A SIMPLE MODEL FOR ELECTROMAGNETIC FIELD IN RFQ CHANNEL

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Abstract

Numerical solution of the RFQ structure optimization problem requires a great amount of computation. Each consecutive step of the numerical optimization includes modification of geometry of the channel and computation of electromagnetic field for modified geometry. Therefore, a simple model describing the field in the channel is needed for the optimization. Such model is proposed in this report. It differs from the commonly used traditional model of the field, which can be applied when profiles of the vanes are described by the harmonic functions of the longitudinal coordinate. Our model is more general and can be applied for arbitrary profiles of the vanes.

INTRODUCTION

Professor D.A. Ovsyannikov proposed an approach which consists in the optimization of accelerator channel based on control theory methods [1-3]. During the optimization the functional characterizing the beam quality and the functional gradient of parameters describing the accelerator channel are computed. After that the parameters change according to the values of functional gradient components. Then iterations repeat until the appropriate structure is found.

Since the channel parameters are changing during the optimization process, the electromagnetic field in the accelerator channel is also changing. Optimization of the RFQ channel requires a great amount of computation [4-9]. Therefore, precise computation of electromagnetic field in the channel at each step of optimization means that optimization process is not executable for a real time. By this reason we should use simple models of electromagnetic field.

Most known model of the electromagnetic field in the RFQ channel was proposed by I.M. Kapchisky [10]. But it is applicable only for the case when the vane modulation is quasi-periodic. Within the framwork of this model the field is described by piecewise harmonic functions.

Here we propose a new simple model applicable in most general case. It allows to compute the electromagnetic field dynamically for each new channel configuration. This article presents the development and the investigation of this model.

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ELECTRIC FIELDS MODELS

Electric field potential u satisfies to the wave equation:

$$\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} - \Delta u = 0,$$

Here t is the time, c is the light velocity.

If electromagnetic oscillation frequency is not very great, the first term in this equation is small and can be neglected and we have quasi-stationary approximation:

$$\Delta u = 0. \tag{1}$$

Assume that on the vanes surfaces the following conditions holds:

$$u_\Gamma = \pm u_0 \cos\omega t. \tag{2}$$

Here $U_0 = V/2$, and V is amplitude of intervane voltage. Assume also that the vane surfaces are described by the equations

$$r^2\cos 2\varphi = \mu(\pm 1 - \frac{4T}{\pi} I_0(kr)\sin\eta), \tag{3}$$

where $\eta(z) = \int\limits_{z_0}^{z} k(z')\,dz'$, $k(z)$ is function specifying dependency of the vane modulation along the longitudinal axis, I_0 is the modified Bessel function of the zeroth order. The interval where η is change from $(i-1)\pi$ to $i\pi$, $i = \overline{1, N}$, corresponds to one cell of the structure, and N is the total number of the cells.

Assume that $k(z)$ and $T(z)$ slowly change when z increases:

$$\frac{dk}{dz} \ll \frac{k(z_i)}{L_i} = \frac{k(z_i)^2}{\pi}.$$

Then length of the $i-$th cell is $L_i = \pi/k(z_i)$, where z_i can be any z inside the cell.

It is easy to see that the solution of the boundary problem (1), (2) is

$$u(r,\varphi,z) = -u_0(\frac{r^2\cos 2\varphi}{\mu} + \frac{4T}{\pi} I_0(kr)\sin\eta)\cos\omega t.$$

Denote the channel aperture by a. It is minimal with respect to all cross-section of the cell distance from the axis to the nearest vane. From the equation (3) we have

$$a^2 = \mu(1 - \frac{4T}{\pi} I_0(ka)). \tag{4}$$

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$$a^2 = \mu(1 - \frac{4T}{\pi} I_0(ka)). \tag{5}$$

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Then

$$\mu = \frac{a^2}{\varkappa}, \qquad \varkappa = 1 - \frac{4T}{\pi} I_0(ka),$$

and the solution of the boundary problem (1), (3) can be written in the form

$$u(r, \varphi, z) = -u_0(\varkappa \frac{r^2}{a^2} \cos 2\varphi + \frac{4T}{\pi} I_0(kr) \sin \eta) \cos \omega t. \tag{6}$$

Denote the distance from the axis to another vane in the section where distance to the nearest vane is minimal. It is maximum distance for this cell. Denote it by $r_{\max}$. It is easy to see from the equation (3) that

$$r_{\max}^2 = \mu(1 + \frac{4T}{\pi} I_0(kr_{\max})). \tag{7}$$

Introducing the modulation coefficient $m = r_{\max}/r_{\min}$, we have

$$m^2 = \frac{1 - \frac{4T}{\pi} I_0(kr_{\min})}{1 + \frac{4T}{\pi} I_0(kr_{\max})},$$

$$T = \frac{\pi}{4} \cdot \frac{m^2 - 1}{m^2 I_0(ka) + I_0(mka)}.$$

The expression for the field potential (6) is valid if vane shape is described by (3), that is the vanes section are hyperbolic. Assume that the potential is described in paraxial area by the same expression also for the case when the vanes sections are not hyperbolic.

In Cartesian coordinates from (6) we have

$$u(x, y, z) = -u_0(\varkappa \frac{x^2 - y^2}{a^2} + \frac{4T}{\pi} I_0(kr) \sin \eta) \cos \omega t. \tag{8}$$

Argument of the function I_0 is less then 1. Approximating it by first two terms of its expansion into the Taylor series $I_0(kr) \approx 1 + (kr)^2/4$, and differentiating we get Cartesian components of the electric field

$$E_x = u_0(\frac{2\varkappa}{a^2} x + \frac{2k^2 T}{\pi} x \sin \eta) \cos \omega t, \tag{9}$$

$$E_y = u_0(-\frac{2\varkappa}{a^2} y + \frac{2k^2 T}{\pi} y \sin \eta) \cos \omega t, \tag{10}$$

$$E_z = u_0 \frac{4kT}{\pi} I_0(kr) \cos \eta \cos \omega t \tag{11}$$

This field model is analogous to the known model of electromagentic field [10], and is widely used for numerical solution of the optimization problem [4-8].

The characteristic feature of this model is piecewise harmonic modulation of the vane.

Here we propose another field model for arbitrary rule of modulation:

$$u(x, y, z) = u_0 \frac{d_y(z) - d_x(z)}{d_x(z) + d_y(z)} + \frac{2u_0}{d_x(z) + d_y(z)}(x - y). \tag{12}$$

Here $d_x(z)$ and $d_y(z)$ are distances from the axis to electrodes along axes x and y corresdpondingly.

The remainder of this article is devoted to the investigation of the approximation quality of this model in a real RFQ channel. It is done by numerical solution of the corresponding boundary problem.

NUMERICAL SOLUTION OF THE BOUNDARY PROBLEM

Assuming that d_x and d_y slowly vary along longitudinal axis, we reduce the boundary problem (1), (2) to a set of two-dimensional boundary problems for various cross-sections of the channel depicted on Fig.1.

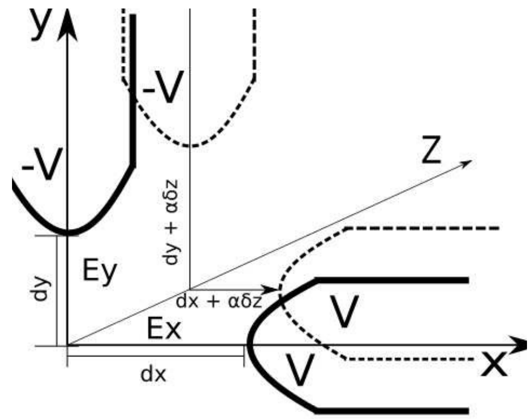

Figure 1: Cross-section of the channel.

Computation is implemented by iteration relaxation method applied for a system of finite difference equations corresponding so some grid in the region under consideration. Computational formulas correspond to various positions of a grid node where field is computed. For, example, for regular inner nodes we have the well known expression

$$U_i^j = \frac{1}{4}(U_{i-1}^j + U_{i+1}^j + U_i^{j-1} + U_i^{j+1}),$$

and for nodes adjacent to the boundary the following expressions:

$$U_i^j = \frac{(\frac{h}{\delta_x} U_0 + U_{i-1}^j)(\delta_y + h) + (\frac{h}{\delta_y} U_0 + U_i^{j+1})(\delta_x + h)}{(\frac{h}{\delta_x} + 1)(\delta_y + h) + (\frac{h}{\delta_y} + 1)(\delta_x + h)}$$

for the case $\delta_y < h$ and $\delta_x < h$,

$$U_i^j = \frac{h(\frac{h}{\delta_x} U_0 + U_{i-1}^j) + \frac{\delta_x + h}{2}(U_i^{j+1} + U_i^{j-1})}{\frac{h^2}{\delta_x} + 2h + \delta_x};$$

for the case $\delta_y \geq h$ and $\delta_x < h$,

$$U_i^j = \frac{2h(\frac{h}{\delta_y} U_0 + U_i^{j+1}) + \frac{\delta_y + h^2}{2}(U_{i+1}^j + U_{i-1}^j)}{\frac{h^2}{\delta_y} + 2h + \delta_y}.$$

for the case $\delta_y < h$ and $\delta_x \geq h$. Here h is a grid step, and δ_x, δ_y are components of displacement of the node from a nearest boundary.

RESULTS

Results of the investigation are presented on the fig.2,3. Dots represent numerically computed values of the potential at distinct cross-section. Lines represent the potential

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values computed according expression (12). Distances between the channel axes and the vanes change linearly in both cases.

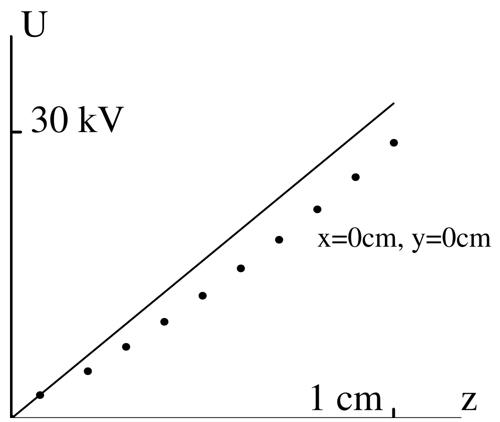

Figure 2: Variation of electric potential along beam axis at $x = 0cm, y = 0cm$.

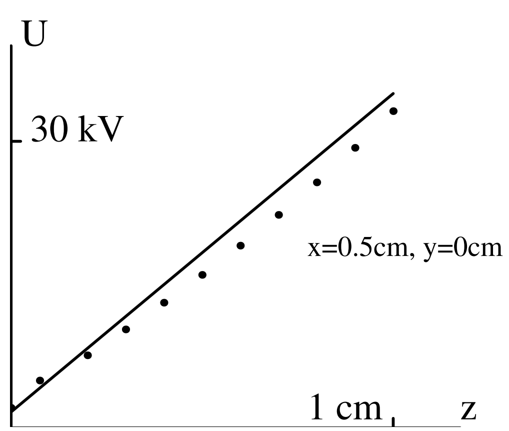

Figure 3: Variation of electric potential along beam axis at $x = 0.5cm, y = 0cm$.

One can see that difference between two models is about 10%. Therefore, the accuracy of the proposed simple model can be estimated as 10%. It means that the proposed simple linear model give good enough approximation, and can be used at an initial stage of optimization. We hope that it is possible to modify this model and to improve its accuracy.

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