CALCULATION OF TOLERANCES AND STATISTICAL TEST

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Abstract

In the paper mathematical methods of tolerance determination of different parameters of accelerating and focusing structures are considered. The determination of tolerances is based on the analytical representation of variation of functional characterizing the beam dynamics. Method of statistical analysis of calculated tolerance values is represented. The purpose of the work is to determine the maximum possible deviations of the real (actual) parameters from nominal, when the qualitative structure function satisfies to the required modes.

INTRODUCTION

In the design of any kind of system, whether is it a linear accelerator or some other system, the nominal (rated) values of parameters are determined. The system must satisfy the specified criterion of the quality according to these parameteres. But accelerator of any type is a very difficult complex structure. It is almost impossible, and sometimes not economical to provide in accelerator the equality of actual parameters values with their nominal values. Deviations from the rated values influence on the quality of the system functioning and cause deviations from the specified quality criterion. These deviations can have a negative impact.

In this work the analytical and statistical method of tolerance determination is considered on the example of the longitudinal motion of charged particles in an accelerator with drift tubes. Thus, the maximum possible deviations of the real (actual) parameters from nominal are found when the qualitative structure function satisfies to the required modes. The statistical analysis of the getting results is carried out.

PROBLEM STATEMENT

In the paper we consider the problem of tolerances calculation for the dynamics of longitudinal motion of charged particles in an accelerator with drift tubes. The equations describing this process have the following form [1], [2]

$$\begin{aligned} \frac{d\varphi}{d\xi} &= \frac{2\pi\gamma}{\sqrt{\gamma^2-1}}, \\ \frac{d\gamma}{d\xi} &= \alpha(\xi)\cos(\varphi), \end{aligned} \tag{1}$$

where $\alpha(\xi)$ — the stepwise function defined on the interval [0, L],

$$\alpha(\xi) = \alpha^i, \quad \xi \in [\mu_{i-1}, \mu_i], \quad i = \overline{1, m}.$$

Here $0 = \mu_0 < \mu_1 < \ldots < \mu_m = L$, m — fixed non-negative integer. Further, we consider tolerances determination only of parameters μ_i, which are coordinates of the drift tubes. We introduce into consideration the functional characterizing the quality of the system functioning according to the μ_i parameters

$$I(\mu) = \int_{M_{L,\mu}} \left(a(\frac{\gamma_L}{\bar{\gamma}} - 1)^2 + b(\varphi_L - \bar{\varphi})^2 \right) d\varphi_L d\gamma_L. \tag{2}$$

Here $\bar{\varphi}$, $\bar{\gamma}$ — average energy and phase, $M_{L,\mu}$ — beam cross-section of paths depending on μ parameters.

The nominal values of the parameters are known, and they are $\mu_0 = (\mu_{01}, \ldots, \mu_{0m})$. It is necessary, according to the given value of $\triangle > 0$, to determinate the tolerances $\triangle_i > 0$, $i = \overline{1, m}$, such that

$$|\triangle I| = |I(\mu_0 + \triangle\mu) - I(\mu)| \le \triangle,$$

where $|\triangle\mu_i| \le \triangle_i$, $\triangle\mu = (\triangle\mu_1, \ldots, \triangle\mu_m)$, and realize statistical analysis of the tollerances value.

ALGORITHM OF THE TOLERENCE ESTIMATE

We assume that the deviations from the nominal values of the parameters are small and that the changes within the tolerances bands are linear. So the total increment of the functional I can be changed by its variation [3]

$$\delta I \approx \sum_{i=1}^{m} \left(\frac{\partial I(\mu_0)}{\partial \mu_i} \right) \triangle\mu_i. \tag{3}$$

There are two basic principles for the determination of the tolerances: the principle of equal influences and the principle of equal tolerances [4]. The sense of principle of equal influences is that the change of each input parameter affects the same way on the output value. So we obtain the formula for the calculation of tolerance:

$$\triangle_i = \triangle(m^{-\frac{1}{2}}) \left| \frac{\partial I(\mu_0)}{\partial \mu_{i0}} \right|^{-1}, \quad i = \overline{1, m}. \tag{4}$$

The sense of principle of equal tolerances is that the all tolerances are equal, i. e. $\triangle_i = \bar{\triangle}, i = \overline{1, m}$. So we

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obtain the formula:

$$\bar{\triangle} = \triangle \left(\sum_{i=1}^{m} \left(\frac{\partial I}{\partial \mu_{i0}} \right)^2 \right)^{\frac{1}{2}}, \quad i = \overline{1,m}. \tag{5}$$

For the tolerance calculation we use the following equation [5]

$$\frac{\partial I(\mu)}{\partial \mu_i} = -\int_{M_{\mu_i,\alpha}} (\alpha_i \psi_2 \cos(\varphi(\mu_i)) - \\ -\alpha_{i+1}\psi_2 \cos(\varphi(\mu_i))) dx_{\mu_i}. \tag{6}$$

Find the ψ_1 and ψ_2 solving the system of differential equations

$$\begin{aligned} \frac{d\psi_2}{d\xi} &= \frac{2\pi\psi_1}{(\gamma^2-1)^{\frac{3}{2}}}, \\ \frac{d\psi_1}{d\xi} &= \alpha(\xi)\sin(\varphi)\psi_2 \end{aligned} \tag{7}$$

with initial conditions

$$\begin{aligned} \psi_2 &= -\frac{2a}{\bar{\gamma}}(\frac{\gamma(L)}{\bar{\gamma}} - 1), \\ \psi_1 &= -2b(\varphi(L) - \bar{\varphi}), \end{aligned} \tag{8}$$

where $\bar{\gamma}, \bar{\varphi}$ — average energy and phase.

STATISTICAL ANALYSIS OF THE TOLERANCE VALUE

After calculations the tolerances were found for each $\mu_i, i = \overline{1,m}$ parameter of the accelerating structure. The structure must function with the required characteristics within tolerances bands. We realize statistical analysis to evaluate the behavior of accelerating structure within tolerances bands.

Assume $\mu_i, i = \overline{1,m}$ are normal random variables with a standart deviation σ_i and mathematical expectation $M[\mu_i] = \mu_{i0}, i = \overline{1,m}$. Set aside the cross-correlation of random variables $\mu_1, \ldots, \mu_m$. Remind that $\mu_{i0}, i = \overline{1,m}$ are nominal (rated) parametres of the system. Acording to the three sigma rule, normal random variable possesses the values in the interval $[\mu_{i0} - 3\sigma, \mu_{i0} + 3\sigma]$ with a probability $0,997$. It is necessary, that all possible values of the parameter μ_i (i. e. all its possible deviations) were within the tolerances bands. In this case the satisfactory system functioning is provided. Thus, the boundary of the tolerance zone must be equal to three sigma, i. e. $\triangle_i = 3\sigma_i$ therefore $\sigma_i = \frac{\triangle_i}{3}$. Model normal distribution for each random value with mathematical expectation $M[\mu_i] = \mu_{i0}$ and standart deviation $\sigma_i = \frac{\triangle_i}{3}$. As a result, all possible values of each $\mu_i, i = \overline{1,m}$ parameter must be in the range of tollerance zone.

Statistical modeling can be realized the following way:

- Generate the values of the random variables $\mu_1, \ldots, \mu_m$.
- Model the dynamics of the longitudinal motion with the new values of the control parameters $\mu_i, \ldots, \mu_m$. Find the functional I corresponding to this dynamics. Also it is suggested, that this functional is a random variable.
- After the implementation N of these experiments we get a sample of the functional values $(I_1, \ldots I_N)$. Find the standart deviation [6]

$$\sigma_I = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(I_M - I_j)^2}. \tag{9}$$

Here I_M — mathematical expectation.

If the standard deviation σ_I is less than the initially specified deviation $\triangle$ of the functional, the tolerances can be increased. Thus, the tolerances zones can be expanded, until σ_I and $\triangle$ are not comparable. Vice versa if σ_I more than $\triangle$, the tolerances bands are reduced.

RESULTS

Based on the principle of equal influences, calculation of the tolerances was realized, and statistical analysis was implemented. We considered, the case where the two parts of the functional (2) are equal to the result. So the weight coefficients have the following meaning $b = 1, a = 2,929e + 006$. Were found the tolerances, according to which the functional deviation shall not exceed 5%, it is equivalent to $\triangle I \leqslant 1,110e - 008$.

Statistical analysis of the tolerances values showed that the standard deviation of the functional $\sigma_I = 2,8838e - 010$ is approximately 0,129%, which is significantly less than 5%. So we can expand the tolerances bands. By increasing the tolerances in 5.9 times, we received the following results $\sigma_I = 7,9668e - 009$, which is about 3,6% (see Fig. 1).

Similarly, we considered cases where functional had another weighting coefficients. Statistical analysis showed that the tolerances can be increased by several times. Measure of the tollerance' increase depends on the type of functionals.

CONCLUSION

In the work method of the tolerances determination was investigated on the example of the longitudinal motion of charged particles in an accelerator with drift tubes. Statistical analysis of the tolerances values was carried out. The same method can be applied to any other accelerating and focusing structure, in which the dynamics of the particles is described by the differential equations. Nowadays, there are many works

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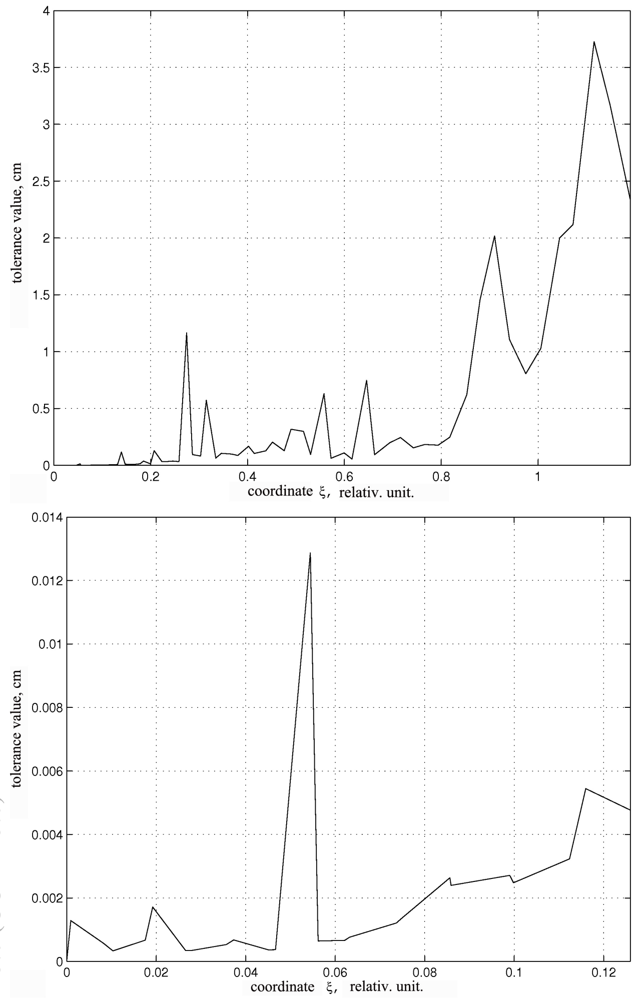

Figure 1: Values of the tolerances.

devoted to beam dynamics optimization. In some of them [1, 7–14] the methods and approaches of finding functional variation were considered. These methods can be used for the tolerance determination.

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