WAKE FIELD COMPONENTS IN A RECTANGULAR ACCELERATING STRUCTURE WITH DIELECTRIC ANISOTROPIC LOADING

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ABSTRACT

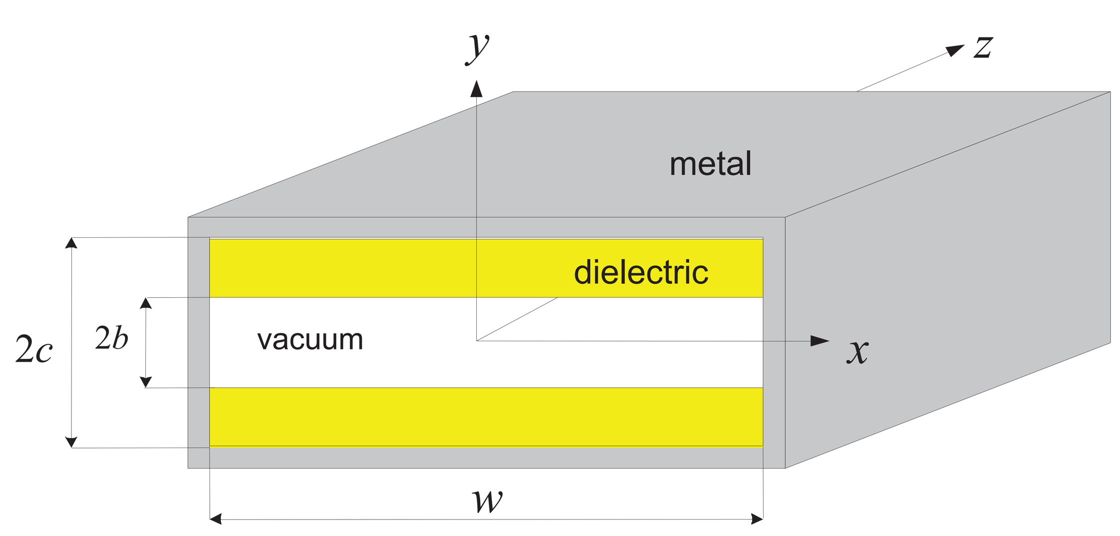
Dielectric lined waveguides are under extensive study as accelerating structures that can be excited by electron beams. Rectangular dielectric structures are used both in proof of principle experiments for new accelerating schemes and for studying the electronic properties of the structure loading material. Some of the materials used for the waveguide loading of accelerating structures possess significant anisotropic properties. General solutions for the fields generated by a relativistic electron beam propagating in a rectangular dielectric waveguide have been derived using the mode expansion method for the transverse operators of the Helmholtz equation. An expression for the combined Cherenkov and Coulomb fields obtained in terms of a superposition of LSM and LSE-modes of rectangular waveguide with anisotropic dielectric loading has been obtained. Numerical modelling of the longitudinal and transverse (deflecting) wake fields has been carried out. It is shown that the dielectric anisotropy influences to excitation parameters of the dielectric-lined waveguide with the anisotropic loading.

DIELECTRIC WAKEFIELD ACCELERATION

Wakefield acceleration principle is based on a generation by high-current charged particles bunch in the waveguide structure of an electromagnetic wave with a longitudinal component of the electric field. This wake field is used to accelerate a following low-current bunch of high energy. In free electron lasers electromagnetic wave generated by high-current electron bunch is extracted from the waveguide and structure is used as an electromagnetic radiation source.

Dielectric wakefield accelerating structures are single or multilayer dielectric waveguides with outer metal covering and vacuum channel along the axis. Along with the longitudinal fields there are transverse fields, leading to bunch deflection from the axis of the waveguide and subsidence of particles on its wall.

RECTANGULAR DIELECTRIC WAKEFIELD STRUCTURE



Rectangular waveguide provides a number of technological and constructive advantages in comparison with a traditional cylindrical waveguide. Such structures (along with cylindrical structures) for generating electromagnetic radiation and producing wakefield acceleration in the frequency range 0.5–1.0 THz are considered. In THz range, the planar geometry can be preferable because of difficulties of precise cylindrical structure manufacture. Rectangular structures can be used for test experiments in analysis of new accelerating systems and for studying the properties of materials effective for producing high acceleration rates and pulsed heating of the structure (diamond, sapphire).

Anisotropic properties are inherent in these materials. It demands research of influence of materials anisotropy on structure excitement.

Field Solution

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_{\parallel}(y) & 0 & 0 \\ 0 & \varepsilon_{\perp}(y) & 0 \\ 0 & 0 & \varepsilon_{\parallel}(y) \end{pmatrix} \qquad \hat{\mu} = \begin{pmatrix} \mu_{\parallel}(y) & 0 & 0 \\ 0 & \mu_{\perp}(y) & 0 \\ 0 & 0 & \mu_{\parallel}(y) \end{pmatrix} \qquad \zeta = z - vt$$

$$E_y = \sum_{n,m} \frac{\psi(x,x_0)}{\varepsilon_0} Y_{E_y n,m}(y) Y^*_{Ed}(y_0) \frac{S_{En,m}(\zeta, z_0)}{\sqrt{|\lambda_E|}}, \qquad H_y = -v \sum_{n,m} \psi'(x,x_0) Y_{H_y n,m}(y) Y^*_B(y_0) \frac{S_{H\,n,m}(\zeta, z_0)}{\sqrt{|\lambda_H|}},$$

$$E_z = -\sum_{n,m}^{\infty} \frac{\psi(x,x_0)}{\varepsilon_0} \left[\frac{G_E(\zeta,z_0) Y_{Ed}(y) Y^*_{Ed}(y_0)}{\left(k_{xn}^2 + \lambda_E\right)} + \frac{k_{xn}^2 \beta^2 G_H(\zeta,z_0) Y_{B_y}(y) Y^*_B(y_0)}{\left(k_{xn}^2 + \lambda_H\right)} \right], \quad H_z = v \sum_{n,m} \psi'(x,x_0). \left[\frac{G_H(\zeta,z_0) Y_{Hd}(y) Y^*_B(y_0)}{\left(k_{xn}^2 + \lambda_H\right)} + \frac{G_E(\zeta,z_0) Y_{D_y}(y) Y^*_{Ed}(y_0)}{\left(k_{xn}^2 + \lambda_E\right)} \right],$$

$$E_x = \sum_{n,m}^{\infty} \frac{\psi'(x,x_0)}{\varepsilon_0} \left[\frac{S_E(\zeta,z_0) Y_{Ed}(y) Y^*_{Ed}(y_0)}{\sqrt{|\lambda_E|}\left(k_{xn}^2 + \lambda_E\right)} - \frac{\lambda_H S_H(\zeta,z_0)\beta^2}{\sqrt{|\lambda_H|}\left(k_{xn}^2 + \lambda_H\right)} Y_{B_y}(y) Y^*_B(y_0) \right], \quad H_x = v \sum_{n,m} \psi(x,x_0) \left[\frac{k_{xn}^2 S_H(\zeta,z_0) Y_{Hd}(y) Y^*_B(y_0)}{\sqrt{|\lambda_H|}\left(k_{xn}^2 + \lambda_H\right)} - \frac{\lambda_E S_E(\zeta,z_0) Y_{D_y n,m}(y) Y^*_{Ed}(y_0)}{\sqrt{|\lambda_E|}\left(k_{xn}^2 + \lambda_E\right)} \right].$$

$$G_{E,H}(\zeta,\zeta_0) = \begin{cases} \cos\left(\sqrt{\lambda_{E,H}}\,(\zeta - \zeta_0)\right)\theta(\zeta_0 - \zeta), \lambda_{E,H} \ge 0; \\ \dfrac{\operatorname{sign}(\zeta - \zeta_0)}{2} e^{-\sqrt{|\lambda_{E,H}|}|\zeta - \zeta_0|}, \lambda_{E,H} < 0, \end{cases} \qquad S_{E,H}(\zeta,\zeta_0) = \begin{cases} -\sin\left(\sqrt{\lambda_{E,H}}\,(\zeta - \zeta_0)\right)\theta(\zeta_0 - \zeta), \lambda_{E,H} \ge 0; \\ \dfrac{\operatorname{sign}(\zeta - \zeta_0)}{2} e^{-\sqrt{|\lambda_{E,H}|}|\zeta - \zeta_0|}, \lambda_{E,H} < 0. \end{cases}$$

$$\psi(x,x_0) = q \sin(k_{xn} x) \sin(k_{xn} x_0)$$

Dispersion equations

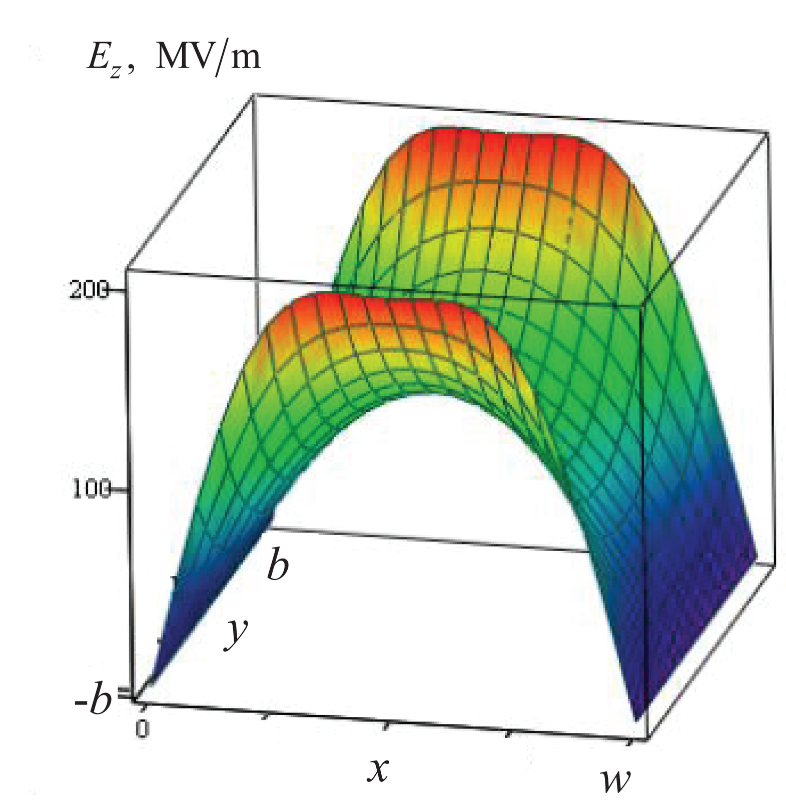
LM modes: $H_y = 0$ **LE modes:** $E_y = 0$

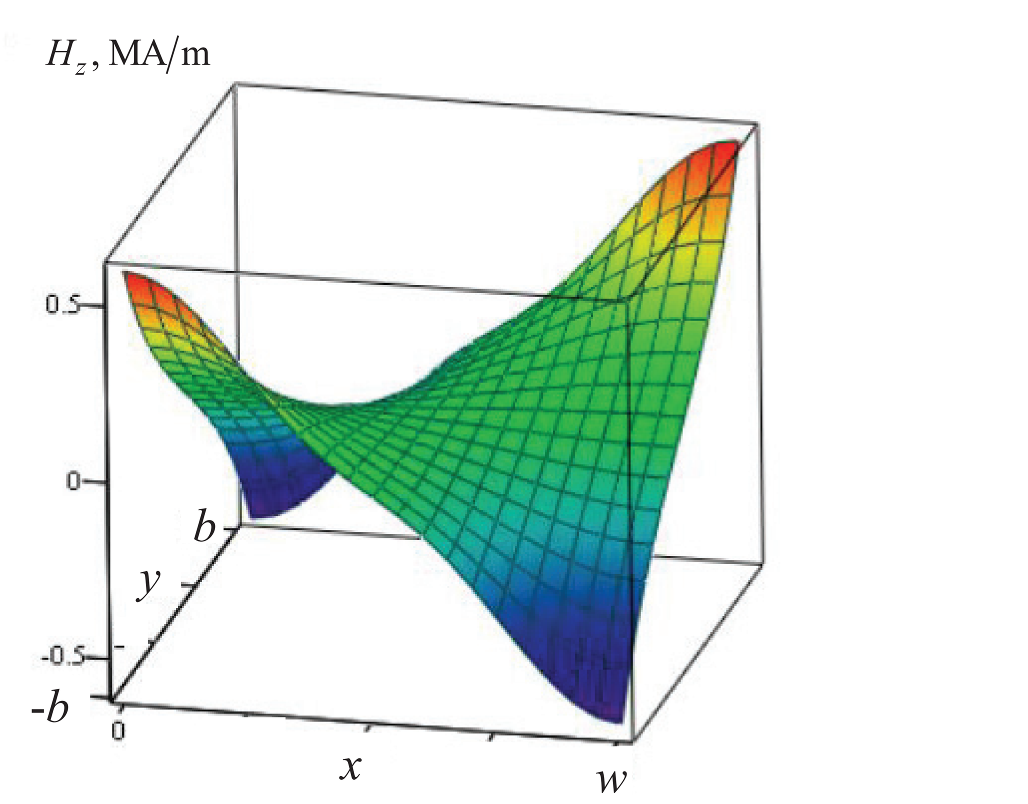
LM modes		LE modes
$\varepsilon_{\parallel 2} k_{y1} \operatorname{th}\left(k_{y1} b\right) - \varepsilon_1 k_{y2E} \operatorname{tg}\left(k_{y2E}(c-b)\right) = 0$	Symmetrical	$\mu_{\parallel 2} k_{y1} \operatorname{th}\left(k_{y1} b\right) + \mu_1 k_{y2H} \operatorname{ctg}\left(k_{y2H}(c-b)\right) = 0$
$\varepsilon_{\parallel 2} k_{y1} \operatorname{cth}\left(k_{y1} b\right) - \varepsilon_1 k_{y2E} \operatorname{tg}\left(k_{y2E}(c-b)\right) = 0$	Anti-symmetrical	$\mu_1 k_{y2H} \operatorname{th}\left(k_{y1} b\right) + \mu_{\parallel 2} k_{y1} \operatorname{tg}\left(k_{y2H}(c-b)\right) = 0$

Transverse functions

$$Y_{E_y}(y) = A_E \begin{cases} \dfrac{1}{\varepsilon_{\perp 2}} \cos(k_{y2E}(c-y)), \\ \begin{bmatrix} \mathrm{ch} \\ \mathrm{sh} \end{bmatrix}(k_{y1} y) \dfrac{\cos(k_{y2E}(c-b))}{\varepsilon_1 \begin{bmatrix} \mathrm{ch} \\ \mathrm{sh} \end{bmatrix}(k_{y1} b)}, \\ \dfrac{1}{\varepsilon_{\perp 2}} \cos(k_{y2E}(c+y)), \end{cases} \quad Y_{Ed}(y) = A_E \begin{cases} \dfrac{k_{y2E}}{\varepsilon_{\parallel 2}} \sin(k_{y2}(c-y)), \\ k_{y1} \begin{bmatrix} \mathrm{sh} \\ \mathrm{ch} \end{bmatrix}(k_{y1} y) \dfrac{\cos(k_{y2}(c-b))}{\varepsilon_{\parallel 1} \begin{bmatrix} \mathrm{ch} \\ \mathrm{sh} \end{bmatrix}(k_{y1} b)}, \\ -\dfrac{k_{y2}}{\varepsilon_{\parallel 2}} \sin(k_{y2}(c+y)), \end{cases}$$

$$Y_{H_y}(y) = A_H \begin{cases} \dfrac{1}{\mu_{\perp 2}} \sin(k_{y2H}(c-y)), \\ \begin{bmatrix} \mathrm{ch} \\ \mathrm{sh} \end{bmatrix}(k_{y1} y) \dfrac{\sin(k_{y2H}(c-b))}{\mu_1 \begin{bmatrix} \mathrm{ch} \\ \mathrm{sh} \end{bmatrix}(k_{y1} b)}, \\ \dfrac{1}{\mu_{\perp 2}} \sin(k_{y2H}(c+y)), \end{cases} \quad Y_{Hd}(y) = A_H \begin{cases} -\dfrac{k_{y2H}}{\mu_{\parallel 2}} \cos(k_{y2H}(c-y)), \\ k_{y1} \begin{bmatrix} \mathrm{sh} \\ \mathrm{ch} \end{bmatrix}(k_{y1} y) \dfrac{\sin(k_{y2H}(c-b))}{\mu_1 \begin{bmatrix} \mathrm{ch} \\ \mathrm{sh} \end{bmatrix}(k_{y1} b)}, \\ \dfrac{k_{y2H}}{\mu_{\parallel 2}} \cos(k_{y2H}(c+y)), \end{cases}$$

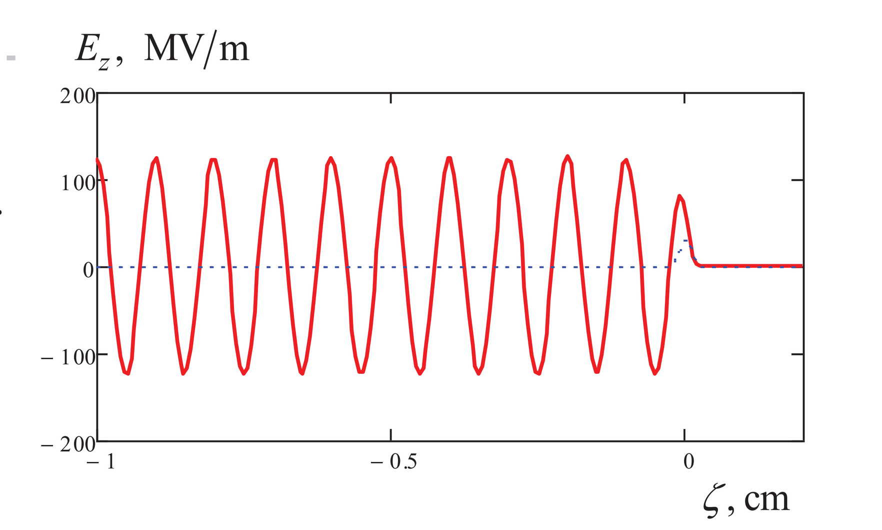




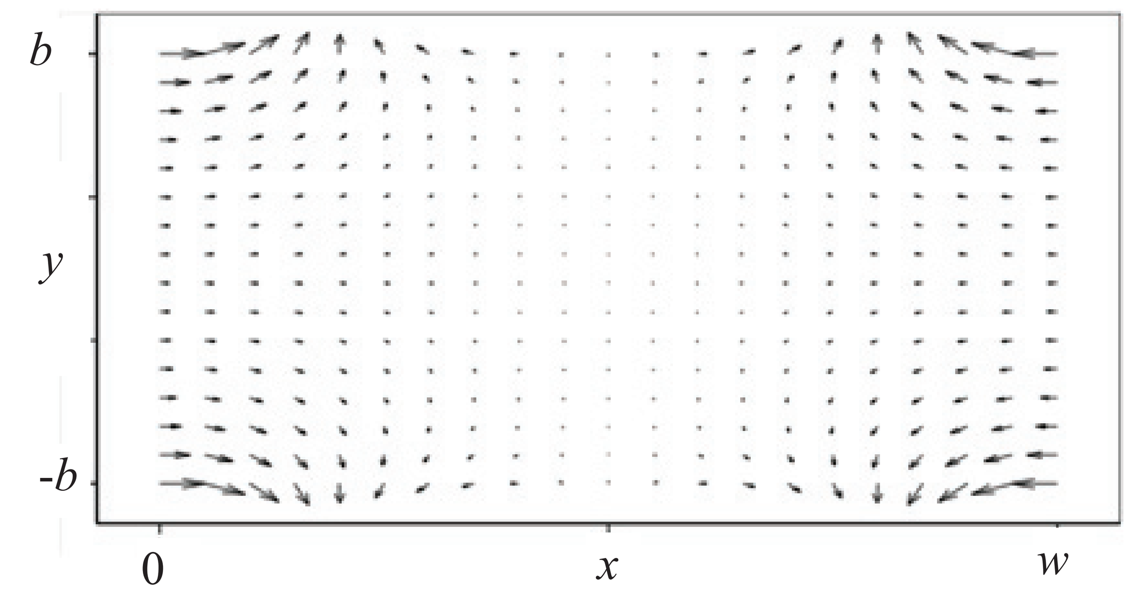
Transverse fields

$$\frac{F_y}{-e} = \sum_{n,m} \frac{\psi(x,x_0)}{\varepsilon_0} \left[\left(\frac{S_E(\zeta,z_0)}{\sqrt{|\lambda_E|}} \left(\frac{1}{\varepsilon_\perp} - \frac{\mu_\parallel \lambda_E \beta^2}{k_{xn}^2 + \lambda_E} \right) \right) Y_{D_y}(y) Y^*_{Ed}(y_0) + \frac{k_{xn}^2 \beta^2 \mu_\parallel S_H(\zeta,z_0) Y_{Hd}(y) Y^*_B(y_0)}{\sqrt{|\lambda_H|}\left(k_{xn}^2 + \lambda_H\right)} \right].$$

$$\frac{F_x}{-e} = \sum_{n,m}^{\infty} \frac{\psi'(x,x_0)}{\varepsilon_0} \left[\frac{S_E(\zeta,z_0) Y_{Ed}(y) Y^*_{Ed}(y_0)}{\sqrt{|\lambda_E|}\left(k_{xn}^2 + \lambda_E\right)} + \frac{S_H(\zeta,z_0) k_{xn}^2 \beta^2 Y_{B_y}(y) Y^*_B(y_0)}{\sqrt{|\lambda_H|}\left(k_{xn}^2 + \lambda_H\right)} \right],$$



$\mathbf{F}_\perp$



SUMMARY

We have proposed an analytic method for calculating wake fields of Vavilov – Cherenkov radiation in a rectangular accelerating structure with anisotropic dielectric filling. Using this method for the AWA / APS accelerator, we have analyzed the sapphire based dielectric structure with a rectangular cross section, in which accelerating gradients higher than 100 MV/m can be attained.