THE INFINITELY THIN FIELD EMITTER MATHEMATICAL MODELING

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Abstract

In this work an axisymmetric diode electron-optical system based on a field emitter is simulated. The field emitter in the form of a thin filament of finite length is located on the flat substrate with the dielectric layer. The anode is a plane. The electrostatic potential distribution was found in an analytical form — in the form of Fourier-Bessel series in the whole area of the system under investigation. The coefficients of Fourier-Bessel series are the solution of the system of linear equations with constant coefficients.

INTRODUCTION

The field emitters as a nanostructured materials with nanometer-scale sharp tips are extensively applied in the various domains of nano-scale electronic devices [1]– [3].

This article is devoted to the modeling of the axially symmetric emission diode system on the field emitter basis [4]. The field emitter is a thin filament of finite length on the flat substrate [5]. The cathode substrate is coated with a dielectric layer. The anode is a plane. Fig. 1 shows a schematic representation of the diode system. The potentials of the emitter and substrate are zeros.

To find the distribution of the electrostatic potential $U(r,z)$ the variable separation method in the cylindrical coordinates (r,z) for the axially symmetric system is used [6].

The problem parameters:

$r = R_1$ — the radius of the system region,

$z = 0$ $(0 \le r \le R_1)$ — the surface of the emitter substrate,

$z = Z_1$ $(0 \le r \le R_1)$ — the boundary between two dielectrics,

Z_2 — the emitter length,

$z = Z_3$ $(0 \le r \le R_1)$ — the anode surface,

$U(r,0) = 0$ $(0 \le r \le R_1)$ — the boundary condition at the substrate,

$U(0,z) = 0$ $(0 \le z \le Z_2)$ — the boundary condition at the emitter,

$U(R_1,z) = f_1(z)$ $(0 \le z \le Z_3)$ — the boundary condition at the surface $r = R_1$,

$U(r,Z_3) = f_2(r)$ $(0 \le r \le R_1)$ — the boundary condition at the anode.

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MATHEMATICAL MODEL

The electrostatic potential distribution $U(r,z)$ is the solution of the boundary value problem for the Laplace equation

$$\begin{aligned} &\triangle\, U(r,z) = 0;\\ &U(r,0) = 0, \quad 0 \le r \le R_1;\\ &U(0,z) = 0, \quad 0 \le z \le Z_2;\\ &U(R_1,z) = f_1(z), \quad 0 \le z \le Z_3;\\ &U(r,Z_3) = f_2(r), \quad 0 \le r \le R_1. \end{aligned} \tag{1}$$

The conditions on the the boundary $z = Z_1$ between two dielectrics with the the dielectric constants ε_1 and ε_0 can be written as:

— continuity conditions of the potential distribution

$$U(r,z)\Big|_{Z_1-0} = U(r,z)\Big|_{Z_1+0}, \tag{2}$$

— the normal derivative of the electric displacement vector continuity conditions

$$\varepsilon_1 \frac{\partial U(r,z)}{\partial z}\Big|_{Z_1-0} = \varepsilon_0 \frac{\partial U(r,z)}{\partial z}\Big|_{Z_1+0}. \tag{3}$$

SOLUTION OF THE BOUNDARY – VALUE PROBLEM

To solve the boundary value problem (1)–(3) the interior of the diode system can be divided into three subdomains:

1 — $(0 \le r \le R_1,\ 0 \le z \le Z_1)$ with dielectric constants ε_1,

2 — $(0 \le r \le R_1,\ Z_1 \le z \le Z_2)$ with dielectric constants ε_0,

3 — $(0 \le r \le R_1,\ Z_2 \le z \le Z_3)$ with dielectric constants ε_0.

Then the potential distribution $U(r,z)$ for $0 \le r \le R_1$ can be represented as:

$$U(r,z) = \begin{cases} U_1(r,z), & 0 \le z \le Z_1,\\ U_2(r,z), & Z_1 \le z \le Z_2,\\ U_3(r,z), & Z_2 \le z \le Z_3. \end{cases} \tag{4}$$

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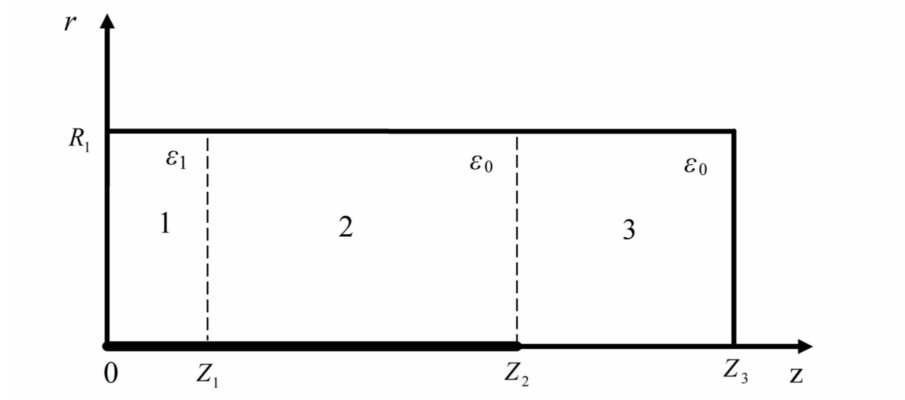

Figure 1: Schematic representation of the diode system.

According to the variable separation method functions (4) $U_i(r,z)$ $(i=1,2,3)$ are the Fourier-Bessel series:

$$U_1(r,z) = \sum_{n=1}^{\infty} A_n \frac{\sinh(\mu_{1,n} z)}{\sinh(\mu_{1,n} Z_1)} J_1(\mu_{1,n} r) + \\ + \sum_{m=1}^{\infty} B_m \frac{I_1(\nu_m r)}{I_1(\nu_m R_1)} \sin(\nu_m z), \tag{5}$$

$$U_2(r,z) = \sum_{n=1}^{\infty} \left[A_n \frac{\sinh(\mu_{1,n}(Z_2 - z))}{\sinh(\mu_{1,n}(Z_2 - Z_1))} + \right. \\ \left. + C_n \frac{\sinh(\mu_{1,n}(z - Z_1))}{\sinh(\mu_{1,n}(Z_2 - Z_1))} \right] J_1(\mu_{1,n} r) + \\ + \sum_{m=1}^{\infty} D_m \frac{I_1(\xi_m r)}{I_1(\xi_m R_1)} \sin(\xi_m (z - Z_1)), \tag{6}$$

$$U_3(r,z) = \sum_{n=1}^{\infty} \left[E_n \frac{\sinh(\mu_{0,n}(Z_3 - z))}{\sinh(\mu_{0,n}(Z_3 - Z_2))} + \right. \\ \left. + F_n \frac{\sinh(\mu_{0,n}(z - Z_2))}{\sinh(\mu_{0,n}(Z_3 - Z_2))} \right] J_0(\mu_{0,n} r) + \\ + \sum_{m=1}^{\infty} G_m \frac{I_0(\eta_m r)}{I_0(\eta_m R_1)} \sin(\eta_m (z - Z_2)), \tag{7}$$

where $J_i(\mu_{i,n} r)$ — Bessel functions of the first kind,
$\mu_{i,n} = \gamma_{i,n}/R_1$, $\gamma_{i,n}$ — the zeros of Bessel functions: $J_i(\gamma_{i,n}) = 0$ $(i = 0,1)$,
$I_1(\nu_m r)$, $I_0(\xi_m r)$, $I_0(\eta_m r)$ — modified Bessel functions of the first kind,
$\nu_m = \pi m / Z_1$,
$\xi_m = \pi m / (Z_2 - Z_1)$,
$\eta_m = \pi m / (Z_3 - Z_2)$.

The coefficients B_m, D_m and G_m for the potential distributions $U_i(r,z)$ (5)–(7) from the boundary conditions (1) at the surface $r = R_1$ can be calculated as:

$$B_m = \frac{2}{Z_1} \int_0^{Z_1} f_1(z) \sin(\nu_m z)\, dz, \\ D_m = \frac{2}{Z_2 - Z_1} \int_{Z_1}^{Z_2} f_1(z) \sin(\xi_m z)\, dz, \\ G_m = \frac{2}{Z_3 - Z_2} \int_{Z_2}^{Z_3} f_1(z) \sin(\eta_m z)\, dz. \tag{8}$$

The coefficients F_n for the potential distributions $U_3(r,z)$ (7) from the boundary conditions (1) at the anode surface $z = Z_3$ can be calculated as:

$$F_n = \frac{2}{R_1^2 J_0^2(\gamma_{0,n})} \int_0^{R_1} r\, f_2(r)\, J_0(\mu_{0,n} r)\, dr. \tag{9}$$

To find the coefficients A_n, C_n, E_n the continuity conditions can be used. The condition (2) is satisfied automatically with the formulas (4)–(6).

The normal derivative continuity of the electric displacement vector continuity conditions (3) can be written as

$$\varepsilon_1 \left. \frac{\partial U_1(r,z)}{\partial z} \right|_{z=Z_1} = \varepsilon_0 \left. \frac{\partial U_2(r,z)}{\partial z} \right|_{z=Z_1}. \tag{10}$$

The additional conditions at the boundary $z = Z_2$ are:

$$U_2(r, Z_2) = U_3(r, Z_2), \tag{11}$$

$$\left. \frac{\partial U_2(r,z)}{\partial z} \right|_{z=Z_2} = \left. \frac{\partial U_3(r,z)}{\partial z} \right|_{z=Z_2}. \tag{12}$$

Making use of the orthogonal systems of eigenfunctions, the formulas (10)–(12) lead to the system of the linear algebraic equations in the unknown coefficients A_n, C_n, E_n.

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CONCLUSION

In this article the axially symmetric emission diode system on the field emitter basis is presented. The diode system is defined as field cathode — a thin filament of finite length on the plane substrate and a plane anode. The cathode substrate is coated with a dielectric layer. The variable separation method is used in the cylindrical coordinates to find electrostatic potential distribution. To solve the boundary value problem (1)–(3) the interior of the diode system can be divided into three subdomains (4). The electrostatic potential distributions for each of the subdomain is found as the Fourier-Bessel series (5)–(7). Some of the coefficients (8), (9) are determined from the boundary conditions (1). The continuity conditions (10)–(12) on the subdomains interfaces are applied to calculate the remaining coefficients of Fourier-Bessel series as the solution of the linear algebraic equations system with the constant coefficients.

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