INVESTIGATION OF A SECOND ORDER METHOD OF RFQ CHANNEL OPTIMIZATION

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Abstract

This report is devoted to a numerical method of solution of the RFQ structure optimization problem. The problem is considered as a control theory problem. Control functions representing geometry of the electrodes and a quality functional describing the beam are introduced. To solve the problem numerically these control functions are parametrized. The presented method is based on the computation of the derivatives of the first and the second order of the quality functional on the parameters. Results of investigation of efficiency of the method relatively to a method including computation of the derivatives only of the first order are presented.

INTRODUCTION

Methods of numerical optimization of accelerator structures were developed in the works of D.A. Ovsyannikov [1, 2, 3]. This theory was applied to the RFQ structure in the works [2, 4, 5, 6, 7, 8].

These methods have the first order, because the optimization process includes computation of derivatives of the first order of a functional describing beam quality over structure parameters. Application of these methods in practice requires a great amount of computations to achive acceptable results.

To reduce a number of computation, the method of the second order was proposed [9]. Here we present results of the second order method investigation, according to which number of computation during the process of optimization is reduced sufficiently.

FORMULATION OF THE PROBLEM

Consider a particles beam describing by the phase density [10, 11] $\varrho(x)$ defined on some surface S in the phase space $\Omega : x \in S \subset \Omega$. Coordinates on S can be taken as coordinates in the phase space. Let that at the initial instant t_0, the particle distribution density is given: $\varrho(t_0, x) = \varrho_{(0)}(x) = \varrho_{(0)\,1\ldots p}(x)\,dx^1 \wedge \ldots \wedge dx^p$, $x \in S_0 \subset \Omega$. At $t > t_0$, density $\varrho(x)$ can be found as the solution of the Vlasov equation [10, 11].

Let the trajectories of particles are described by the equation

$$\frac{dx}{dt} = f(t, x, u),$$

where $t \in T_0 = [t_0, T]$, $u \in U \subset R^r$ is a control function.

Also, introduce the following functional

$$\Phi(u) = \int\limits_S g(x_T)\varrho(T, x_T), \qquad (1)$$

where $g(x)$ is a piecewise continuous function. This functional characterizes quality of the beam at outlet of the accelerating channel. The problem of minimizing of functional (1) over control function $u \in U$ is called the terminal problem of charged particles beam control.

METHOD

Consider the equation for the first variation of x

$$\frac{d\delta x^i}{dt} = \frac{\partial f^i}{dx^j}\delta x^j + \delta_u f^i, \quad \delta x^i(t_0) = 0. \qquad (2)$$

Here and further, we apply the Einstein summation rule over repeated upper and low indices. The problem (2) has the following solution

$$\delta x^i(t) = \int\limits_{t_0}^{t} G^i_j(t, t')\delta_u f^j(t')dt',$$

where $G(t, t')$ is the Green matrix of the system (2), satisfying to the equation

$$\frac{dG^i_j(t, t')}{dt'} = G^i_k(t, t')\frac{\partial f^k}{\partial x^j},$$

and $G(t, t) = E$, where E is identity matrix.

Then variation of the functional (1) can be written as

$$\delta_u \Phi = \int\limits_{t_0}^{T}\int\limits_{\Omega} \frac{\partial g}{\partial x} G(T, t')\delta_u f(t, x)\varrho(t, x)\, dt. \qquad (3)$$

Consider the differential form

$$\psi(t, x) = -\left.\frac{\partial g}{\partial x}\right|_{x=x_T} G(T, t).$$

It satisfies the following equation and conditions

$$\frac{d\psi}{dt} = -\psi\frac{\partial f}{\partial x}, \qquad \psi(T) = -\left.\frac{\partial g}{\partial x}\right|_{x=x_T}.$$

Then expression (3) can be rewritten as follows

$$\delta_u \Phi = -\int\limits_{t_0}^{T}\int\limits_{\Omega} \psi(t, x)\delta_u f(t, x)\varrho(t, x)\, dt.$$

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ISBN 978-3-95450-181-6

Assume that the control function can be represented in parameterized form, depending on a finite number of parameters. Consider the second derivatives with respect to the same parameter $\partial^2\Phi/\partial u_i^2$. Then

$$\frac{\partial x^j}{\partial u_i}(t) = \int_{t'}^{t} G_k^j(t,t')\frac{\partial \delta_u f^k}{\partial u_i}(t')\,dt'. \qquad (4)$$

Assume that $\partial^2\Phi/(\partial x^i \partial x^j) = 0$, if $i \neq j$. And then

$$\frac{\partial^2\Phi}{\partial u_i^2} = \int_{\Omega} \varrho(T)\left[\frac{\partial\Phi}{\partial x^j}\frac{\partial^2 x^j}{\partial u_i^2}(T) + \frac{\partial^2\Phi}{\partial (x^j)^2}\left[\frac{\partial x^j}{\partial u_i}(T)\right]^2\right].$$

Passing to the summation on macroparticles we get

$$\frac{\partial^2\Phi}{\partial u_i^2} = \sum_{k=1}^{N}\left[\frac{\partial\Phi}{\partial x^j}\frac{\partial^2 x^j_{(k)}}{\partial u_i^2}(T) + \frac{\partial^2\Phi}{\partial (x^j)^2}\left[\frac{\partial x^j_{(k)}}{\partial u_i}(T)\right]^2\right],$$

where the first derivatives are expressed by (4).

It can be shown that when f^j are linear on the control parameters u_i, second variation of x can be represented in the form

$$\delta^2 x^j(t) = \int_{t_0}^{t} (D^j_{kl}(t,t')\delta_u f^k(t') + G^j_k(t,t')\times$$

$$\times\delta_u\left(\frac{\partial f^k}{\partial x^l}\right)\bigg|_{t'} \int_{t_0}^{t'} G^l_m \delta_u f^m(t'')\,dt'')\,dt',$$

where components of the tensor D satisfy to the system of differential equations

$$\frac{\partial D^j_{kl}(t,t')}{\partial t'} = -2D^j_{km}(t,t')\frac{\partial f^m}{\partial x^l}(t') + G^i_m(t,t')\frac{\partial^2 f^m}{\partial x^k \partial x^l}(t')$$

and the condition

$$D^j_{kl}(t,t) = 0,\;\; j = \overline{1,m},\;\; k = \overline{1,m},\;\; l = \overline{1,m}.$$

OPTIMIZATION

Let the longitudinal component of electric field in the RFQ channel is

$$E_z = U_0\frac{4kT}{\pi}\cos\eta\cos\omega t, \quad \eta(z) = \int_{z_0}^{z} k(z')\,dz'. \qquad (5)$$

Here $2U_0$ is the amplitude of the intervane voltage, ω is the frequency, a is the aperture, $k = \pi/L$, L is the cell length, $\eta(z)$ is the phase of electrode modulation, T is acceleration effiency.

Denote the difference between phase of synchronous particle φ_s and the phase of space modulation of the vanes η by Φ_s :

$$\Phi_s = \varphi_s - \int \overline{k}\,d\zeta. \qquad (6)$$

Here $\zeta = z/\lambda$, $\overline{k} = \lambda k$, $\lambda = 2\pi c/\omega$. Take function $u_1(\zeta) = d\Phi_s/d\zeta$ as the first control function, and T as the second control function: $u_2(\zeta) = T(\lambda\zeta)$. In this model, longitudinal motion can be considered separately from the transverse one. The equations of longitudinal dynamics for a beams of low density are

$$\frac{d\varphi}{d\zeta} = 2\pi\gamma(\gamma^2-1)^{-1/2}, \qquad (7)$$

$$\frac{d\gamma}{d\zeta} = C_L(2\pi\gamma_s(\gamma_s^2-1)^{-1/2} - u_1)u_2\cos\eta\cos\varphi, \qquad (8)$$

where $C_L = 2eU_0/(\pi mc^2)$. Equation for η has form [6]

$$\frac{d\eta}{d\zeta} = 2\pi\gamma_s(\gamma_s^2-1)^{-1/2} - u_1.$$

Consider the case of a single scalar control function $u = T$. Then

$$\frac{dG^\varphi_\varphi}{d\zeta} = -G^\varphi_\gamma C_L\overline{k}T\cos\eta\sin\varphi, \quad \frac{dG^\varphi_\gamma}{d\zeta} = G^\varphi_\varphi\frac{2\pi}{(\gamma^2-1)^{3/2}},$$

$$\frac{dG^\gamma_\varphi}{d\zeta} = -G^\gamma_\gamma C_L\overline{k}T\cos\eta\sin\varphi, \quad \frac{dG^\gamma_\gamma}{d\zeta} = G^\gamma_\varphi\frac{2\pi}{(\gamma^2-1)^{3/2}},$$

$$\frac{\partial D^\varphi_{\varphi\varphi}(\zeta,\zeta')}{\partial\zeta'} = (2D^\varphi_{\varphi\gamma}\sin\varphi - G^\varphi_\gamma\cos\varphi)C_L\overline{k}T\cos\eta,$$

$$\frac{\partial D^\varphi_{\varphi\gamma}(\zeta,\zeta')}{\partial\zeta'} = -4\pi D^\varphi_{\varphi\varphi}(\gamma^2-1)^{3/2} - G^\varphi_\gamma C_L\overline{k}T\cos\eta\cos\varphi,$$

$$\frac{\partial D^\varphi_{\gamma\varphi}(\zeta,\zeta')}{\partial\zeta'} = 2D^\varphi_{\gamma\gamma}C_L\overline{k}T\cos\eta\sin\varphi,$$

$$\frac{\partial D^\varphi_{\gamma\gamma}(\zeta,\zeta')}{\partial\zeta'} = -\frac{4\pi D^\varphi_{\gamma\varphi}}{(\gamma^2-1)^{3/2}} + G^\varphi_\varphi\frac{6\pi\gamma}{(\gamma^2-1)^{5/2}},$$

$$\frac{\partial D^\gamma_{\varphi\varphi}(\zeta,\zeta')}{\partial\zeta'} = (2D^\gamma_{\varphi\gamma}\sin\varphi - G^\gamma_\gamma\cos\varphi)C_L\overline{k}T\cos\eta,$$

$$\frac{\partial D^\gamma_{\varphi\gamma}(\zeta,\zeta')}{\partial\zeta'} = -\frac{4\pi D^\gamma_{\varphi\varphi}}{(\gamma^2-1)^{3/2}},$$

$$\frac{\partial D^\gamma_{\gamma\varphi}(\zeta,\zeta')}{\partial\zeta'} = 2D^\gamma_{\gamma\gamma}C_L\overline{k}T\cos\eta\sin\varphi,$$

$$\frac{\partial D^\gamma_{\gamma\gamma}(\zeta,\zeta')}{\partial\zeta'} = -\frac{4\pi D^\gamma_{\gamma\varphi}}{(\gamma^2-1)^{3/2}} + G^\varphi_\gamma\frac{6\pi\gamma}{(\gamma^2-1)^{5/2}}.$$

For brevity, introduce the function $\overline{K}(\zeta) = \overline{k}\cos\eta(\zeta)\cos\varphi(\zeta)$ Second derivatives of the functional are

$$\frac{\partial^2\Phi}{\partial T_j^2} = \sum_{i=1}^{N}\Big\{\frac{\partial^2\Phi}{\partial\varphi^2}C_L^2\Big[\int_{\zeta_{j-1}}^{\zeta_j} G^\varphi_\gamma(\zeta_M,\zeta')\overline{K}(\zeta')\,d\zeta'\Big]^2 +$$

$$+\frac{\partial^2\Phi}{\partial\gamma^2}\Big[\int_{\zeta_{j-1}}^{\zeta_j} G^\gamma_\gamma(\zeta_M,\zeta')\overline{K}(\zeta')\,d\zeta'\Big]^2 +$$

ISBN 978-3-95450-181-6

$$+\frac{\partial \Phi}{\partial \varphi}\int\limits_{\zeta_{j-1}}^{\zeta_j} D^{\varphi}_{\gamma\varphi}(\zeta_T,\zeta')\overline{K}(\zeta')\int\limits_{\zeta_{j-1}}^{\zeta'} G^{\varphi}_{\gamma}(\zeta',\zeta'')\overline{K}(\zeta'')\,d\zeta''\,d\zeta' +$$

$$+\frac{\partial \Phi}{\partial \varphi}\int\limits_{\zeta_{j-1}}^{\zeta_j} D^{\varphi}_{\gamma\gamma}(\zeta_T,\zeta')\overline{K}(\zeta')\int\limits_{\zeta_{j-1}}^{\zeta'} G^{\gamma}_{\gamma}(\zeta',\zeta'')\overline{K}(\zeta'')\,d\zeta''\,d\zeta' +$$

$$+\frac{\partial \Phi}{\partial \gamma}\int\limits_{\zeta_{j-1}}^{\zeta_j} D^{\gamma}_{\gamma\varphi}(\zeta_T,\zeta')\overline{K}(\zeta')\int\limits_{\zeta_{j-1}}^{\zeta'} G^{\varphi}_{\gamma}(\zeta',\zeta'')\overline{K}(\zeta'')\,d\zeta''\,d\zeta' +$$

$$+\frac{\partial \Phi}{\partial \gamma}\int\limits_{\zeta_{j-1}}^{\zeta_j} D^{\gamma}_{\gamma\gamma}(\zeta_T,\zeta')\overline{K}(\zeta')\int\limits_{\zeta_{j-1}}^{\zeta'} G^{\gamma}_{\gamma}(\zeta',\zeta'')\overline{K}(\zeta'')\,d\zeta''\Big]\,d\zeta'\Big\}.$$

RESULTS

Test calculations were performed for accelerator structure with frequency 433 MHz and initial energy of protons 60 keV. The final energy is about 1852 MeV. Dependencies of energy of particles that have passed the accelerating structure on longitudinal coordinate after optimiztion are presented on the Fig.1

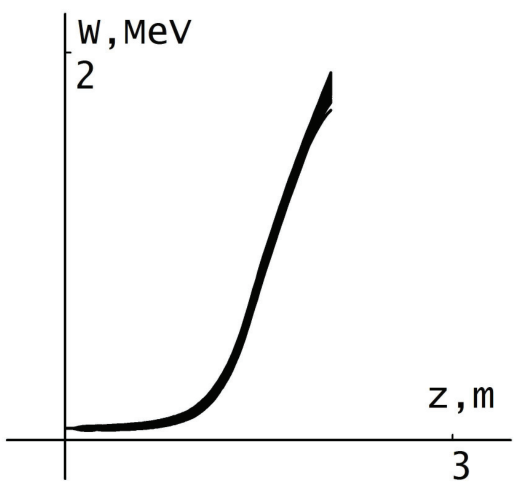

Figure 1: Energy of particles in accelerator channel

Comparison between the second order method and a method using only first order derivatives is presented on the Fig.2, and Fig.3. The dependencies of the functional quality on iteration number are shown.

The process of optimization of only one cell is shown on the Fig.2. One can see that using of the second order method speeds up the process sufficiently, as it should be.

The process of optimization of the entire structure is shown on the Fig.3. The black line describes descent by the first order method with constant step. The red line corresponds to the first order method with adaptive step. It means that the value of optimization step changes depending on changing of the functional value on this step. The

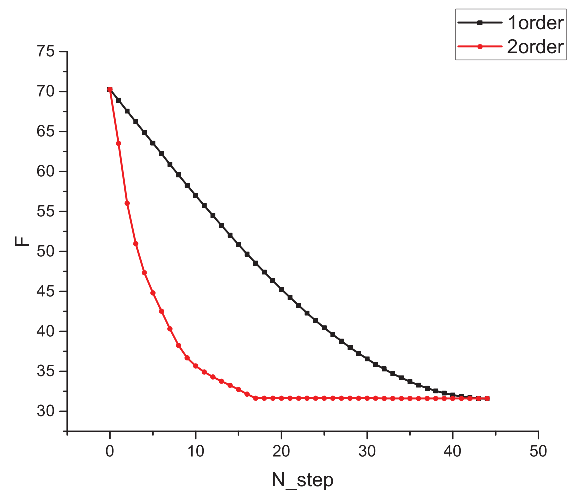

Figure 2: Changing of the quality functional during the process of optimization of the last cell

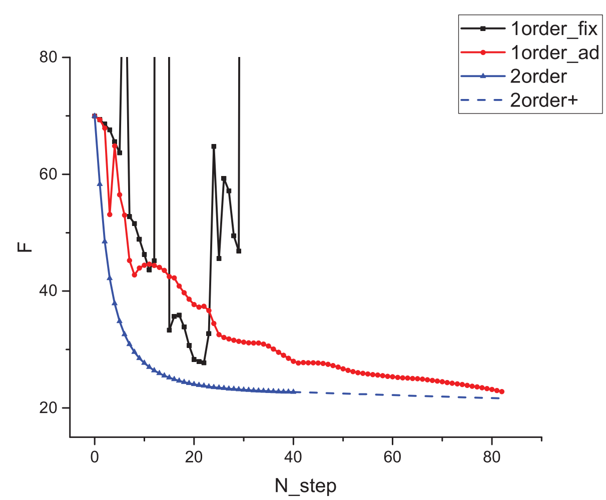

Figure 3: Changing of the functional quality during the process of optimization on the entire structure in same cases

blue line corresponds to the optimization with use of the second order method. Stroke line denotes extrapolation of the solid line.

Analyzing this result, one can see that the difference betwen the first order method an the second order one is expressed clearly. Using the second order method allows sufficiently reduce the number of computation.

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ISBN 978-3-95450-181-6