numerical Results and Discussion

Straightforward evaluation of the dispersion coefficients $v'_{gn} = \mathrm{d} v_{gn} / \mathrm{d} k$ shows that in a relatively wide range of plasma parameters one of them, say v'_{g2} is several orders of magnitude smaller than the other two, and therefore can be neglected. The equation for C_2 possesses a simple solution of the form $C_2 = g_2 \mathrm{e}^{-i \Psi(x;t)}$, where g_2 is a constant, while the phase Ψ can be determined, provided C_1 and C_3 are known. This implies that our initial system (41) can be reduced to a simpler system of two coupled nonlinear Schrodinger equations [19]. Similar to the preceding Section, we seek traveling wave solutions through the standard ansatz

$$C_1 = \mathrm{e}^{i(\mu_1 \xi + \mu_2 \eta)} \mathcal{P}_1(\eta), \qquad C_3 = \mathrm{e}^{i \mu_3 (\xi + \eta)} \mathcal{P}_3(\eta). \tag{42}$$

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The resulting system of nonlinearly coupled Duffing equations for $\mathcal{P}_1$ and $\mathcal{P}_3$ are solved by employing the method of formal series of Dubois-Violette [16, 19]. Figures 3 and 4

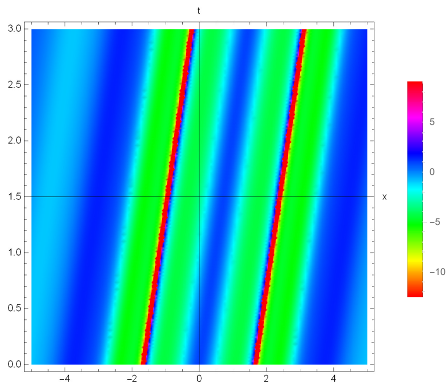

Figure 3: Evolution of the fully relativistic traveling wave amplitude $\mathcal{P}_1$ for the case, where $\bar{\omega}_e = 1$ $k = 1$, $\mu_1 = -1$, $\mu_3 = 1$ and $g_2 = 0$.

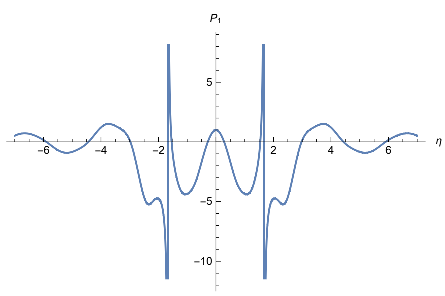

Figure 4: Evolution of the fully relativistic traveling wave amplitude $\mathcal{P}_1$ as a function of η for the case, where $\bar{\omega}_e = 1$ $k = 1$, $\mu_1 = -1$, $\mu_3 = 1$ and $g_2 = 0$.

describe the fully relativistic case, for which the contribution of the mode self-coupling terms Π_{mm} has been taken into account. The traveling wave solution represents $1/\eta$-damped quasi-periodic oscillations of the whistler mode amplitudes fading away with respect to the travelling wave variable η. The solitary-like wave crests with respect to the spatial variable for both $\mathcal{P}_1$ and $\mathcal{P}_3$ are almost monolithic structures, which are stable in time and are symmetrically located on both sides of the line $x = v_{g3}t$. Note also that, in both the non relativistic and the fully relativistic case, the whistler mode amplitudes $\mathcal{P}_1$ and $\mathcal{P}_3$ at a fixed location in the longitudinal direction x decay rapidly in time [19]. According to Eq. (40), the plasma response to the induced whistler waves consists in transverse velocity redistribution, which follows exactly the nonlinear behaviour of the whistlers. This means that the electron current flow is well confined and localized in the transverse direction, such that on a scale $3 \sim 4\ c/\omega_e$ the tails of the electron density distribution can be considered as practically completely subdued.

CONCLUDING REMARKS

The principle of generating super-strong electric accelerating fields has been demonstrated using a simple and illustrative physical model. An exact relativistic hydrodynamic closure of equations describing the dynamics of various species in a quasi-neutral plasma has been obtained. As expected, the warm fluid dynamic equations are invariant under Lorentz transformation. Further, a system comprising a vector nonlinear Schrodinger equation for the transverse envelopes of the self-consistent plasma wakefield coupled to a scalar nonlinear Schrodinger equation for the electron current velocity envelope has been derived. The numerical results presented in Figure 2 show that the traveling wave solution of the coupled nonlinear Schrodinger equations represents a damping quasi-periodic wave, which on a scale of $3 \sim 4\ c/\omega_e$ can be considered as practically completely subdued. The analysis performed here clearly demonstrates generation of nonlinear electromagnetic waves driven by an external radiation source. These waves possess a solitary (shock) and multipeak structure and are possibly related to recent experiments on the so-called "shock acceleration" [21]. The ELI-NP facility will provide focused laser beams with intensities above $10^{25}\ W/m^2$. A number of dedicated experiments [22] has been proposed, which could confirm the relevance of the theory developed here.

Utilizing a technique known as the hydrodynamic substitution, a relativistic hydrodynamic system of equations describing the dynamics of various species in a cold quasi-neutral plasma immersed in an external solenoidal magnetic field has been obtained. Based on the method of multiple scales, a further reduction of the macroscopic fluid and the wave equations for the self-consistent electromagnetic fields has been performed. As a result, a system comprising three coupled nonlinear Schrodinger equation for the three basic whistler modes has been derived. An intriguing feature of our description is that whistler waves do not perturb the initial uniform density distribution of plasma electrons. The plasma response to the induced whistler waves consists in transverse velocity redistribution, which follows exactly the behaviour of the whistlers [12]. The electron current flow is well localized in the transverse direction, such that on a spatial scale of $3 \sim 4\ c/\omega_e$ the tails of the electron density distribution can be considered as practically completely faded away. This property may have an important application for transverse focusing of charged particle beams in future laser plasma accelerators. According to the adopted geometry the direction of the laser light propagation coincides with the direction of the external solenoidal field. After the charged particles are accelerated in a section free of magnetic field, the divergent bunch enters the solenoid, where it is focused by the nonlinear transverse whistler waves.

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