STABILIZATION OF THE EQUILIBRIUM POSITION OF A MAGNETIC CONTROL SYSTEM WITH DELAY*

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Abstract

A model of magnetic suspension control system of a gyro rotor is studied. A delay in the feedback control scheme and dissipative forces occurring due to energy losses at the interaction of the magnetic field with currents in the control loops are taken into account. Two approaches to the synthesis of controls stabilizing the equilibrium position of the considered system are proposed. The results of a computer simulation are presented to demonstrate effectiveness of the approaches.

INTRODUCTION

Nonlinear oscillatory systems are widely used for the modeling charge particles motions in cyclotrons in neighborhoods of equilibrium orbits [1–3]. They are also applied for the analysis and synthesis of magnetic control devices [4, 5]. In particular, magnetic systems of retention of power gyro rotors are used in modern control systems of spacecraft orientation with long periods of autonomous operation. An actual problem for such systems is stabilization of their operating modes.

It is worth mentioning that realistic models of numerous control systems should incorporate delay in feedback law [6]. It is well-known that delay may seriously affect on system's dynamics. Therefore, it is important to obtain restrictions for delay values under which stability can be guaranteed.

In this paper, analytical and numerical investigations of stability of the equilibrium position for a nonlinear oscillatory system are presented. The system can be treated as a mathematical model of magnetic suspension control system of a gyro rotor [5]. A delay in the feedback control scheme and dissipative forces occurring due to energy losses at the interaction of the magnetic field with currents in the control loops are taken into account.

Two approaches to the synthesis of stabilizing controls are proposed. With the aid of a computer simulation of dynamics of closed-loop systems, a comparison of these approaches is fulfilled, and their features and conditions of applicability are determined.

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STATEMENT OF THE PROBLEM

Consider the control system

$$\begin{cases} \ddot{x}(t) - p(x(t), y(t))y(t) = u_x, \\ \ddot{y}(t) + p(x(t), y(t))x(t) = u_y. \end{cases} \quad (1)$$

Here $(x(t), y(t))^T$ is the state vector, u_x and u_y are control variables, and function $p(x, y)$ is defined by the formula

$$p(x, y) = \alpha + \beta(x^2 + y^2),$$

where α and β are constant parameters. Thus, the considered system is affected by non-conservative forces and control ones. Equations of the form (1) are used, for instance, for modeling rotor dynamics in magnetic suspension system [5].

In the present paper, we will assume that $\alpha = 0$, and non-conservative forces are generated in the electromechanical system with a certain delay $\tau \geq 0$. The reason for arising the delay is an inertia in response of the magnetic suspension control system on rotor deviations from the equilibrium position. It should be noted that the value of delay might be unknown. In addition, we will assume that the system is affected by a dissipative force $(F_x, F_y)^T$ depending only on the velocities.

Thus, the rotor dynamics is described by the equations

$$\begin{cases} \ddot{x}(t) - \beta\left(x^2(t-\tau) + y^2(t-\tau)\right) y(t-\tau) \\ \qquad + F_x(\dot{x}(t), \dot{y}(t)) = u_x, \\ \ddot{y}(t) + \beta\left(x^2(t-\tau) + y^2(t-\tau)\right) x(t-\tau) \\ \qquad + F_y(\dot{x}(t), \dot{y}(t)) = u_y. \end{cases} \quad (2)$$

We assume that initial functions for solutions of (2) belong to the space $C^1([-\tau, 0], R^2)$ of continuously differentiable functions $\varphi(\theta) = (\varphi_x(\theta), \varphi_y(\theta))^T : [-\tau, 0] \to R^2$ with the uniform norm

$$\|\varphi\|_\tau = \max_{\theta \in [-\tau, 0]} \left(\|\varphi(\theta)\| + \|\dot{\varphi}(\theta)\|\right),$$

and $\|\cdot\|$ denotes the Euclidean norm of a vector.

For the desired position of the rotor axis we have $x = y = 0$. It is known, see [7], that if $\tau = 0$ and $u_x = u_y = 0$, then the equilibrium position

$$x = y = \dot{x} = \dot{y} = 0 \quad (3)$$

of system (2) is unstable. Therefore, we should to design a feedback control law stabilizing the equilibrium position.

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In the present paper, we will study the stabilization problem for two types of dissipative forces:
(i) linear forces;
(ii) nonlinear homogeneous forces.

SYNTHESIS OF STABILIZING CONTROLS

First, consider the case where dissipative forces are linear ones. Then system (2) takes the form

$$\begin{cases} \ddot{x}(t) - \beta\left(x^2(t-\tau)+y^2(t-\tau)\right)y(t-\tau) \\ \qquad +b_{11}\dot{x}(t)+b_{12}\dot{y}(t)=u_x, \\ \ddot{y}(t) + \beta\left(x^2(t-\tau)+y^2(t-\tau)\right)x(t-\tau) \\ \qquad +b_{21}\dot{x}(t)+b_{22}\dot{y}(t)=u_y. \end{cases} \quad (4)$$

Here $b_{11}, b_{12}, b_{21}, b_{22}$ are constant coefficients such that the matrix $B = \{b_{ij}\}_{i,j=1}^2$ is positive definite.

With the aid of the approach proposed in [8, 9], we obtain that the following theorem is valid.

Theorem 1 *Let*

$$u_x = -\frac{\partial\Pi(x(t),y(t))}{\partial x}, \quad u_y = -\frac{\partial\Pi(x(t),y(t))}{\partial y}. \quad (5)$$

Here $\Pi(x,y)$ *is an arbitrary positive definite homogeneous form of the forth order. Then the equilibrium position* (3) *of system* (4) *is asymptotically stable for any* $\tau \geq 0$.

Remark 1 Theorem 1 remains valid in the case where there is a delay in the feedback law, i.e., where

$$u_x = -\frac{\partial\Pi(x(t-\tau_1),y(t-\tau_1))}{\partial x},$$

$$u_y = -\frac{\partial\Pi(x(t-\tau_1),y(t-\tau_1))}{\partial y}.$$

Here $\tau_1 = \text{const} > 0$.

Next, consider the case where system (2) is of the form

$$\begin{cases} \ddot{x}(t) - \beta\left(x^2(t-\tau)+y^2(t-\tau)\right)y(t-\tau) \\ \qquad +b_1\left(\dot{x}^2(t)+\dot{y}^2(t)\right)^\gamma\dot{x}(t)=u_x, \\ \ddot{y}(t) + \beta\left(x^2(t-\tau)+y^2(t-\tau)\right)x(t-\tau) \\ \qquad +b_2\left(\dot{x}^2(t)+\dot{y}^2(t)\right)^\gamma\dot{y}(t)=u_y. \end{cases} \quad (6)$$

Here b_1, b_2 and γ are positive parameters. Thus, we assume that dissipative forces are essentially nonlinear and homogeneous ones.

Using the results of [8–10], we arrive at the following theorem.

Theorem 2 *Let*

$$u_x = g\beta\dot{y}(t), \qquad u_y = -g\beta\dot{x}(t), \quad (7)$$

where $g = \text{const} > 0$. *If* $0 < \gamma < 1$, *then the equilibrium position* (3) *of system* (6) *is asymptotically stable for any* $\tau \geq 0$.

Remark 2 Condition $0 < \gamma < 1$ of Theorem 2 is essential one. Really, if $\gamma = 1$, $\tau = 0$, and $b_1 = b_2 = \beta = 1/g > 0$, then the corresponding system (6) closed by control (7) admits the following family of solutions:

$$x(t) = c\sin t, \quad y(t) = c\cos t,$$

where c is an arbitrary constant. Hence, the equilibrium position (3) of the system is not asymptotically stable.

RESULTS OF A NUMERICAL SIMULATION

The results of a computer simulation are presented in Figs. 1–4. It is assumed that $\beta = 1$, $b_{11} = b_{22} = 1$, $b_{12} = b_{21} = 0$, $b_1 = b_2 = 1$, $\gamma = 1/2$, $\tau = 1$. Initial conditions of solutions are chosen as follows: $t_0 = 0$, $\varphi_x(\theta) = \varphi_y(\theta) = 0.25$, $\dot{\varphi}_x(\theta) = \dot{\varphi}_y(\theta) = 0$ for $\theta \leq 0$.

First, consider system (4) closed by the control

$$u_x = -10x^3(t), \qquad u_y = -10y^3(t). \quad (8)$$

According to Theorem 1, Fig. 1 illustrates stabilization process.

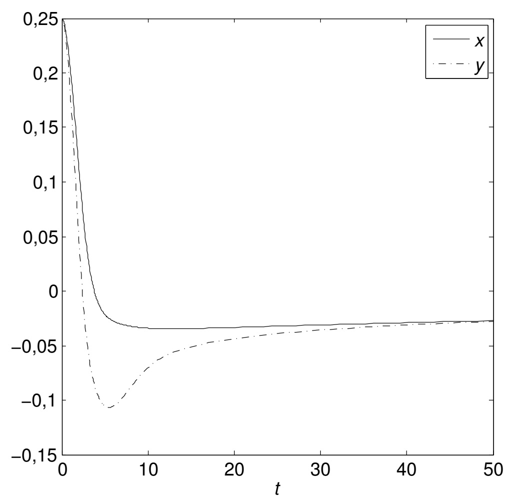

Figure 1: State response of system (4) closed by control (8).

Next, consider system (4) with a control of the form (7). Let

$$u_x = 0.242\dot{y}(t), \qquad u_y = -0.242\dot{x}(t). \quad (9)$$

In this case, Fig. 2 demonstrates unstable behavior of the solution.

In Fig. 3, trajectory of the solution of system (6) closed by the control

$$u_x = \dot{y}(t), \qquad u_y = -\dot{x}(t) \quad (10)$$

is shown. The figure demonstrates the effectiveness of Theorem 2.

Finally, consider system (6) closed by a control of the form (5) with a delay in the feedback law. Let

$$u_x = -100x^3(t-1.05), \quad u_y = -100y^3(t-1.05). \quad (11)$$

Figure 4 shows that the corresponding solution does not converge to the origin as $t \to +\infty$.

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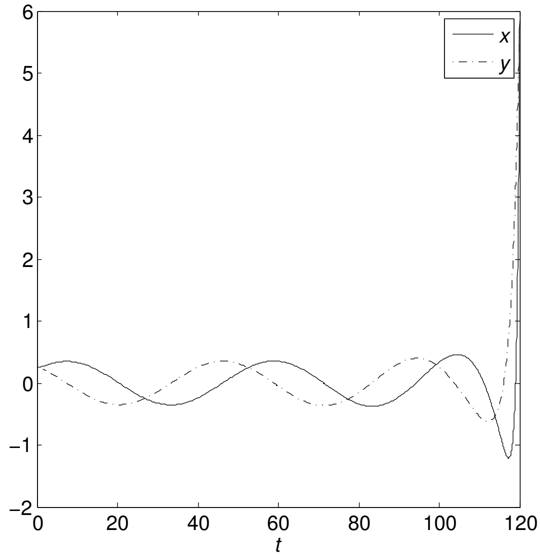

Figure 2: State response of system (4) closed by control (9).

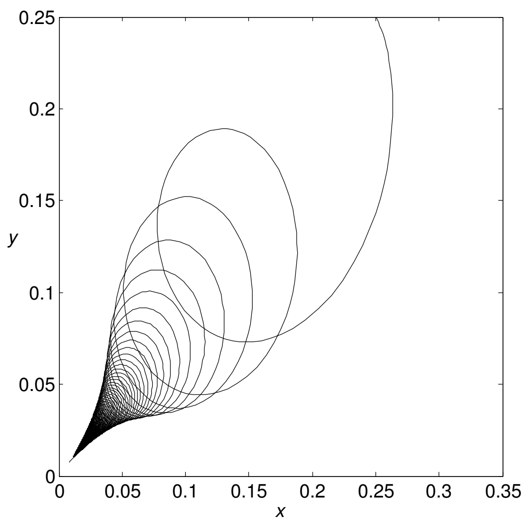

Figure 3: The trajectory of system (6) closed by control (10).

CONCLUSION

In the paper, two approaches to the synthesis of stabilizing controls are proposed for a nonlinear oscillatory system with time delay. The first one is based on the using of potential control forces. It is applicable in the case of linear dissipative forces. The second approach is efficient for systems with essentially nonlinear homogeneous dissipative forces. For this case, gyroscopic control forces are constructed. It should be noted that the application of the proposed approaches provides asymptotic stability of the corresponding closed-loop systems for any nonnegative delay.

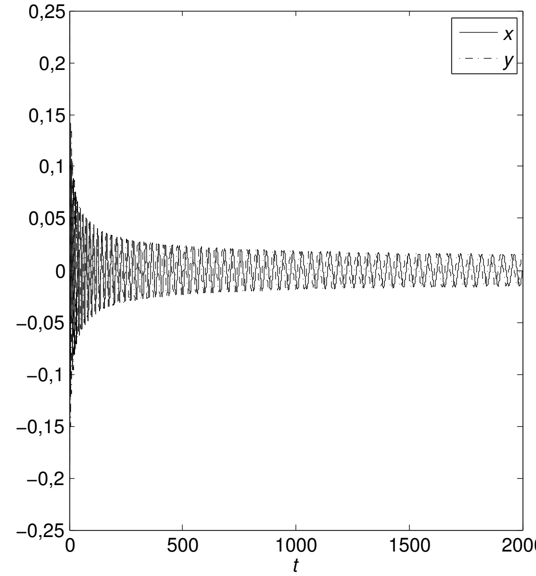

Figure 4: State response of system (6) closed by control (11).

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