

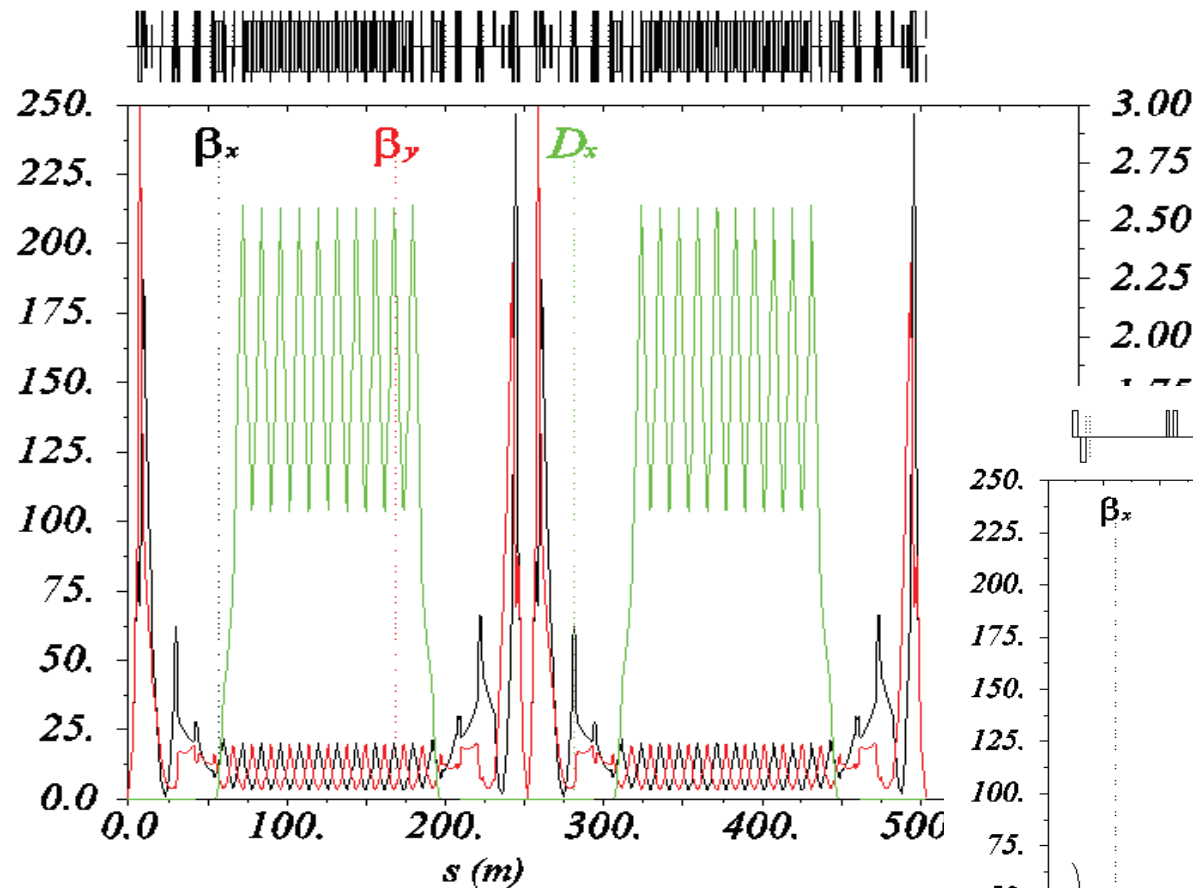
Dynamic aperture optimization of the NICA collider

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Outline

- Nonlinear quadrupole fringe fields and their effect on dynamic aperture
- Quadrupole fringe fields and octupoles
- Comparison of TrackKing and MAD-X simulation codes
- Dynamic aperture optimization
 - Working point optimization
 - Octupole optimization
 - Linear lattice optimization

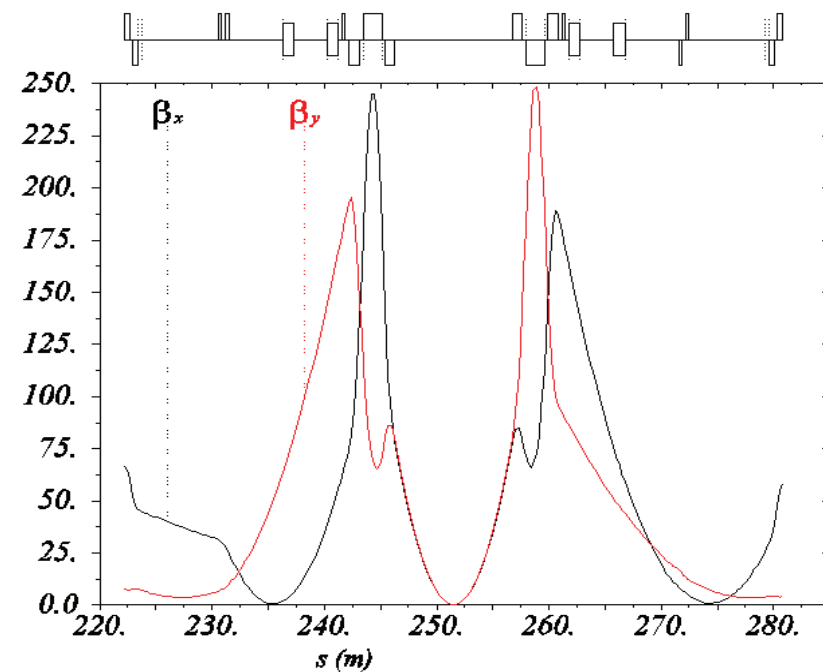
Initial lattice optics



$$\beta_{\max} = 250 \text{ m}$$

Interaction region

Whole ring



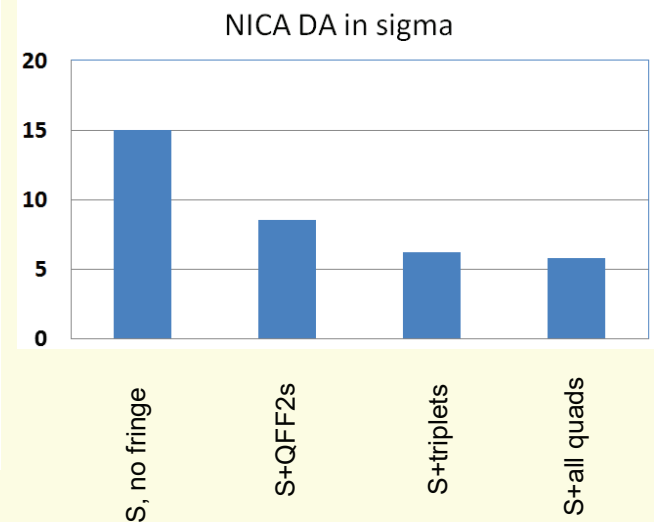
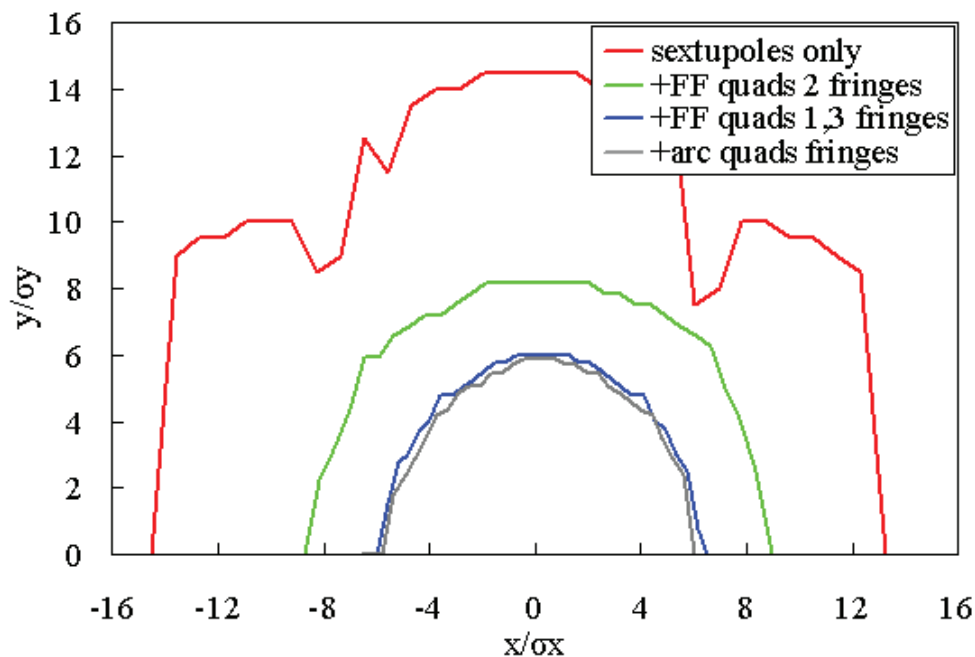
DA reduction due to nonlinear quadrupole fringe fields



NICA transverse on-momentum DA is limited mainly by quadrupoles fringe field nonlinearity.

Among all the quads the most severe effect comes from the FF triplet.

Among the FF quads the central one prevails.



Quadrupole fringe fields and octupoles

Hamiltonian for charged particle moving in a lattice with quadrupole and octupole fields:

$$H = \frac{p_x^2 + p_y^2}{2} + k_1(s) \frac{x^2 - y^2}{2} + p_x k_1'(s) \frac{x^3 + 3xy^2}{12} - p_y k_1'(s) \frac{y^3 + 3yx^2}{12} + k_3(s) \frac{x^4 - 6x^2y^2 + y^4}{24}$$

Nonlinear quadrupole fringe fields:

$$\left\{ \begin{array}{l} (\bar{x})_{\text{fringe}} = x + \Delta k_1 \frac{x^3 + 3xy^2}{12} \\ (\bar{y})_{\text{fringe}} = y - \Delta k_1 \frac{y^3 + 3yx^2}{12} \\ (\bar{p}_x)_{\text{fringe}} = \frac{p_x \left(1 - \Delta k_1 \frac{x^2 + y^2}{4} \right) + p_y \Delta k_1 \frac{xy}{2}}{1 - (\Delta k_1)^2 \frac{(x^2 - y^2)^2}{16}} \\ (\bar{p}_y)_{\text{fringe}} = \frac{p_y \left(1 + \Delta k_1 \frac{x^2 + y^2}{4} \right) - p_x \Delta k_1 \frac{xy}{2}}{1 - (\Delta k_1)^2 \frac{(x^2 - y^2)^2}{16}} \end{array} \right.$$

Octupoles:

$$\left\{ \begin{array}{l} (\bar{p}_x)_{\text{oct}} = p_x - (k_3 l) \frac{x^3 - 3xy^2}{6} \\ (\bar{p}_y)_{\text{oct}} = p_y - (k_3 l) \frac{y^3 - 3yx^2}{6} \end{array} \right.$$

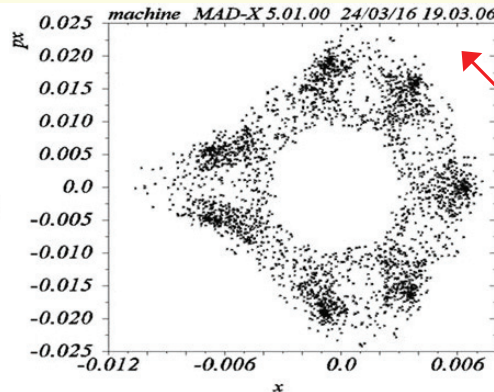
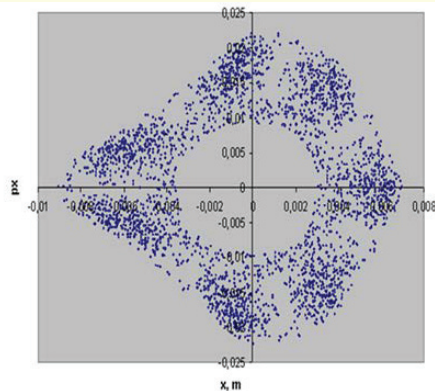
Terms coming from fringes and octupoles have similar structure but different signs. Higher order terms also come from fringes. So, effect of nonlinear quadrupole fringe fields can be partially cured with octupoles.

Cross-check with MAD-X (sextupoles only)

First we track particles with sextupoles only and cross-check the results with MAD-X.

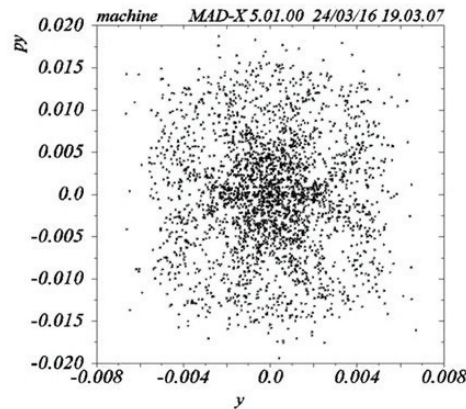
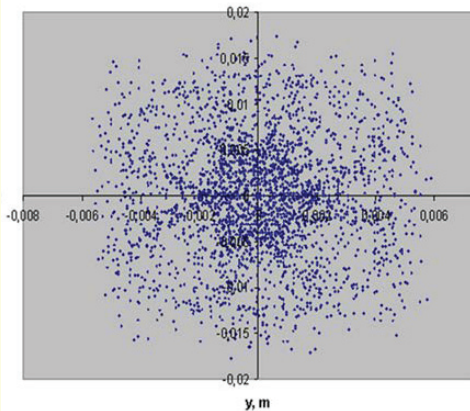
TrackKing

MAD-X



Results from TrackKing and MAD correspond very well.

7-spot structure hints to the 7-order horizontal resonance which, indeed, can take place according to $7Q_x = 7 \cdot 9.44 \approx 66$.

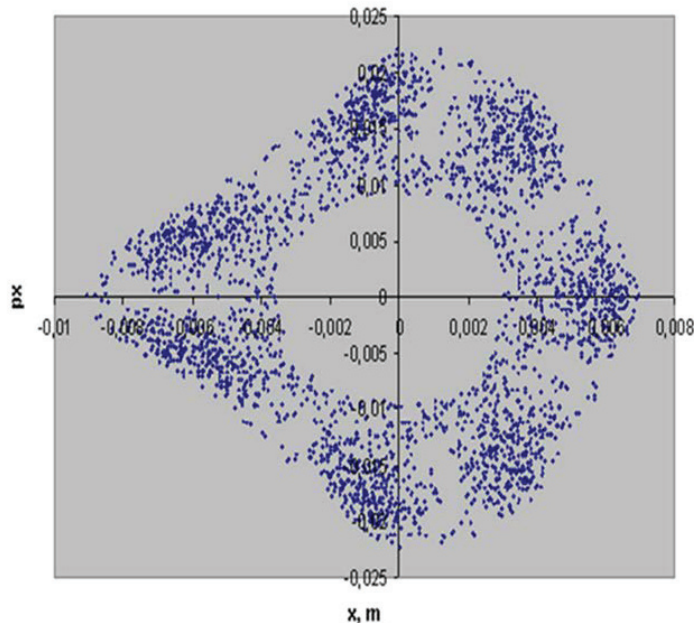


A serious problem is that under the applied initial values $X_0 \neq 0$ $Y_0 = 0$ the vertical motion must be exactly zero due to the symmetry of the sextupole potential.

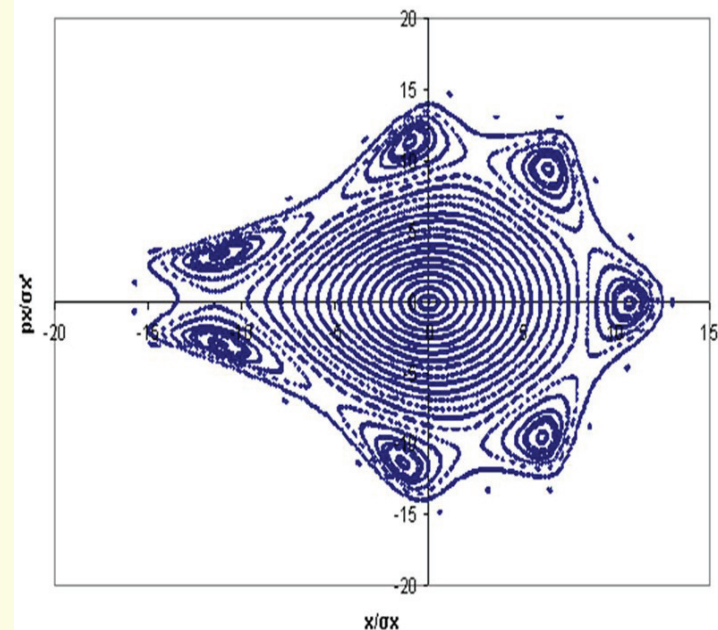
Tilt correction

As it was found, the problem is in definition of the vertical bend separating two collider rings. The vertical bend in the original MAD lattice is defined as rotation of the horizontal bend with the command $\text{TILT} = 1.570796327 = \pi/2 + 2.051 \cdot 10^{-10}$. The inevitable rounding error $\sim 10^{-10}$ produces very small starting seed of the vertical motion which, at the coupling resonance, excites large amplitude vertical oscillation which couples with the horizontal motion and obscures it. When this error was corrected in TrackKing, the effect has disappeared.

Without TILT correction

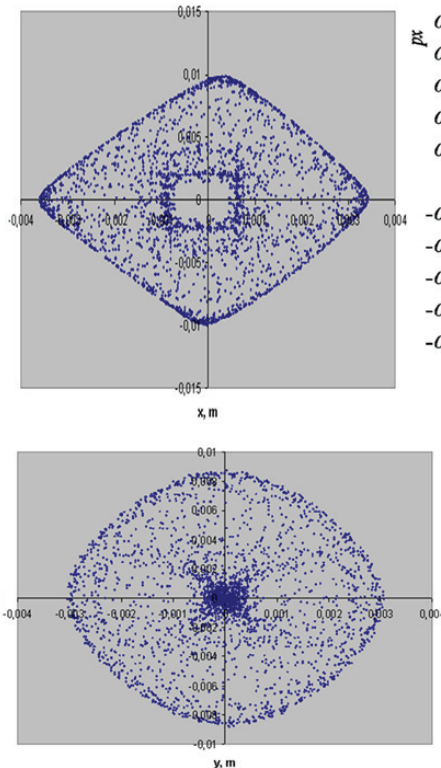


With TILT correction

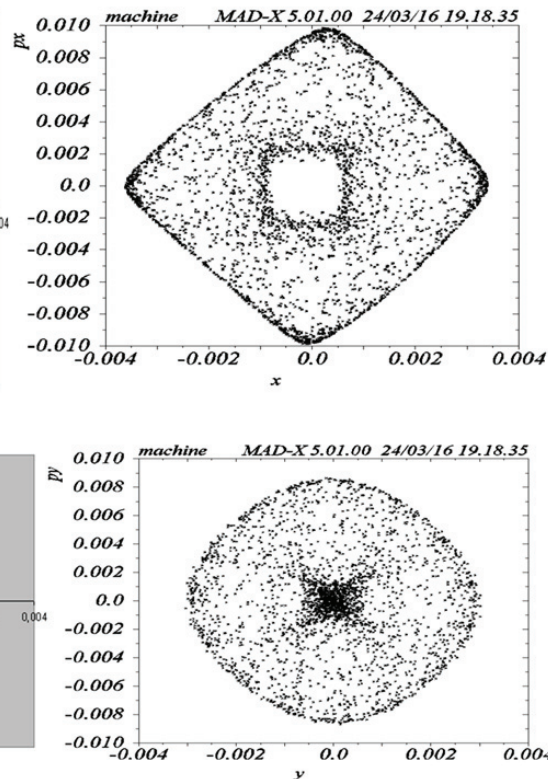


Cross-check with MAD-X (sextupoles + fringes)

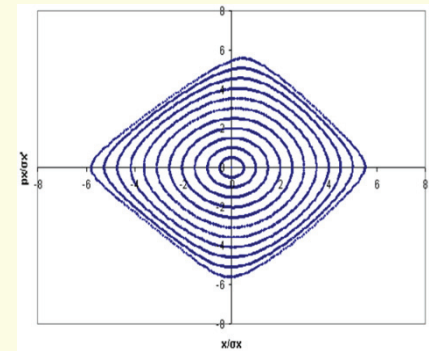
TrackKing



MAD-X



TrackKing with TILT correction



Combined effect of the chromatic sextupoles+quadrupole fringe field is reproduced identically by both MAD and TrackKing.

And again the TILT correction turns smeared trajectories to the regular ones.

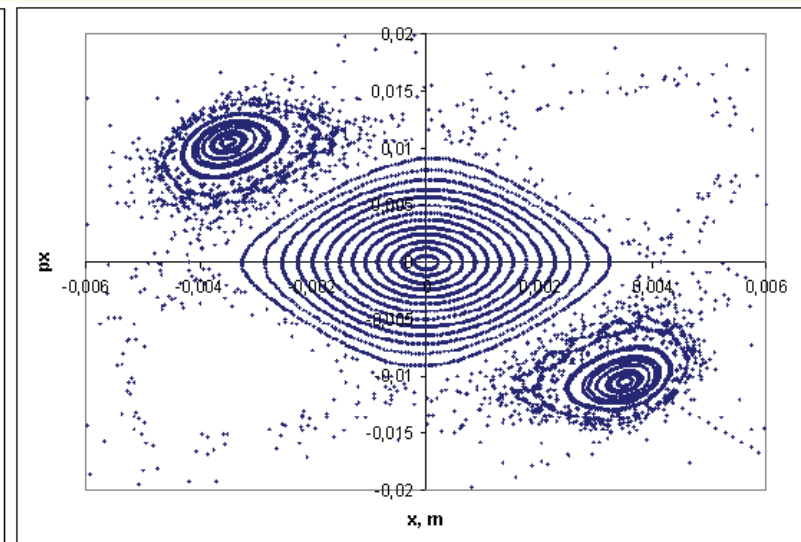
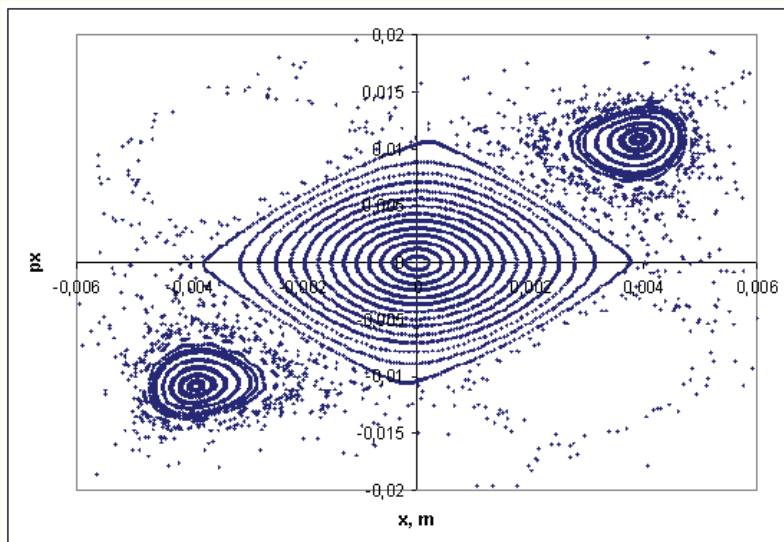
A little information can be extracted from phase space portraits without the tilt correction because they are smeared by nonlinear coupling (effect looks similar to stochastic motion) and no regular curves are visible.

Typical resonance structure

Phase portraits with fringes have diamond-like form because of a pair of 2nd order resonances driven by quasi-octupole terms. This can be easily explained in case of one-dimensional motion with octupole-like perturbation:

$$\begin{pmatrix} x \\ p_x \end{pmatrix}_1 = \begin{pmatrix} \cos \mu_x + \alpha_x \sin \mu_x & \beta_x \sin \mu_x \\ -\gamma_x \sin \mu_x & \cos \mu_x - \alpha_x \sin \mu_x \end{pmatrix} \begin{pmatrix} x \\ p_x \end{pmatrix} + \begin{pmatrix} 0 \\ kx^3 \end{pmatrix} = - \begin{pmatrix} x \\ p_x \end{pmatrix} \Rightarrow \begin{cases} x = \pm \sqrt{\frac{2}{k\beta_x}} \operatorname{ctg} \frac{\mu_x}{2} \\ p_x = -\frac{1}{\beta_x} \left(\alpha_x + \operatorname{ctg} \frac{\mu_x}{2} \right) x \end{cases}$$

Horizontal phase portraits with tilt correction for $p_{x,0} = \pm c x_0$:



Working point optimization

Scanning along the
coupling resonance
 $Q_x = Q_y$.

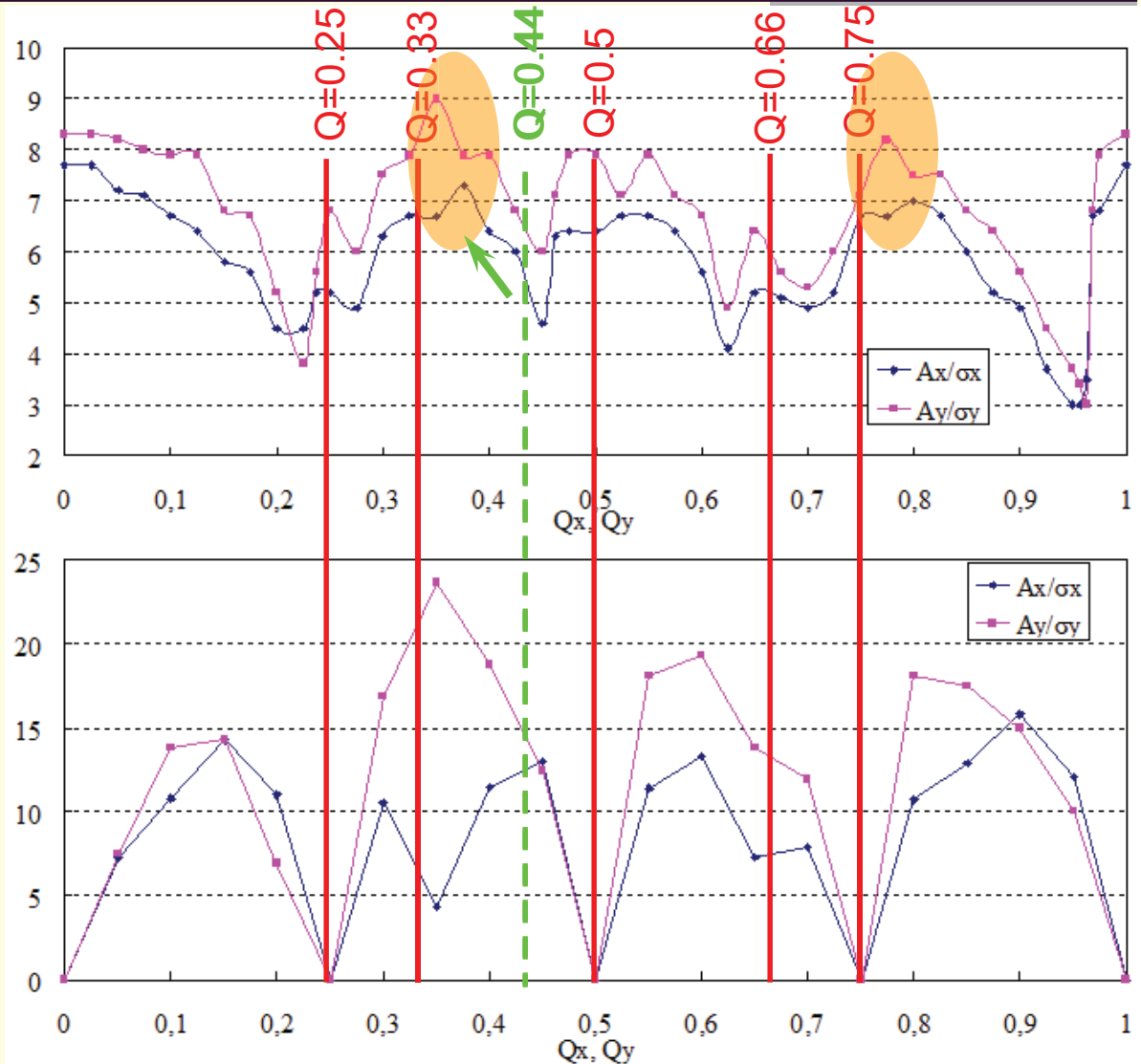
Sextupoles + fringes:

Strong resonances

Current working point

Good areas

Sextupoles only:

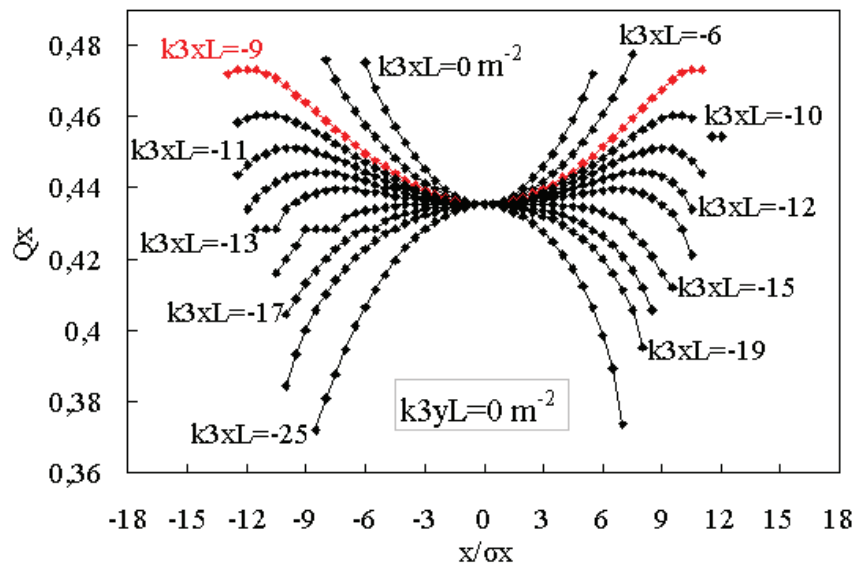


Octupole optimization

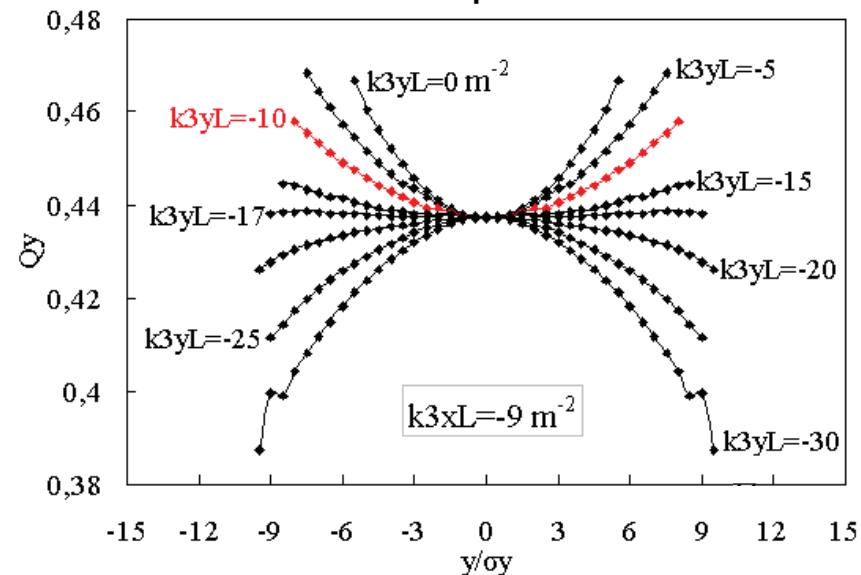
$$\begin{cases} \Delta Q_x \approx \frac{1}{16\pi} \sum_o (k_{3,o} l_o) (\beta_{x,o}^2 I_x - 2\beta_{x,o} \beta_{y,o} I_y) + \frac{1}{8\pi} \sum_q (k_{1,q} l_q) k_{1,q} (\beta_{x,q}^2 I_x + 2\beta_{x,q} \beta_{y,q} I_y) \\ \Delta Q_y \approx \frac{1}{16\pi} \sum_o (k_{3,o} l_o) (\beta_{y,o}^2 I_y - 2\beta_{x,o} \beta_{y,o} I_x) + \frac{1}{8\pi} \sum_q (k_{1,q} l_q) k_{1,q} (\beta_{y,q}^2 I_y + 2\beta_{x,q} \beta_{y,q} I_x) \end{cases}$$

We place 2 families of thin octupoles into the chromatic correction sextupoles. We optimise x-family first (step 1), because pure horizontal motion does not disturb vertical one, then optimize y-family (step 2).

Step 1:



Step 2:



Linear lattice optimization

■ Constraints:

- $\alpha_{IP} = 0$
- $\beta_{max} : 250m \rightarrow 200m$
- $Q_{x,y} : 0.43 \rightarrow 0.39$
- $\alpha_{x,y}, \beta_{x,y}$ are unchanged at the arc entrances

■ Variables:

- 3+3 FF quads (individually on each side)
- 6+6 other quads in straight section

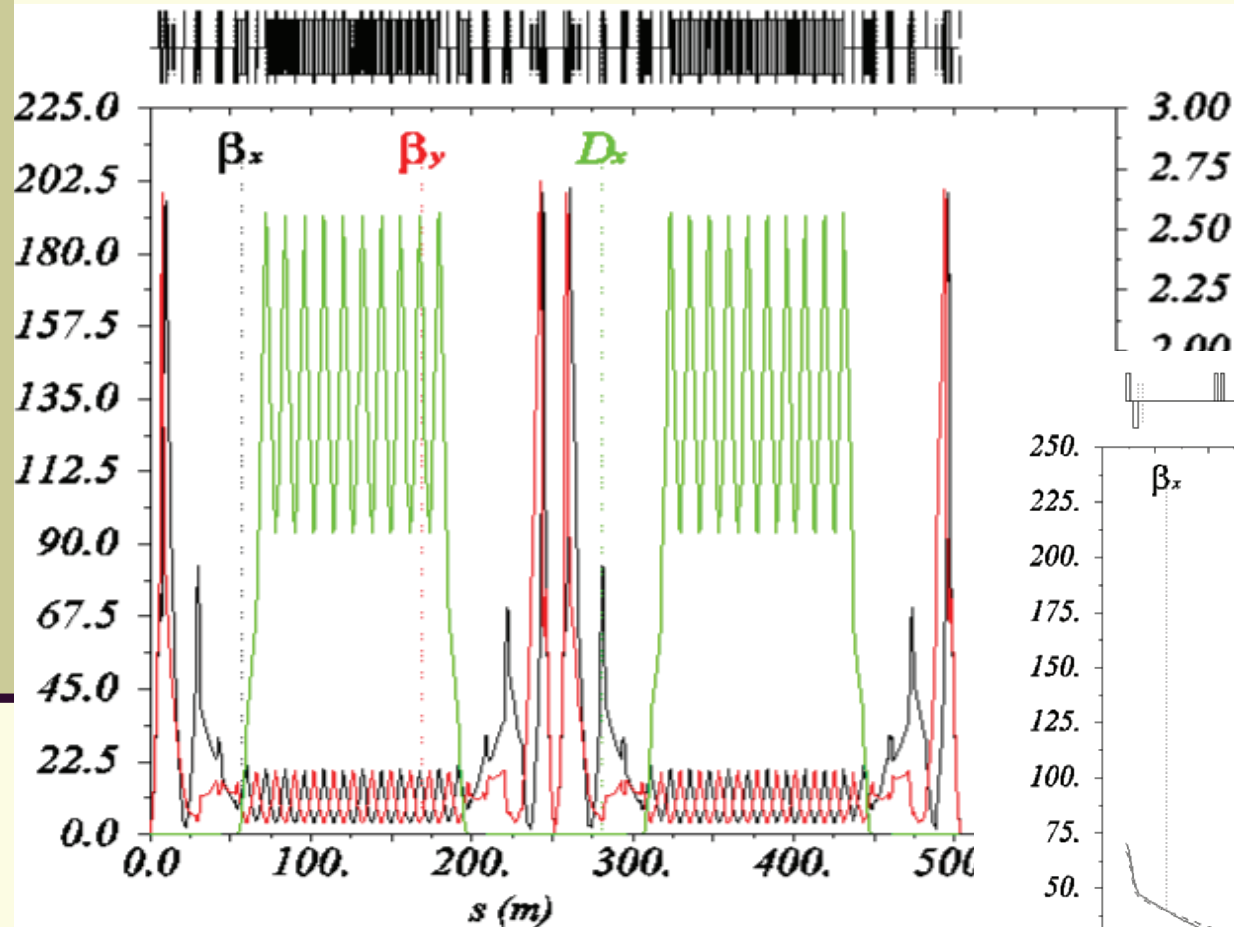
■ Side effects:

- $\beta_{IP} : 0.35m \rightarrow 0.41m$

Quadrupole strengths

	initial, m ⁻¹	modified, m ⁻¹	Δ , %
kqdt6i	0,282	0,257	-8,85
kqft6i	-0,329	-0,311	-5,45
kqdt7i	-0,331	-0,371	+12,09
kqft7i	0,094	0,131	+39,13
kqt9i	0,391	0,312	-20,26
kqt10i	0,000	0,033	-----
kqff3i	-0,416	-0,436	+4,76
kqff2i	0,437	0,431	-1,39
kqff1i	-0,433	-0,391	-9,62
kqff1	0,414	0,363	-12,43
kqff2	-0,409	-0,402	-1,72
kqff3	0,407	0,448	+10,12
kqt10	0,000	-0,081	-----
kqt9	-0,256	-0,231	-9,65
kqft7	0,488	0,417	-14,62
kqdt7	-0,514	-0,407	-20,84
kqft6	-0,161	-0,170	+5,80
kqdt6i	0,069	0,004	-94,10

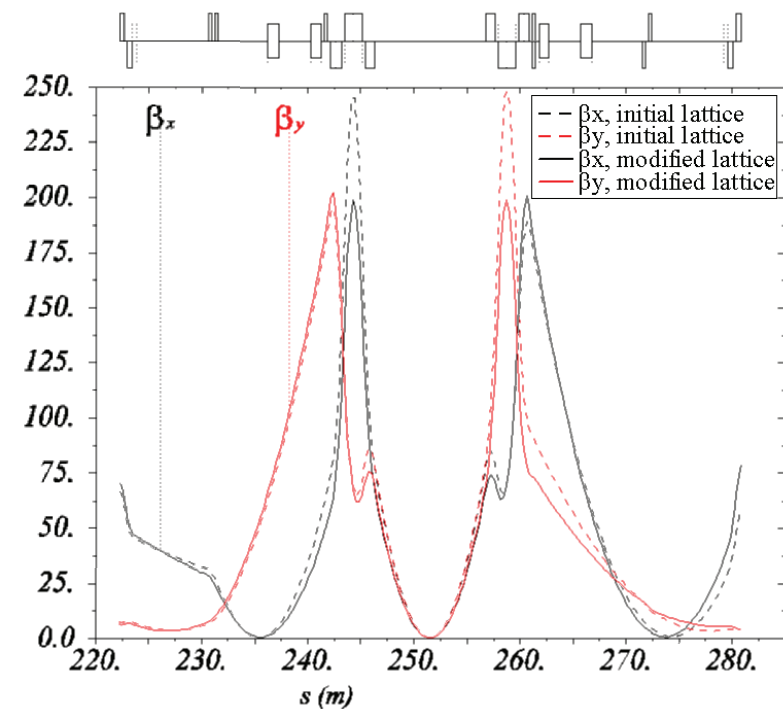
Modified lattice optics



$$\beta_{\max} = 200 \text{ m}$$

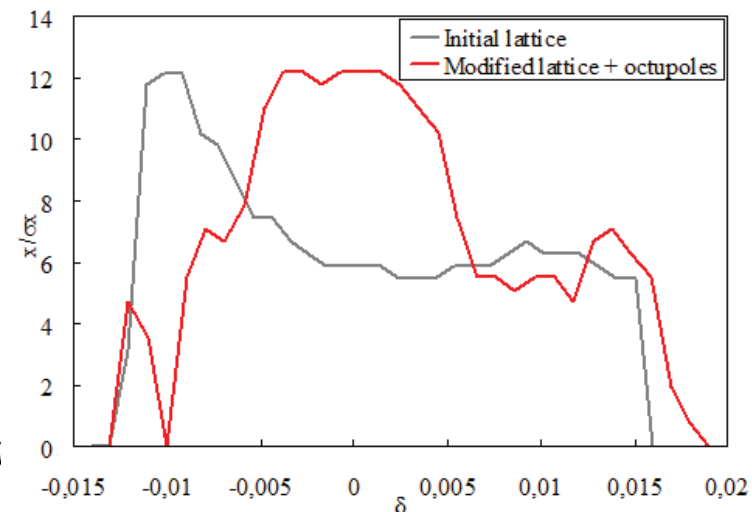
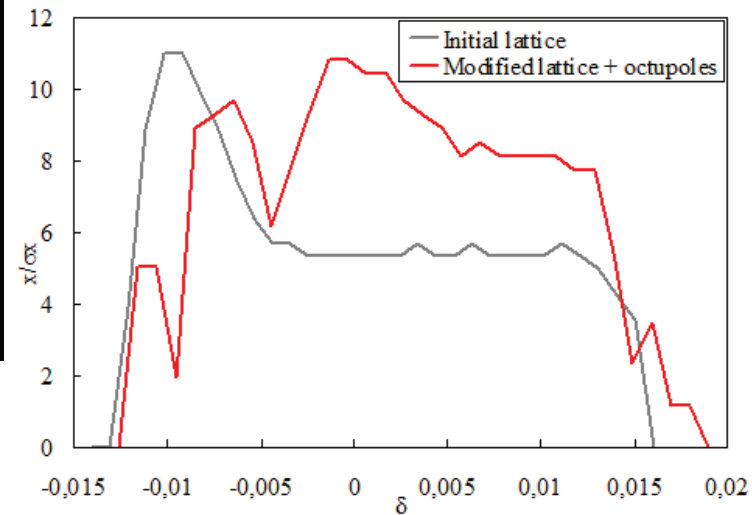
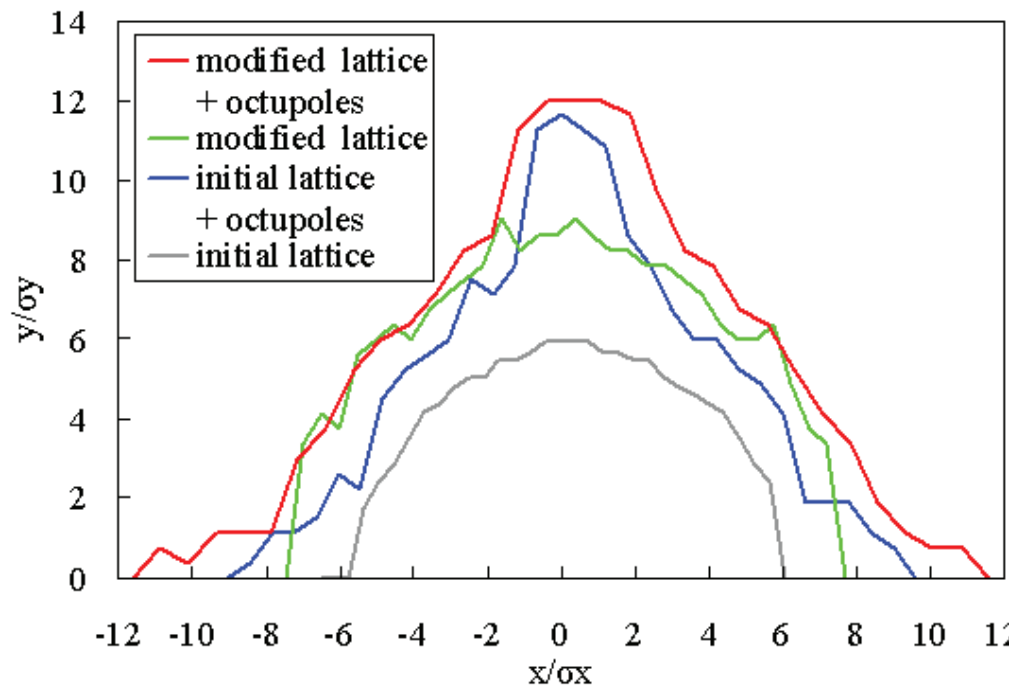
Interaction
region

Whole ring



Optimized dynamic aperture

	Initial lattice	Modified lattice
Transversal DA	$5.8\sigma_x \times 5.8\sigma_y$	$11.5\sigma_x \times 12\sigma_y$
Energy acceptance	-0.013...0.016	-0.01...0.019



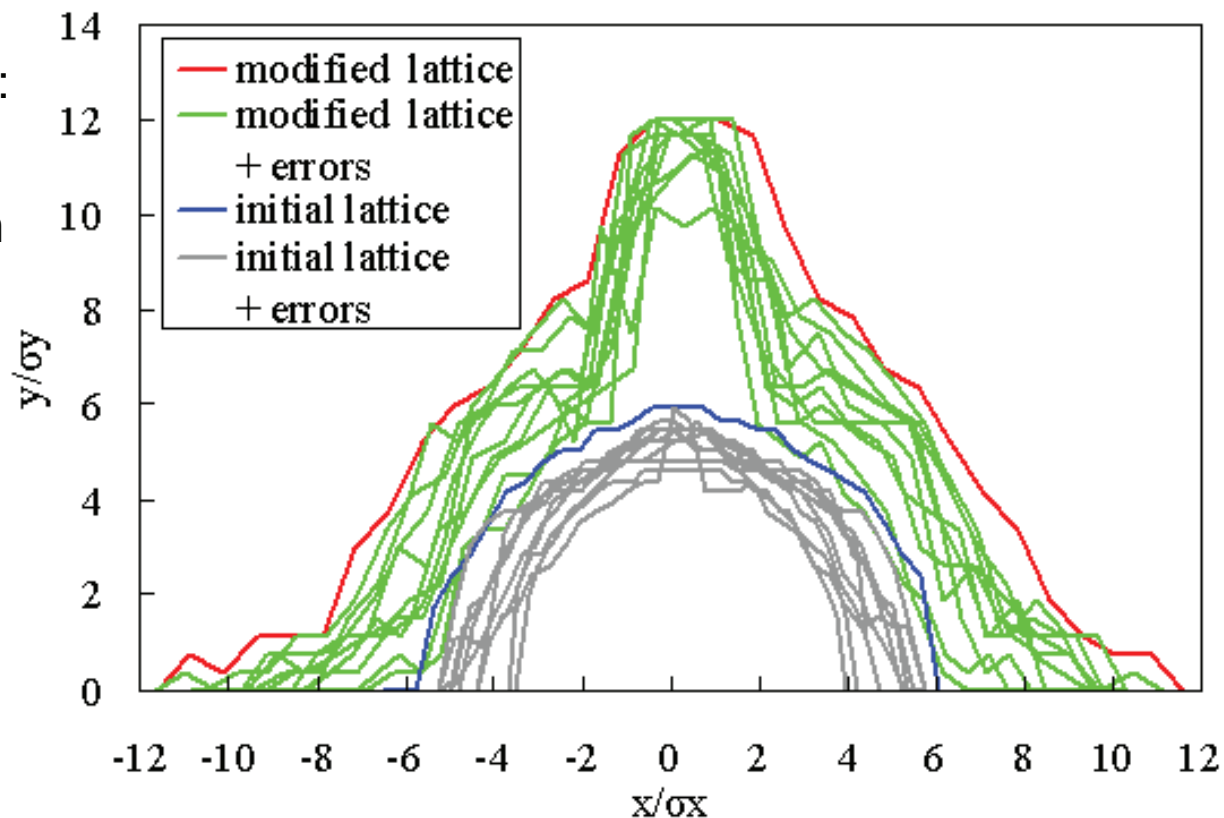
Octupole errors and misalignments

Octupole errors in quads:

Gaussian with cut at 2σ ,
 σ is 10^{-4} of quad strength
at $r = 3$ cm.

Sextupole+octupole
misalignments:

Gaussian with cut at 2σ ,
 $\sigma = 0.1$ mm.



Future plans

- Implement closed orbit correction module.
- Assign realistic misalignment distribution.
- Implement intrabeam scattering (IBS) simulation.
- Implement space charge simulation.
 - Symplectic transversal & longitudinal kick.
- Perform DA optimization with >2 octupole families.
 - Make use of genetic algorithm.
- Implement quadrupoles with “soft fringes”.
 - Use design quadrupole field maps.

Summary

- Nonlinear fringe fields of FF quad triplet reduce DA to $<6\sigma$.
- Octupoles may partially cure this effect.
- Sextupole misalignment & octupole field errors may reduce DA further to $<4\sigma$.
- Linear lattice optimization is needed to reduce strength of FF quadrupoles and β -functions in them.
- Working point optimization is needed.
- Optimal octupole distribution along the lattice should be found to maximize DA.



Thank you for your kind attention!