SUPERCONDUCTING STORED ENERGY RF LINAC AS FREE ELECTRON LASER DRIVER

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Abstract

Due to cavity losses in multi pass free electron laser (FEL), generation starts in it from definite threshold of driving electron beam current. Depending on generation wave range the threshold current strikes from fraction of ampere to dozens of amperes. In order to rich laser saturation, from hundreds to thousands electron bunches are required. Simple estimations give the value from units up to tens joules of bunches train energy in order to rich FEL saturation for infrared wave range (approximately 20 – 25 Mev of bunches energy and 3 A of pick current, bunch length being 1 cm). A beam with parameters mentioned might be obtained in rf superconducting linac operating in stored energy mode. The advantage of such approach is simplified linac power supply since dozens watts cw rf generator is required only to rich necessary accelerating voltage. At the same time the energy spread arising from beam loading may be compensated by additional cavities exited at shifted frequencies. In this paper Maxwell equations are used for beam-cavity interaction analysis. The bunch energy loss or the same the voltage induced by radiating bunch is expressed in terms of cavity external parameters. The detailed analysis of beam energy spread compensation is carried out followed by an example showing the reality of FEL schema suggested.

INTRODUCTION

Four decades of free electron lasers (FEL) development prove clearly their impressive and power status for human activity in scientific and other application. This is especially true for FELs in roentgen, ultraviolet and infrared range inaccessible for classical lasers. It is necessary to underline the main features of FELs – large power and tune ability – that are inherent to these devices due to physical mechanism of radiation generation. On the other hand FELs still are complicated and costly devises and for this reason not available for many research groups. There is no evident solution for accelerator scheme to match ridged parameters specification from light generation part and reduce simultaneously beam driver cost. Multi frequency superconducting linac that operates in stored energy mode [1] seems to be appropriate approach to drive FEL. This linac operation mode does not requires power rf source since dozens watts are necessary only in order to reach high gradient in multi cell superconducting cavity while the rf energy stored in the cavity (dozens joules) is quit sufficient to accelerate electron bunches train for laser excitation. It had been shown as well [1] that bunch train energy spread arising from beam loading might be compensated by additional cavities operating at slightly shifted frequencies.

Bunch energy and energy spread within individual bunches and in bunch train, bunch current and train length, undulator and laser cavity parameters – all these items are necessary to determine linac main parameters. For this reason we start from laser description, followed by appropriate formulae for linac parameters specification. Then appropriate formulae for bunch energy losses in cavity are derived based on solution of Maxwell equations and expressed over cavity external parameters. At last, multi frequency rf linac scheme with beam loading effect compensation is discussed followed by appropriate calculation.

FEL PARAMETERES SPECIFICATION

The processes in FEF are similar to those in traveling wave amplifier or rf linac. In latter devises electrons interact with longitudinal electric field of traveling electromagnetic wave and in the case of synchronism deliver part of their energy to field (amplifier) or take away it from a wave (accelerator). The difference is that in FEL electron interact with transverse electromagnetic wave with longitudinal electric field being equal to zero. The electrons must have transverse velocity in order to exchange their energy with the wave and for this reason a device that forces the beam to move in transverse direction – undulator - is necessary in beam environment. More over the synchronism and the energy exchange is effective when phase lag 2π between the electron and the wave takes place on one undulator period. Fig. 1 is a simple demonstration of the device just described.

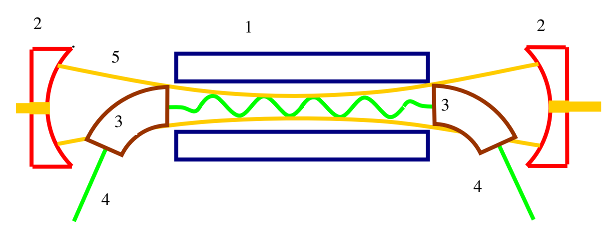

Figure 1: FEL layout. 1 – undulator, 2 – mirror, 3 – magnet, 4 – electron beam, 5 – optical mode envelope, 6 – laser radiation.

One pass of electron bunch in undulator area provides wave amplification. To transmit more energy to light train two mirror cavity is used. Twice reflected light bunch moves again in the same direction and acquires a portion of energy from new electron bunch entering the

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undulator. The wave amplification continues up to the level when nonlinearity occurs and saturation takes place.

The formula for one pass gain that together with the operation principles just described is sufficient for formulation of linac parameters specification. Appropriate expression for gain G looks like [2]

$$G = 4\sqrt{2}\pi N\lambda_w^{1/2}\lambda_s^{3/2}\frac{K^2}{(1+K^2)^{3/2}}\frac{i}{si_A}\left(\frac{\Delta\omega_s}{\omega_s}\right)_t^{-2} \quad (1)$$

Here λ_w - undulator period, λ_s - generated radiation wavelength, K - undulator parameter.

$$\lambda_s = \frac{\lambda_w}{2\gamma^2}(1+K^2)\,, K = eH_{W0}\lambda_w / 2\pi mc^2\,, \quad (2)$$

γ is relativistic factor, H_{W0} - the amplitude of periodic magnetic field on undulator axis. $\Delta\omega$ is line widening, caused by undulator finite length and non zero bunch length and its longitudinal emittance (non zero energy spread). s is bunch cross section, $i_A = mc^3 / e = 17\kappa A$ and

$$\left(\frac{\Delta\omega_s}{\omega_s}\right)_t^2 = \left(\frac{\Delta\omega_s}{\omega_s}\right)_h^2 + \left(\frac{\Delta\omega_s}{\omega_s}\right)_i^2 \quad (3)$$

$$\left(\frac{\Delta\omega_s}{\omega_s}\right)_h^2 = \left(\frac{1}{N}\right)^2 + \left(\frac{\lambda_s}{2l_e}\right)^2, \left(\frac{\Delta\omega}{\omega}\right)_i = 2\frac{\Delta E}{E} \quad (4)$$

Keeping in mind paper motivation and running a few steps forward let us make some estimation for infrared FEL (10 µm). To make estimation more transparent we consider the contribution of finite undulator length end beam energy spread in line widening be equal end neglect the widening coursed by finite bunch length. In this case gain G_{est}

$$G_{est} \cong \pi N^3 \lambda_w^2 \frac{K^2}{\gamma^3}\frac{i}{si_A} \quad (5)$$

Supposing

$N = 50, \lambda_W = 3\,cm, K = 1, \gamma = 50, s = \pi \times 0.01\,cm^2$

we have $G_{est} \cong 0.05i\ (A)$. Taking into account cavity losses at the level 3-5 percent we arrive at the conclusion that bunch current of order 1 A is the FEL excitation threshold and might be referred to as reference value.

CHARGE RADIATION FIELD IN RF CAVITY

We consider that radiating charge is frozen in both direction of motion that is does not change its velocity within a cavity. To find out the field that moving charge radiates in a RF cavity we will use strict electromagnetic approach based on the theory had been developed in [3]. In this theory vortex electrical $\vec{E}(\vec{r},t)$ and magnetic $\vec{H}(\vec{r},t)$ fields are represented as derivatives of vector potential $\vec{A}(\vec{r},t)$ on time t and space $\vec{r}$ coordinates:

$$\vec{E}(\vec{r},t) = -\frac{\partial\vec{A}(\vec{r},t)}{\partial t},\ \ \vec{H}(\vec{r},t) = \frac{1}{\mu_0}rot\vec{A}(\vec{r},t) \quad (6)$$

where μ_0 is magnetic permeability of free space. Here and later SI units are used. Vector potential satisfies the wave equation

$$\Delta\vec{A}(\vec{r},t) - \frac{1}{c^2}\frac{\partial^2\vec{A}(\vec{r},t)}{\partial t^2} = -\mu_0\vec{j}(\vec{r},t) \quad (7)$$

$\vec{j}(\vec{r}.t)$, c being current density and the light velocity.

To find out the expressions for vector potential we will use the most direct way. Namely, we represent vector potential as an expansion on the infinite sum of RF cavity eigen functions $\vec{A}_\lambda(\vec{r})$ with time dependent coefficients $g_\lambda(t)$:

$$\vec{A}(\vec{r},t) = \sum_{\lambda=1}^{\infty} g_\lambda(t)\vec{A}_\lambda(\vec{r}) \quad (8)$$

with the boundary conditions $\vec{A}_\lambda|_\Sigma = 0$ on cavity surface.

Starting from the equation (7) and taking into account (8) one can easily obtain the equations for cavity vector eigen functions and appropriate time dependent coefficients (fields amplitudes):

$$\Delta\vec{A}_\lambda(\vec{r}) + k_\lambda{}^2\vec{A}_\lambda(\vec{r}) = 0 \quad (9)$$

$$\frac{d^2 g_\lambda(t)}{dt^2} + \omega_\lambda{}^2 g_\lambda(t) = \int_V \vec{j}(\vec{r},t)\vec{A}_\lambda(\vec{r})dV \quad (10)$$

Here $k_\lambda = \omega_\lambda / c$ are eigen values of boundary values problems (4), the specific solutions for RF cavities are called cavity modes, ω_λ being the eigen angular frequencies of appropriate modes, c is light velocity. Integration in formula is assumed to be performed over cavity volume V. Last equation can be generalized up to the next one

$$\frac{d^2 g_\lambda(t)}{dt^2} + \frac{\omega_\lambda}{Q_\lambda}\frac{dg_\lambda}{dt} + \omega_\lambda{}^2 g_\lambda(t) = \int_V \vec{j}(\vec{r}.t)\vec{A}_\lambda(\vec{r})dV \quad (11)$$

if losses in cavity and outside are taking into account. Here Q_λ stands for cavity quality factor:

$$Q_\lambda = \frac{\omega_\lambda W_\lambda}{P_\lambda} \quad (12)$$

where W_λ is the electromagnetic energy in the mode λ, stored in cavity volume and P_λ represents the total RF power losses that besides ohm losses in cavity walls includes the external losses due to cavity coupling with

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external circuits. It is supposed that eigen functions are normalized by the condition

$$\int_V A_\lambda^2 = \mu_0 c^2 = 1/\varepsilon_0 \quad (13)$$

Here μ_0 and ε_0 are magnetic and electric permeability respectively.

For the analysis followed we will use the cavity excitation equation in the form with small RF losses, and this has no any influence on generality of results to be obtained. Then, all calculations will be made for a single charged particle with charge value q of zero dimensions in all directions entering cavity at moment $t = 0$. In such a case the total current density

$$\vec{j}(\vec{r},t) = q\vec{v}(\vec{r},t)\delta(x,y,vt)), \quad (14)$$

where $\vec{v}(\vec{r},t)$ stands for particle velocity being assumed constant within the cavity, and $\delta()$ is Dirac delta function: $\delta(x) = \infty$ for $x = 0$, $\delta(x) = 0$ for $x \neq 0$

$$\int_{-\infty}^{\infty} \delta(x)dx = 1 \quad (15)$$

We suppose also the case that is the most interesting for accelerator based applications – the particle moves along cavity axis where

$$x = 0, y = 0, z = vt \quad (16)$$

Starting with such assumptions and following[1,4] we arrive finally at the for the voltage induced on equivalent gap:

$$U = -\frac{q\omega}{2}\frac{R}{Q_0}\exp(-\omega t/2Q)\cos\omega t \quad (17)$$

Where R, Q_0 are cavity shunt impedance and quality factor. Very often, current value I averaged over RF period is used instead of charge value

$$U = -\pi I\frac{R}{Q_0}\exp(-\omega t/2Q)\cos\omega t \quad (18)$$

STORED ENERGY RF LINAC AS FEL DRIVER

If charge train is accelerated in a cavity that operates in stored energy mode charges in charge sequence move in the field that is the sum of the amplitude of stored field and radiated one. The radiated field amplitude is increased with any new bunch entering the cavity and due to this linear energy decrease takes place. To compensate this energy decrease we had suggested the linac scheme consisting of two cavities [1]. The second cavity operates at slightly higher frequency and any new charge sees higher voltage. If appropriate condition is satisfied

$$\frac{\Delta\omega}{\omega} = \frac{U_{lost}}{2\pi U_m (N-1)h}, \quad (19)$$

full compensation of energy decrease takes place. Here U_m and U_{lost} are amplitude of stored mode in the second cavity and total amplitude induced in two cavities.

It can be shown that the energy lost (in voltage units) of n-th bunch in the sequence of bunches in second rf cavity is

$$U_{lost} = -\pi I\frac{R}{Q}\times\frac{\sin\left[(n-1)\pi\left(\frac{h\Delta\omega}{\omega}\right)\right]}{\sin\pi(\frac{h\Delta\omega}{\omega})}\cos\left[\pi(n-2)(\frac{h\Delta\omega}{\omega})\right] \quad (20)$$

One has to set $\Delta\omega = 0$ for the main cavity and to sum both expressions. Thus the total induced voltage is

$$U_{lost} = \pi(n-1)I\frac{R_1}{Q_1} + \pi(n-1)I\frac{R_2}{Q_2} \quad (21)$$

if the following condition takes place

$$\pi(n-1)(\frac{h\Delta\omega}{\omega}) << 1 \quad (22)$$

Here $\omega = h\Omega$, are the main cavity frequency, the number of periods per bunch, the bunch repetition rate accordingly, $\omega + \Delta\omega$ is the frequency of the second cavity

CONCLUSION

Resonator TESLA developed for International Linear Collider one has the following parameters

$Q = 10^{10}\ R/Q = 1kOm, U = 25\,MV, W \approx 80\,J.$

Using half of stored energy one has the beam of 20 MeV approximately. With an optical cavity length equal to 2.3 m (10 wavelength of TESLA cavity) one has h=20 Above mentioned bunch current 1A corresponds approximately to 50 mA current averaged over rf period. At these parameters $5*10^4$ bunches may be generated.

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