ON MODELING AND OPTIMIZATION OF INTENSE QUASIPERIODIC BEAM DYNAMICS

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Abstract

The paper is devoted to quasiperiodic beam dynamics investigation. Particle density is modeled bv trigonometric polynomial. Space charge field is represented in the similar form. This approach is applied to beam dynamics investigation in klystron-type buncher. algorithm of polynomial coefficients Numerical calculation from the positions and impulses of model particles is formalized. As a result Coulomb field intensity is expressed in the form of integral over the set of particle phase states. Integro-differential beam evolution model is presented. Analytical expression of the variation of beam dynamics quality criterion is obtained. It makes possible directed methods using for beam dynamics optimization.

BEAM DYNAMICS EQUATIONS

Consider quasiperiodic beam dynamics in accelerator or some beam forming system. Let us take klystron buncher as an example (the bunching process is supposed to be adiabatic). The channel is supposed to be cylindrical tube of radius a. Let us introduce the cylindrical coordinates r, θ, z with Oz axis coincided the channel axis.

Beam evolution is simulated on the basis of particle-incell method. Model particles are supposed to be "thick" disks with radius R. Dynamics equations are as follows:

$$\frac{dz_i}{d\tau} = \frac{p_i}{\sqrt{1+p_i^2}}, \quad \frac{dp_i}{d\tau} = -\frac{e}{m_0 c^2} \left(E_i^{(RF)} - E_i^{(int)} \right). \quad (1)$$

Here $\tau = ct \in [0,T]$, *t* is the time, *c* is the velocity of light; z_i and p_i are longitudinal coordinate and reduced impulse of *i*-th particle; *e* and m_0 are absolute charge value and rest mass of electron; $E_i^{(RF)}$ and $E_i^{(int)}$ are the intensity functions characterizing the action on model particle of RF and Coulomb fields correspondingly.

PARTICLE INTERACTION ACCOUNT

Assume that independent variable value τ is fixed. We suppose beam spatial quasiperiod to be cylinder $[0, R] \times [0, 2\pi) \times [z_c - H, z_c + H)$ where z_c is center coordinate; the cylinder is charged uniformly across the radius. We presume the beam to be periodic when calculate space charge forces. Coulomb field calculation algorithm is as follows.

1. Introduction of longitudinal coordinate grid

 $\left\{ \xi_j = z_c - H + 2jh, \ j = \overline{0, 2M} \right\}$, where 2h = H/M.

2. Calculation of grid cell charges q_j , $j = \overline{0, 2M - 1}$ with the use of clouds-in-cells method. It is supposed that $q_{2M} = q_0$ due to beam spatial periodicity. Approximation of bunch charge density by piecewise constant function $\{\tilde{S}(z, z_c) = q_j / (2h\pi R^2), z \in [\xi_j - h, \xi_j + h], j = \overline{0, 2M - 1}\}$.

3. Approximation of the function $\tilde{S}(z, z_c)$ by trigonometric polynomial $S(z - z_c)$ taking the values $S_j = q_j / (2h\pi R^2), \ j = \overline{0, 2M - 1}$ at grid points [1-3]:

$$S(\varsigma) = \sum_{m=0}^{M} \left[A_m \cos(m\pi\varsigma/H) + B_m \sin(m\pi\varsigma/H) \right].$$
(2)

The coefficients $A_m, B_m, m = \overline{0, M}$ are expressed by trigonometric interpolation formulae:

$$A_{m} = \frac{(-1)^{m}}{M \nu_{m}} \sum_{j=0}^{2M-1} S_{j} \cos(m \ j \ \pi/M), \ m = \overline{1, M},$$

$$B_{m} = \frac{(-1)^{m}}{M} \sum_{j=0}^{2M-1} S_{j} \sin(m \ j \ \pi/M), \ m = \overline{0, M}, \quad (3)$$

$$A_{0} = Q/(2H\pi R^{2}), \ \nu_{m} = [1 + m/M],$$

where Q is bunch charge value, v_m is the integer part of the value 1 + m/M.

4. Calculation of potential field intensity characterizing the periodic beam action on the model particle. The intensity expression is derived on the basis of potential function obtained by Poisson equation solving with righthand part proportional the polynomial $S(z-z_c)$ [2-4]. The intensity calculation formula is as follows:

$$E_{i}^{(int)} = C \sum_{k=1}^{\infty} D_{k} \sum_{m=1}^{M} C_{km}(p_{c}) \times \\ \times \left[A_{m} \Gamma_{sm}(z_{i}, z_{c}, p_{c}) - B_{m} \Gamma_{cm}(z_{i}, z_{c}, p_{c}) \right],$$
(4)

where

$$C = \frac{(2\pi Ra)^2 H}{\varepsilon_0}; D_k = \frac{J_1^2(\mu_k R/a)}{\mu_k^2 J_1^2(\mu_k)};$$
$$C_{km}(p) = \frac{m}{(m\pi a)^2 + \mu_k^2 H^2(1+p^2)};$$

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$$\Gamma_{cm}(z_i, z_c, p) = \int_{z_i-d}^{z_i+d} \cos\left(\frac{m\pi}{H}(z-z_c)\right) \chi(z-z_i, p) dz;$$

$$\Gamma_{sm}(z_i, z_c, p) = \int_{z_i-d}^{z_i+d} \sin\left(\frac{m\pi}{H}(z-z_c)\right) \chi(z-z_i, p) dz.$$

Here ε_0 is electric constant, $J_1(x)$ is first order Bessel function, $\mu_k, k = 1, 2, ...$ are the zeros of Bessel function $J_0(x)$; p_c is the average reduced impulse of the bunch; $\chi(x, p_c)$ is model particle form coefficient; 2d is cloud size.

Intense beam dynamics was investigated for klystrontype buncher with following main characteristics: initial energy of electrons $W_0 = 0.5$ MeV, average beam current I = 15 A [2]. Beam dynamics simulation code was developed in cooperation with B.S. Zhuravlev. Two interaction account modes were realized: "disks-in-cells" model [3] using Fourier-Bessel series and described above model using trigonometric polynomial. The analysis of numerical results obtained confirms both modes validity. Clearly, "trigonometric" model provides a significant smoothing of the processes under study and is preferred to use when electron bunches are mostly formed.

COULOMB FIELD INTEGRAL REPRESENTATION

Coulomb field representation in terms of model particle positions

Let us suppose form coefficient $\chi(x, p_c)$ to be continuous function taking zero values outside the cloud. Note that the coefficients $A_m, B_m, m = \overline{0, M}$ in formulae (2)-(4) are expressed in terms of grid cell charges. In their turn, grid cell charges may be expressed in terms of model particles positions $z_n, n = \overline{1, N}$, where N is particles number. For any j = 0, ..., 2M - 1

$$q_j = 2h\pi R^2 Q/N \sum_{n=1}^N \int_{-\infty}^{\infty} \chi(x - z_n, p_c) \Pi(x - \xi_j) dx, \quad (5)$$

So where $\Pi(x)$ is interpolation function defining the rule of Charge distribution in grid cells.

In view of Eq. (3)-(5), beam action on the i-th model particle has the representation of the following form:

$$E_i^{(int)} = \frac{1}{N} \sum_{n=1}^N V_n = \frac{1}{N} \sum_{n=1}^N V(z_i, z_c, p_c, z_n),$$
(6)

where $V(z_i, z_n, z_c, p_c)$ is the smooth function.

Phase density and Coulomb field integral form

Now we will take into account external field dependence on control vector **u**. Assume that τ value and vector function **u** are fixed. Let us consider particle distribution to be continuous; let $M_{\tau,u}$ be the domain of bunch particles phase states.

Let particle phase state (z, p) be a random variable with the values in the domain $M_{\tau,u}$ and $\rho(\tau, z, p)$ be probability density. Consequently, the expected value of any function U(z, p) defined on beam trajectories may be presented in the form $\int_{M_{\tau,u}} U(z_{\tau}, p_{\tau})\rho(\tau, z_{\tau}, p_{\tau})dz_{\tau}dp_{\tau}$.

Relying formula (6) and the law of large numbers, we can argue that

$$E_{i}^{(int)} \xrightarrow{P} \int_{M_{\tau,u}} V(z_{i}, \mathbf{x}_{c}, \hat{z}_{\tau}) \rho(\tau, \hat{z}_{\tau}, \hat{p}_{\tau}) d\hat{z}_{\tau} d\hat{p}_{\tau}, \quad (7)$$

$$N \to \infty \quad {}^{M_{\tau,u}}$$

where $\mathbf{x}_c = (z_c, p_c)$. The integral in right-hand part of formula (7) provides mathematical model of quasiperiodic beam Coulomb field.

INTEGRO-DIFFERENTIAL BEAM DYNAMICS MODEL

Consider beam evolution description with due account of the fields excited by the beam itself basing on the research conducted by Dmitri Ovsyannikov and his colleagues [5-11].

We will describe quasiperiodic beam dynamics by integro-differential equations. Mathematical models of such a class are widely applied in treatment of beam dynamics modeling and optimization problems [5,9,12-16].

Let us generalize beam dynamics model (1) taking into account the fields induced by moving beam itself [12]. Dynamic controlled process is described by the equations

$$\frac{d\mathbf{x}}{d\tau} = \mathbf{f}(\tau, \mathbf{x}, \mathbf{u}, \mathbf{F}(\mathbf{u})) = \mathbf{f}_{1}(\tau, \mathbf{x}, \mathbf{u}, \mathbf{F}(\mathbf{u})) + \int_{M_{\tau,\mathbf{u}}} \mathbf{f}_{2}(\tau, \mathbf{x}, \mathbf{y}_{\tau}) \rho(\tau, \mathbf{y}_{\tau}) d\mathbf{y}_{\tau},$$
(8)

$$\frac{\partial \rho}{\partial \tau} + \frac{\partial \rho}{\partial \mathbf{x}} f(\tau, \mathbf{x}, \mathbf{u}, \mathbf{F}(\mathbf{u})) + \rho \, div_{\mathbf{x}} \mathbf{f}(\tau, \mathbf{x}, \mathbf{u}, \mathbf{F}(\mathbf{u})) = 0 \quad (9)$$

with initial conditions

$$\mathbf{x}(0) = \mathbf{x}_0 \in M_0, \ \rho(0, \mathbf{x}) = \rho_0(\mathbf{x}).$$
 (10)

Here $\tau \in [0,T]$ is independent variable, T is constant, **x** is phase vector; **u** is control; vector function **f**₁ is

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determined by the method of external fields modeling; **F**(**u**) is the vector of values of functionals defined on beam trajectories; vector function \mathbf{f}_2 is determined by the method of particle interaction account; $\rho(\tau, \mathbf{x}, \mathbf{y})$ is phase density defined on system (8) trajectories; M_0 is the set of initial particle phase states; $\rho_0(\mathbf{x})$ is initial phase density; $M_{\tau,\mathbf{u}} = \{\mathbf{x}_{\tau} = \mathbf{x}(\tau, \mathbf{x}_0, \mathbf{u}) : \mathbf{x}_0 \in M_0\}$. Vector **F**(**u**) components are the values of the functionals

$$F_l(\mathbf{u}) = \int_{0}^{T} \int_{M_{\tau,u}} \Lambda_l(\tau, \mathbf{x}_{\tau}, \mathbf{u}) \rho(\tau, \mathbf{x}_{\tau}) d\mathbf{x}_{\tau} d\tau, \quad l = \overline{1, L},$$

describing the characteristics of RF fields excited by moving beam.

All the functions in the Eq. (8)-(10) are supposed to be rather smooth to obtain quality functional variation and gradient [5,12].

Mathematical model (8)-(9) may be applied to describe beam control process in klystron buncher. In this case we have $t = \tau$; $\mathbf{x} = (z, p)^T$. Besides, we suppose $p_c = p_{c0}$, $z_c = z_{c0} + \tau p_c / \sqrt{1 + p_c^2}$, where z_{c0} and p_{c0} are initial coordinate and initial reduced impulse of bunch centre. So the integrand in formula (7) may be presented as $V(\tau, z_i, \hat{z})$. Control vector \mathbf{u} is the vector of device parameters (resonator mismatches and drift tube lengths); the components of vector $\mathbf{F}(\mathbf{u})$ are the coefficients of Fourier-series expansion of induced current in resonators;

$$\mathbf{f}_{1} = \left(p / \sqrt{1 + p^{2}}, -\frac{e}{m_{0}c^{2}} E^{(RF)}(\tau, z, \mathbf{u}, \mathbf{F}(\mathbf{u})) \right)^{T},$$
$$\mathbf{f}_{2}(\tau, z, \hat{z}) = \left(0, \frac{e}{m_{0}c^{2}} V(\tau, z, \hat{z}) \right)^{T}.$$

The detailed RF fields description is given in [12,15-17].

OPTIMIZATION PROBLEM

The approach suggested by D.A. Ovsyannikov makes it possible to formulate different beam dynamics optimization problems as trajectory ensemble control problems; under certain conditions one can obtain quality criterion gradient. Such an approach is successfully applied by many researchers [9,13,18-27].

Let us estimate the controlled process (8)-(9) quality by the values of functional

$$I(\mathbf{u}) = \int_{0}^{T} \int_{M_{\tau,\mathbf{u}}} \Phi(\tau, \mathbf{x}_{\tau}, \mathbf{u}) \rho(\tau, \mathbf{x}_{\tau}) d\mathbf{x}_{\tau} d\tau \qquad (11)$$

with smooth integrand.

For example, when optimizing klystron buncher parameters, one can construct the integrand $\Phi(\tau, \mathbf{x}, \mathbf{u})$ to be positive for the particles satisfying the requirements imposed at device exit and to be zero (or negative) otherwise. Functional (11) is to be maximized.

Using the results [5,12] we obtain nonclassical variation of the functional (11):

$$\delta I(\mathbf{u}, \Delta \mathbf{u}) = -\int_{0}^{T} \int_{M_{\tau, u}} \left[\psi^{T}(\tau, \mathbf{x}_{\tau}) \Delta_{\mathbf{u}} f_{1}(\tau, \mathbf{x}_{\tau}, \mathbf{u}, \mathbf{F}) - \Delta_{\mathbf{u}} \Phi(\tau, \mathbf{x}_{\tau}, \mathbf{u}) - \mathbf{G}(\mathbf{u}) \Delta_{\mathbf{u}} \Lambda(\tau, \mathbf{x}_{\tau}, \mathbf{u}) \right] \rho(\tau, \mathbf{x}_{\tau}) d\mathbf{x}_{\tau} d\tau.$$
(12)

Here $\Delta \mathbf{u}$ is control \mathbf{u} variation; $\Delta_{\mathbf{u}}$ designates the increment of any function with respect to argument \mathbf{u} only; vector function $\psi(\tau, \mathbf{x})$ satisfies on the trajectories of dynamic process (8)-(9) the auxiliary system of integro-differential equations

$$\frac{d\Psi}{d\tau} = \left(\frac{\partial\Phi(\tau, \mathbf{x}(\tau))}{\partial\mathbf{x}}\right)^{T} - \left(\frac{\partial\mathbf{f}(\tau, \mathbf{x}(\tau), \mathbf{u}(\tau), \mathbf{F}(\mathbf{u}))}{\partial\mathbf{x}}\right)^{T} \Psi - \int_{M_{\tau,\mathbf{u}}} \left(\frac{\partial f_{2}(\tau, \mathbf{y}_{\tau}, \mathbf{x}(\tau))}{\partial\mathbf{x}}\right)^{T} \Psi(\tau, \mathbf{y}_{\tau})\rho(\tau, \mathbf{y}_{\tau})d\mathbf{y}_{\tau} - (13) - \left(\frac{\partial\mathbf{A}(\tau, \mathbf{x}(\tau), \mathbf{u})}{\partial\mathbf{x}}\right)^{T} \mathbf{G}^{T}(\mathbf{u})$$

with the following condition at $\tau = T$:

$$\boldsymbol{\psi}(T, \mathbf{x}(T)) = 0.$$

Vector G(u) in Eq. 12, 13 is the vector of values of functionals defined on beam trajectories:

$$\mathbf{G}(\mathbf{u}) = \int_{0}^{T} \int_{M_{\tau,u}} \boldsymbol{\psi}^{T}(\tau, \mathbf{x}_{\tau}) \frac{\partial f_{1}(\tau, \mathbf{x}_{\tau}, \mathbf{u}, \mathbf{F}(\mathbf{u}))}{\partial F} \rho(\tau, \mathbf{x}_{\tau}) d\mathbf{x}_{\tau} d\tau.$$

The analytical representation (12) of quality criterion variation makes it possible to use the directed optimization methods in beam dynamics optimization problems. It may be beneficial to combine the gradient optimization with random search [28].

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