

# WAKEFIELD PRODUCED BY A SMALL BUNCH MOVING IN COLD MAGNETIZED PLASMA ALONG THE EXTERNAL MAGNETIC FIELD\*

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## Abstract

Plasma wakefield acceleration (PWFA) is a promising tool for acceleration of charged particles to high energies at relatively small lengths. Knowledge about the structure of the electromagnetic field produced by the driver bunch in plasma plays the essential role for the realization of this accelerating scheme. Constant external magnetic field which can be used for focusing the driver bunch affects the field structure essentially because plasma acquires both anisotropy and gyrotropy. However, the field in the latter case has not been practically investigated until present. Here we study the field produced by point charge and small bunch moving in cold magnetized plasma along the external magnetic field. We note the singular behavior of some components of the wave field produced by point charge near the charge trajectory. We also analyze the influence of the external magnetic field and bunch size on the field components.

## INTRODUCTION

Cherenkov radiation in a cold magnetized plasma has been investigated for the first time in the early fifties [1], but the detailed analysis of the electromagnetic field structure in this situation has not been performed until present. However, this question is of essential interest in the context of wakefield acceleration method [2] and especially plasma wakefield acceleration (PWFA) method [3], which has achieved a 40 GeV/m gradient for now [4]. Outcomes of the present paper concerning the peculiarities of the electromagnetic field of small bunch moving in the considered medium can be used for further development of PWFA technique.

We consider cold electron plasma under the external magnetic field  $H_{ext}$  described by permittivity tensor [5]

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_1 & -i\varepsilon_2 & 0 \\ i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix}, \quad (1)$$

$$\varepsilon_1 = 1 - \frac{\omega_p^2}{\omega^2 - \omega_h^2}, \quad \varepsilon_2 = \frac{-\omega_p^2 \omega_h}{\omega(\omega^2 - \omega_h^2)}, \quad \varepsilon_3(\omega) = 1 - \frac{\omega_p^2}{\omega^2}, \quad (2)$$

where  $\omega_p^2 = 4\pi N e^2 / m$  is a plasma frequency ( $N$  is an electron density,  $e$  and  $m$  are an electron charge and a mass respectively),  $\omega_h = |e| H_{ext} / (mc)$  is a ‘‘gyrofrequency’’ and  $c$  is the light speed in vacuum.

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## FIELD OF POINT CHARGE

The electromagnetic field generated by point charge  $q$  moving with constant velocity  $v = \beta c$  along  $H_{ext}$  in (1) is given in the ultrarelativistic case  $\gamma \gg 1$  ( $\gamma$  is Lorentz factor) for  $\zeta < 0$  ( $\zeta = z - vt$ ) and  $|\zeta| \gg c/(\gamma \omega_p)$  by the following formula

$$H_\varphi = \int_{\omega_p}^{\omega_\Sigma} h_\varphi(\omega) J_1(\rho s_e) \sin(\omega \zeta / v) d\omega \quad (3)$$

for the azimuthal magnetic component (the cylindrical frame with  $z$  axis coinciding with  $H_{ext}$  is used) and by similar ones for the rest of components [1, 6]. Here

$$h_\varphi = \frac{qc\beta}{\omega_p^2 \omega_h} \frac{\omega_h^2 - \omega^2}{\sqrt{\omega^2 - \omega_c^2}} \left[ \frac{\omega^2}{c^2} \left( \varepsilon_2^2 - \varepsilon_1^2 + \frac{\varepsilon_1}{\beta^2} \right) + \varepsilon_1 s_e^2 \right] s_e, \quad (4)$$

$$s_e^2 = \frac{(\beta^2 - 1)(u - u_1)(u - u_3)(u - u_4)}{\beta^2 c^2 (u - u_2)}, \quad u = \sqrt{\omega^2 - \omega_c^2}, \quad (5)$$

$$\omega_c^2 = \omega_p^2 - \omega_h^2 [1 - \beta^2]^2 / (4\beta^2), \quad \omega_\Sigma = \sqrt{\omega_p^2 + \omega_h^2}, \quad (6)$$

$$u_{1,2} = \frac{\omega_h(1 \mp \beta^2)}{2\beta}, \quad u_{3,4} = \frac{\omega_h \beta}{2} \mp \frac{1}{2\beta} \sqrt{\frac{\omega_h^2}{4} - \frac{\omega_p^2 \beta^2}{1 - \beta^2}}. \quad (7)$$

Two analytical approaches have been applied to calculation of (3). First one gives the field representation in the far-field zone within the Cherenkov cone  $\rho s_{e1,2} \gg 1$ ,  $|\zeta| > \zeta_{\min}(\rho)$ :

$$H_\varphi \approx \left[ -h_\varphi(\omega_{s1}) \sin(\rho s_{e1} - \omega_{s1} |\zeta| / v) / \sqrt{s_{e1} |s_{e1}'|} + h_\varphi(\omega_{s2}) \cos(\rho s_{e2} - \omega_{s2} |\zeta| / v) / \sqrt{s_{e2} |s_{e2}'|} \right] / \rho, \quad (8)$$

where  $s_{e1,2} = s_e(\omega_{s1,2})$ ,  $\omega_{s1,2}$  are solutions of  $ds_e/d\omega = |\zeta| / (\rho v)$ ,  $\zeta_{\min}(\rho) = \rho v s'_{e\min}$ ,  $s'_{e\min} = ds_e(\omega_{s0})/d\omega$ ,  $\omega_{s0}$  is solution of  $d^2 s_e / d\omega^2 = 0$ . In the special case of  $\gamma \gg 1$  and  $\omega_h \ll \omega_p$  one obtains:

$$\omega_{s0} \approx \omega_p \left[ 1 + \omega_h^2 / (8\omega_p^2) \right], \quad s'_{e\min} \approx 4\omega_p^2 / \omega_h^2. \quad (9)$$

For  $|\zeta| \gg \zeta_{\min}$  we have

$$\omega_{s1}^2 \approx \omega_p^2 + \left[ \rho \omega_p^2 / (2|\zeta| \sqrt{\omega_h}) \right]^{4/3}, \quad (10)$$

$$\omega_{s2}^2 \approx \omega_p^2 + \left[ \omega_h - \left[ \rho \omega_p \omega_\Sigma / (2|\zeta| \sqrt{\omega_h}) \right]^{2/3} \right]^2. \quad (11)$$

Formula (8) predicts the beating behavior of the field.

Another approach describes the field in the vicinity of the charge motion line behind the charge [6]:

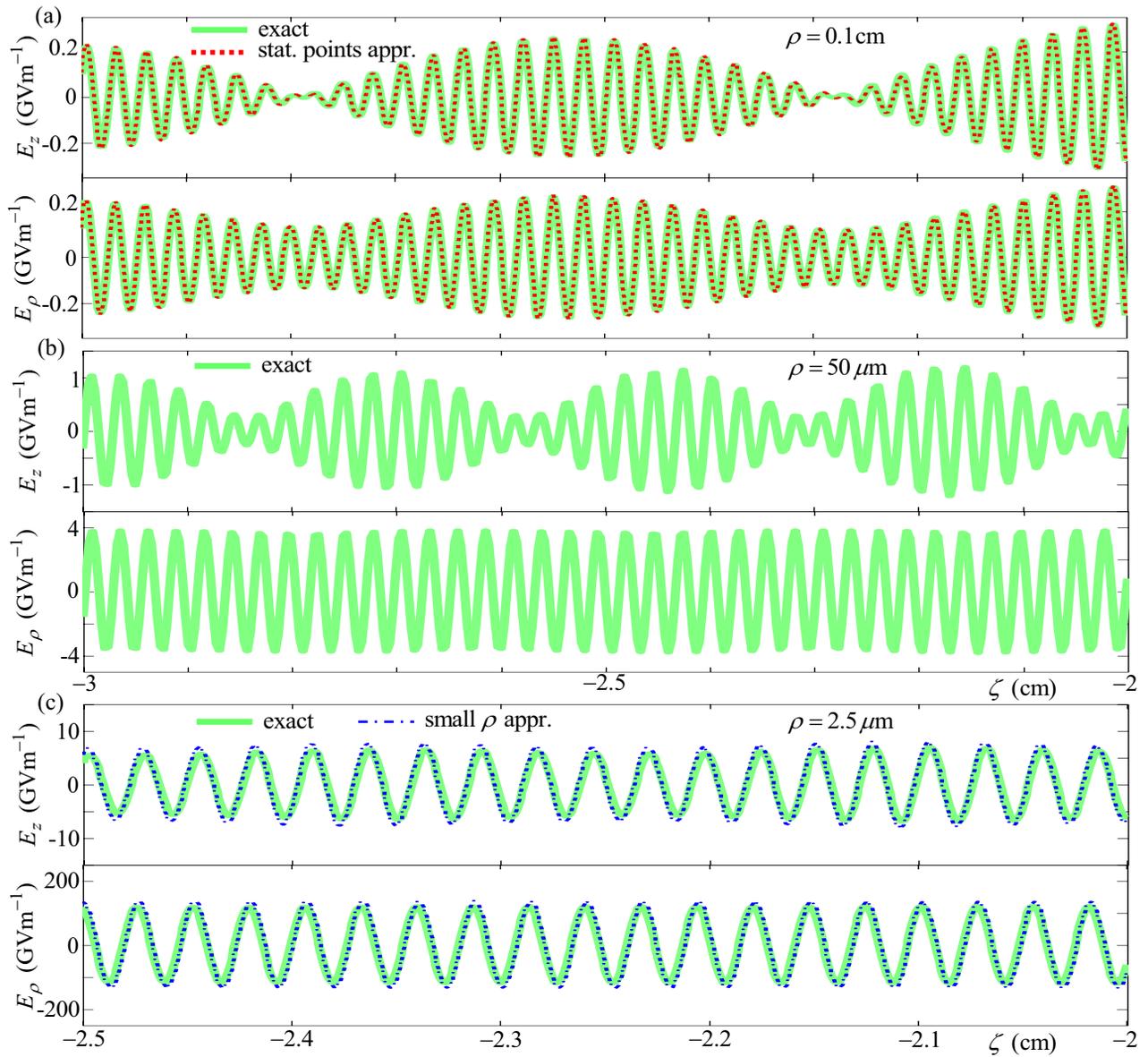


Figure 1: Field of point charge calculated with the use of exact numerical approach (solid green line), stationary point approach (dashed red line) and small  $\rho$  approximation (dash-dotted blue line). Plot (a) corresponds to the far field zone, plot (b) corresponds to the middle zone, while plot (c) corresponds to the zone of small values of  $\rho$ . Calculation parameters are:  $q = -1$  nC,  $\omega_p = 2\pi \times 10^{12}$  s $^{-1}$ ,  $\omega_h = 0.5\omega_p$  ( $B_{ext} \approx 18$  T),  $\gamma = 22$ .

$$E_\rho \approx E_{\rho 0} \sin(\omega_\Sigma \zeta / v), \quad E_z \approx E_{z 0} \cos(\omega_\Sigma \zeta / v), \quad (12)$$

$$H_z \approx H_{z 0} \sin(\omega_\Sigma \zeta / v),$$

$$E_{\rho 0} = 2q\omega_p^2 (v\omega_\Sigma \rho)^{-1}, \quad E_{z 0} = 2q\omega_p^2 v^{-2} \ln(\rho\omega_p / c), \quad (13)$$

$$H_{z 0} = 2q\omega_p^2 \omega_h (v\omega_\Sigma)^{-1} \ln(\rho\omega_p / c).$$

As one can see,  $E_\rho$  possesses strong inverse proportional singularity, while  $E_z$  and  $H_z$  possess weaker logarithmical ones. The rest of components vanishes  $\sim \rho \ln \rho$  as  $\rho \rightarrow 0$ . Moreover, all components behave harmonically with frequency  $\omega_\Sigma$ . Possibilities of decreasing the orthogonal electric component and

enlarging the longitudinal magnetic one by increasing the external magnetic field have been shown in [6].

Along with two aforementioned analytical approaches an effective numerical algorithm has been developed for computation of the field components for arbitrary distance from the charge. Figure 1 shows the dependence of the longitudinal and orthogonal components of the electric field on  $\zeta$  for different offsets  $\rho$  from the charge trajectory. For relatively large offset (Fig. 1,a) the total field calculated numerically is in a good agreement with that calculated via the stationary point approach (8). Both components are of the same magnitude, and the beating behavior is brightly expressed. For relatively small offset

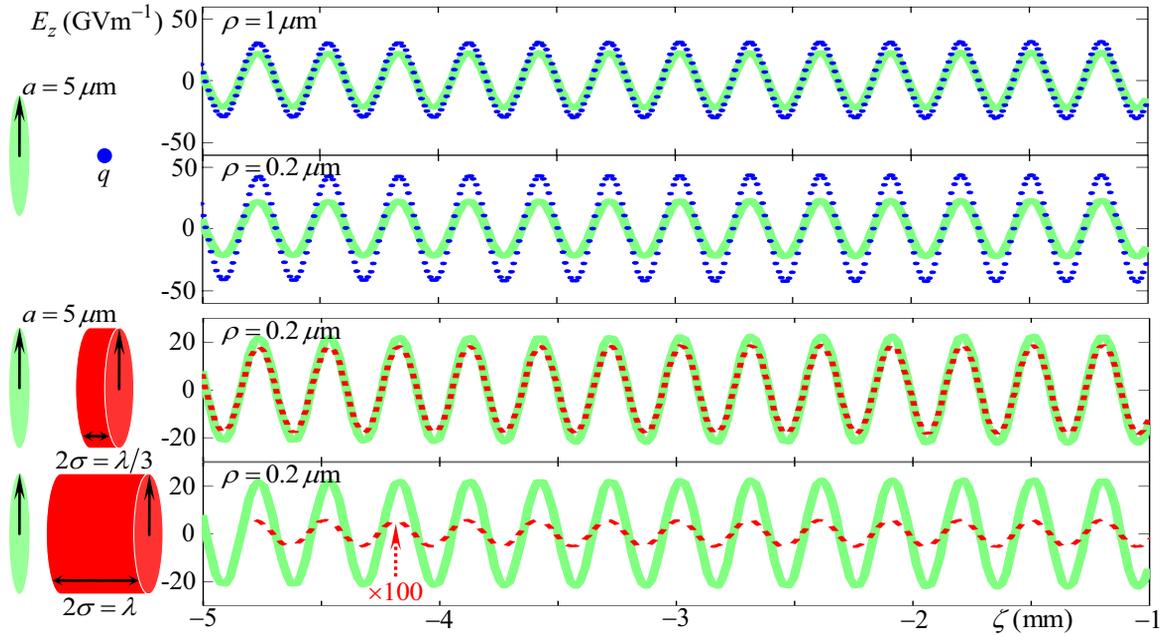


Figure 2: Fields produced by point charge (dotted blue line), thin disc (solid green line) and cylindrical bunch (dashed red line) versus  $\zeta$  for  $q = -1\text{nC}$ ,  $\omega_p = 2\pi \times 10^{12}\text{s}^{-1}$ ,  $\omega_h = 0.14\omega_p$  ( $B_{\text{ext}} = 5\text{T}$ ),  $\gamma = 22$  and  $\lambda = 2\pi\beta c/\omega_\Sigma = 300\ \mu\text{m}$ .

(Fig. 1,c) the total field calculated numerically is again in a good agreement with that calculated via small  $\rho$  approximation formulas (12). Both components behave harmonically at frequency  $\omega_\Sigma$  and  $E_\rho$  is about one order larger compared with  $E_z$ . The region of middle offsets (Fig. 1,b) cannot be covered by any of discussed analytical approximations. Here  $E_z$  component is still in the beating regime, while  $E_\rho$  is already in the harmonic one; magnitudes of components are comparable.

## FIELD OF FINITE BUNCHES

As was shown, field components (12) have singularities on the charge trajectory. Therefore, the analysis of the small bunch field near the line  $\rho = 0$ , which is of most interest in the context of PWFA, requires accounting the finite size of the bunch. We can easily calculate the field of different bunches using the field of the point charge (12) as the Green function. This approach is based on computing the convolution integral with analytical functions and therefore requires neither expensive numerical simulation suites nor a large amount of computer resources.

Figure 2 compares fields produced by point charge, charged disc and charged cylinder. For relatively large  $\rho$  (comparable with the disc radius) the longitudinal electric field produced by point charge practically coincides with that produced by charged disc. With decrease in  $\rho$  the difference becomes essential. Moreover, short cylinder (in the wavelength  $\lambda = 2\pi\beta c/\omega_\Sigma$  scale) produces the field which practically coincides with that of disc. The field of long cylinder is several orders smaller compared with the

field of disc which is explained by non-coherency of the fields produced by separate particles of the long bunch.

## CONCLUSION

In this paper we have analytically described the field of a small bunch moving in a cold electron plasma along the external magnetic field both in the far-field zone and in the vicinity of the bunch trajectory. We have suggested the combined numerically-analytical method for computation of wakefields produced by small (in the orthogonal direction) bunches of arbitrary shape and charge distribution which can be applied to improvement of PWFA technique. Moreover, we have shown that the use of longitudinally magnetized plasma provides certain advantages for this scheme, such as focusing action of the external magnetic field (which can be additionally enhanced by the longitudinal magnetic component of the wakefield) and suppressing of the transversal electric field which causes the bunch spreading.

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