ACCELERATION OF THE OPPOSSITELY CHARGED PARTICLES IN THE SINGLE STREAM

A.S.Chikhachev, All-Russian Electrotechnical Institute, Moscow, Russia

Abstract

One of the problems arising at extraction of heavy ions from plasma is removal of electrons from a stream of particles. Therefore possibility of simultaneous acceleration in one direction as ions (electric field), and electrons (pressure gradient) is represented rather interesting. In work when using the hydrodynamic description in the accelerating interval conditions of cold ions and hot electrons are studied. Possibility of excess by ions of speed of an ionic sound is shown, and the ratio of sizes of streams of electrons and ions can be any.

INTRODUCTION

In work [1] acceleration of heavy ions in the presence of a counter flow of electrons was studied. Thus streams of charged particles were considered equal in size. We will note, however, that flows oppositely charged particles can be accelerated in one direction if electrons except force from electric field are affected by rather big force caused by pressure gradient.

MAXWELLIAN DISTRIBUTION OF THE ELECTRONS

Let at there is a source of electrons at $x = 0$ and once ionized ions, and potential $\phi(x)|_{x=0} = \phi_0$. Ions are considered cold, that is $T_i \to 0$ and them only force affects, from electric field, electrons, except a field pressure gradient ΔP works. In the stationary mode of the equation, describing system, have an appearance:

$$M\frac{\upsilon^2}{2} = -e\phi + M\frac{\upsilon_0^2}{2}, \qquad (1)$$

$$m\upsilon_e\frac{d\upsilon_e}{dx} = e\frac{d\phi}{dx} - \frac{1}{n_e}\frac{dP}{dx}. \qquad (2)$$

Here M - the mass of an ion, m - the mass of an electron, υ_i, υ_e - speeds of ions and electrons, respectively,υ_0 - the initial speed of ions, P - pressure of electronic gas. In an isothermal case,$P = nT, T \equiv const$ - temperature of electronic gas. The equation (2) can be integrated:

$$m\frac{\upsilon_e(x)^2}{2} - e\phi(x) + T\ln\frac{n_e(x)}{n_*} = 0, \qquad (3)$$

where n_* - any constant, dimensional density. We will put that there is a stream of ions Γ and a stream of electrons $\Gamma_e = g\Gamma$. Then follows from the equations of a continuity $\upsilon_e = \frac{g\Gamma}{n_e}$ and $\upsilon_i = \frac{\Gamma}{n_i}$. We will designate $\frac{e\phi}{T} = \varphi$ and will set entry conditions: at $x = 0 n_e = n_i = n_0, \varphi = \varphi_0$ Then $\Gamma = n_0\upsilon_0$, where υ_0 - the speed of ions at x =0. From (3) follows: $\varphi - \varphi_0 = \ln\left(\frac{n_e}{n_0}\right) - \frac{\upsilon g^2}{M}\frac{\upsilon_0^2}{2\upsilon_s^2}\left(1 - \frac{n_0^2}{n_e^2}\right)$. Poisson's equation has an appearance:

$$\frac{d^2\phi}{dx^2} = 4\pi(n_i - n_e). \qquad (4)$$

Using the expression for potential following from (3), we will receive from (4) equation containing only density of electrons. Passing to dimensionless variables $y = \frac{x}{\lambda}, \lambda^2 = \frac{T}{8\pi e^2 n_0}, \eta = \frac{n_e}{n_0}$ and having designated $a = \frac{m}{M}\frac{\upsilon_0^2}{\upsilon_s^2}g^2, S(\eta) = 1 - 2\frac{\upsilon_s^2}{\upsilon_0^2}\left(\ln\eta + a\frac{\upsilon_s^2}{\upsilon_0^2}\right)$, we will receive:

$$2\frac{d}{dy}\left[\frac{1}{y}\frac{d\eta}{dy}\left(1 - \frac{a}{\eta^2}\right)\right] = \eta - \frac{1}{\sqrt{S(y)}}. \qquad (5)$$

Speed of a stream of ions from the above-stated expressions is defined as $\upsilon_i = \upsilon_0\sqrt{S(\eta)}$. The equation (5) has integral of the following look:

$$\left[\frac{1}{\eta}\frac{d\eta}{dy}\left(1 - \frac{a}{\eta^2}\right)\right]^2 = C_0 + \left(\frac{a}{\eta} + \eta\right) + + \frac{\upsilon_0^2}{\upsilon_s^2}\sqrt{1 - 2\frac{\upsilon_0^2}{\upsilon_s^2}\left(\ln\eta + \frac{a}{2\eta^2}\right) + a\frac{\upsilon_0^2}{\upsilon_s^2}}. \qquad (6)$$

At a zero stream of electrons (i.e. at $g = 0$) the integral (7) passes into the integral used in [2] for studying of criterion Bohms. The system of the equations describing particles a field according to definition [2], is the Hamilton, having integral system of the equations. This property remains and for more difficult case in the presence of nonzero stream of electrons. We will enter a new variable:$\eta = \exp(-z)$ also we will construct dependence

$$q(z) = a\exp(z) + \exp(-z) +$$

$$+\frac{\upsilon_0^2}{\upsilon_s^2}\sqrt{1 - 2\frac{\upsilon_s^2}{\upsilon_0^2}\left(-z + \frac{a}{2}\exp(2z)\right) + a\frac{\upsilon_s^2}{\upsilon_0^2}}.$$

At $a = 10^{-6}, \upsilon_0 = 0.1\upsilon_s$ this dependence has the appearance represented in fig. 1. The phase trajectory, the equation for which follows from (5):

$$\frac{dz}{dy} = \frac{\sqrt{C_0 + q(z)}}{|1 - 10^{-6}\exp(2z(y))|}, \qquad (7)$$

at $C_0 = -0.2935$, it is represented in fig. 2. We will note that value of a constant C_0 is chosen we conceal in a way that the phase trajectory concerned an axis $\dot{z} = 0$. If $C_0 < -0.2935$, the phase trajectory breaks up to two

ISBN 978-3-95450-170-0

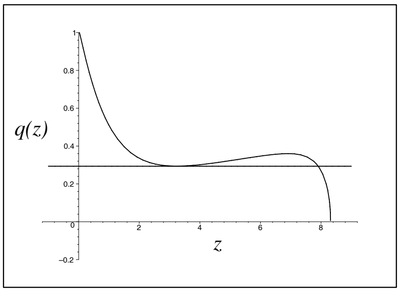

Figure 1: Dependence $q(z)$ at $a = 10^{-6}, v_0 = 0.1v_s$.

not connected areas that doesn't allow to translate a stream from small speeds to rather big. At the chosen value C_0 the initial field is minimum.

We will bring, further dependence $z(y)$, following from the decision (7) under a condition $z(0) = 0$. Because of existence of feature in the equation (5) dependence finds a characteristic break a rupture of a derivative. In compliance with this dependence density of electrons $n_e = n_0 \exp(-z)$ very quickly exponential decreases with growth of value of coordinate.

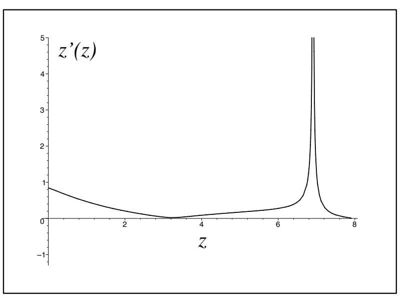

Figure 2: The phase trajectory $z'(z)$ at $C_0 = -0.2935$.

We will provide the schedule of the dependence $u(y) = \frac{v_i}{v_s} = 0.1\sqrt{S(\eta(y))}$, determining the speed of a stream of ions (fig. 3). Existence of a break at $y \approx 60$ is also explained by feature in (5) at $z \approx 8$. In this point the stream speed relation to the speed of a sound maximum and equally, i.e. the speed of a stream of ions by ≈ 3.5 times exceeds ion-sound speed.

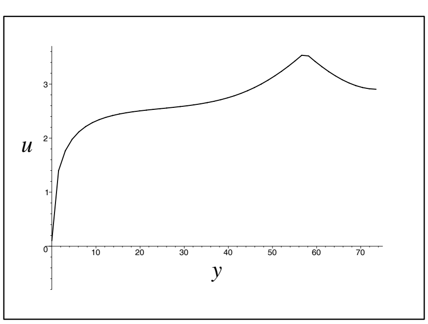

Figure 3: Dependence of ion velocity from coordinate.

We will note importance of creation of certain entry conditions for streams of particles here. Size g is the relation of initial speeds and is defined by entry conditions. Parameter a meets a condition $a << 1$ in wide area of parameters, including at $g = 1$, i.e. at equality of streams of ions and electrons. Thus, it is shown that perhaps simultaneous acceleration of ions and electrons in a flat interval ions electric field, and electrons pressure gradient at rather big difference of electronic density, and the speed of ions can surpass ion-sound speed.

ADIABATIC DISTRIBUTION OF THE ELECTRONS

Unlike the previous section, we will consider the cold ions and electrons which are characterized by adiabatic distribution here. In work [2] acceleration of heavy ions at counter flows of ions and electrons was studied, and electrons were characterized by the isothermal equation of a state $P = n_e T$, where p - pressure,T - temperature of electrons,n_e - density of electrons. In the section states with the electrons characterized by the adiabatic equation of a state will be considered, considering that streams of particles coincide in size and the direction. We will consider electrons as the one-nuclear ideal gas described by the state equation:

$$P = Cn_e^{\gamma}. \qquad (8)$$

where γ - an adiabatic curve indicator. For one-nuclear gas $\gamma = 5/3$. It is convenient to Constant Still there are fair equations (1), (2) and (4), and instead of (3) will receive:

$$\frac{mv_e^2}{2} - e\varphi + T_0\left[\left(\frac{n_e}{n_0}\right)^{\gamma-1} - \kappa\right] = 0. \qquad (9)$$

In (9) size κT_0 is an integration constant. From the equations (1), (2), (4) and (9) we will receive at $v_0 = 0$ and at equality of streams of ions and electrons:

$$\frac{e}{T}\frac{d^2\varphi}{dx^2} = y - \sqrt{\frac{M}{2m}}\frac{1}{\sqrt{\kappa - \left(\frac{y}{y_0}\right)^{\gamma-1} - \frac{1}{2y^2}}}. \qquad (10)$$

Substituting $\frac{e\varphi}{T_0}$ from (9) we come to the equation wich has the integral similar (7):

$$\left[\frac{dy}{dx}\left(\frac{1}{y^2} - \frac{\gamma-1}{y_0}\left(\frac{y}{y_0}\right)^{\gamma-2}\right)\right]^2 = C_* + \left(\frac{1}{y} - \frac{\gamma-1}{\gamma}\left(\frac{y}{y_0}\right)^{\gamma}\right) + \sqrt{\frac{2M}{m}}\sqrt{\kappa - \left(\frac{y}{y_0}\right)^{\gamma-1} - \frac{1}{3y^2}}. \qquad (11)$$

We will solve further (11) considering $\gamma = 5/3, y_0 = 10, \kappa = 1$, and assuming that $\sqrt{\frac{2M}{m}} \approx 697$, i.e. considering plasma of once ionized xenon. We will study behavior

ISBN 978-3-95450-170-0

of the right member of equation (11) at these values of parameters and at $C_* = 0$. Fig. 4. shows behavior of function $G(y)$:

$$G(y) = \frac{\sqrt{F(y)}}{\frac{1}{y^2} - \frac{\gamma-1}{y_0}\left(\frac{y}{y_0}\right)^{\gamma-1}},$$

where $F(y) = \left(\frac{1}{y} - \frac{\gamma-1}{\gamma}\left(\frac{y}{y_0}\right)^{\gamma}\right) + \sqrt{\frac{2M}{m}}\sqrt{\kappa - \left(\frac{y}{y_0}\right)^{\gamma-1} - \frac{1}{3y^2}}$.

From drawing it is possible to see that in the equation (11) there is a gap at $y \approx 2.1$,, and y decreases from size $y = 9.92$.

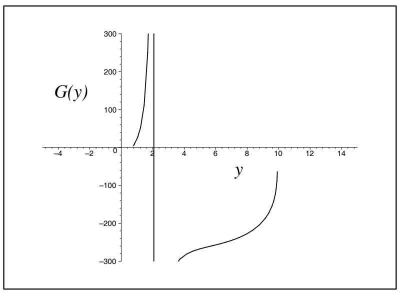

Figure 4: Dependence $G(y)$ at $C_* = 0$.

Fig. 5 shows the solution of the equation (11) with initial value. The curve of I shows behavior of electronic density, and the curve of II represents 10^{-3} density of ions. The gradient of pressure accelerates electrons in the same direction in which electric field accelerates ions. The size of a gradient of pressure is sufficient to exceed the slowing-down force from electric field. Density of ions in all interval significantly exceeds electrons.

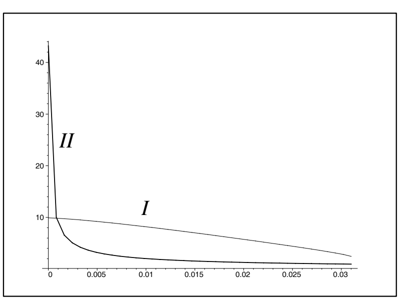

Figure 5: Dependence of the electron density from coordinate (I) and ion density$\times 10^{-3}$ from coordinate (II).

According to an equation of a state of ideal gas temperature of an electronic stream is function of coordinate: $T_e = T_0\frac{\gamma-1}{\gamma}\left(\frac{y(x)}{y_0}\right)^{\gamma-1}$. Speed of an ionic stream is defined by expression:

$$\upsilon_i(x) = \sqrt{\frac{2T_0}{M}}\sqrt{1 - \left(\frac{y(x)}{y_0}\right)^{2/3} - \frac{1}{2y(x)^2}}$$

For comparison of ion-sound speed with the speed of an ionic stream in Fig.6 dependences $\upsilon_i(x)\sqrt{\frac{M}{2T_0}}$ (a curve of I) and $\sqrt{\frac{T_e}{T_0}}$ are shown (a curve of II). Speed of an ionic stream grows and reaches ion-sound speed at the end of an accelerating interval.

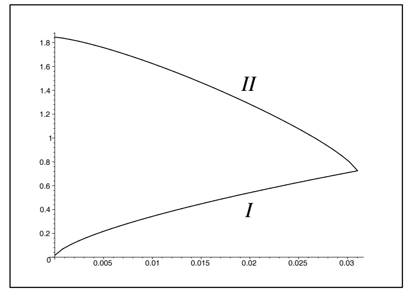

Figure 6: Dependence of the ion speed and temperature from coordinate.

CONCLUSION

Thus, it is shown that, both in case of isothermal distribution and in case of the electrons described by adiabatical distribution perhaps simultaneous acceleration of streams of particles of an opposite sign of a charge in one direction. The described problems were studied in works [3], [4] earlier.

REFERENCES

[1] 1. Yu.A.Kovalenko, T. V. Chernyshev, A. S. Chikhachev, News of the Russian Academy of Sciences, Energy drink series, 2011, No. 4, page 24-28

[2] 2.N.Sternberg, V. Godyak//IEEE Transaction on Plasma Science, Vol. 35, No 5. October 2007.

[3] 3. Yu.A.Kovalenko, T. V. Chernyshev, A. S. Chikhachev, ZhTF, 2013, T.83, No. 4, page 142-145.

[4] 4.A.S.Chikhachev, Journal of Modern Physics, 2011, 2, 1550-1552

ISBN 978-3-95450-170-0