Physics of coherent radiation by relativistic electron bunches

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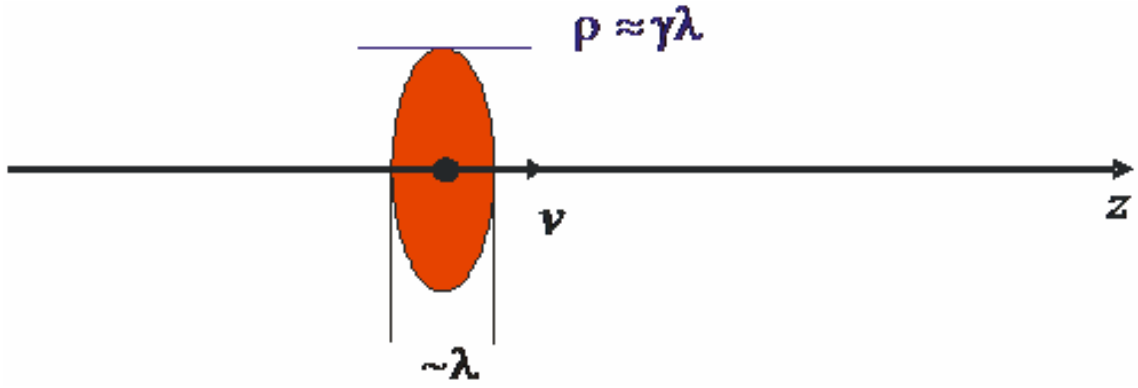
- How does Electron Radiate?
- Coherent beam - beam radiation
- Backward Compton scattering = undulator radiation
- On radiationless motion of accelerated electrons bunch
- Positron source

Relativistic Electron Field Potential



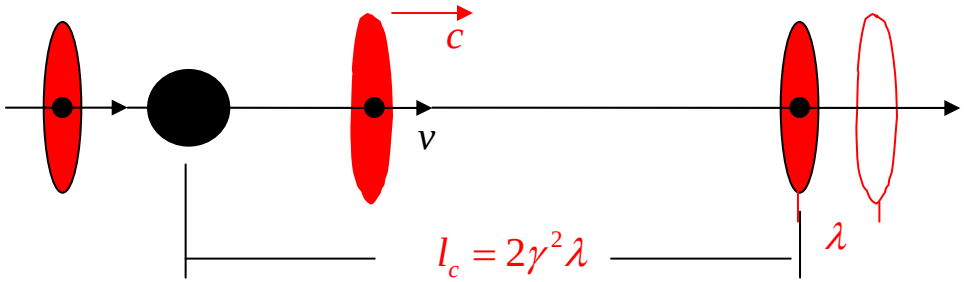
$$c_{\vec{k}} = e^{-ik_z vt} \int d^3 r e^{i\vec{k}\vec{r}} \frac{e}{\sqrt{z^2 + \rho^2/\gamma^2}} =$$

$$= \frac{4\pi\gamma e}{k_\perp} e^{-ik_z vt} \int_0^\infty dz \cos k_z z e^{-\gamma k_\perp z} = 4\pi e \cdot e^{-ik_z vt} \int_0^\infty \rho \, d\rho \; J_0\left(k_\perp \rho\right) K_0\left(k_z \rho/\gamma\right)$$

How Does Electron Radiate?

The excitation is small

$\gamma\theta_{scat} << 1$

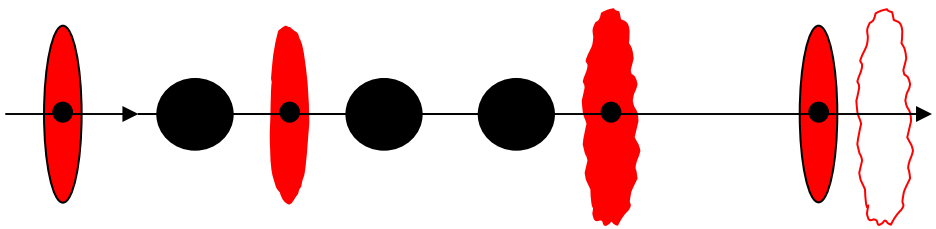


$$v_{rel} = c - v$$

$$\lambda = (c - v)\Delta t_c \rightarrow \Delta t_c \approx 2\gamma^2\lambda$$

The excitation is increased

$\gamma\theta_{scat} \approx 1$

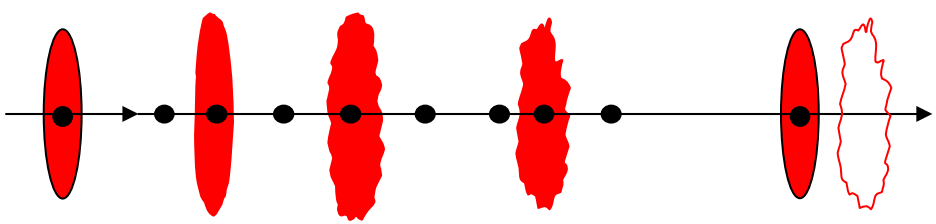


$$l_c = 2\gamma^2\lambda$$

$$\varepsilon = 5GeV$$

The excitation is strong

$\gamma\theta_{scat} >> 1$



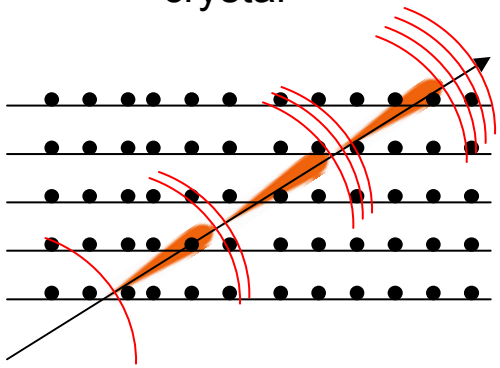
$$l_c \sim 10^{-3} cm \quad for \quad \omega \sim 1MeV$$

$$l_c \sim 1cm \quad for \quad \omega \sim 1KeV$$

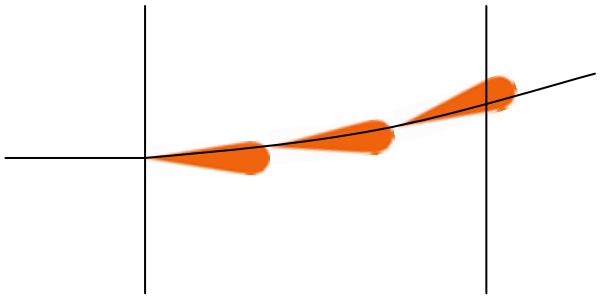
Coherence Length

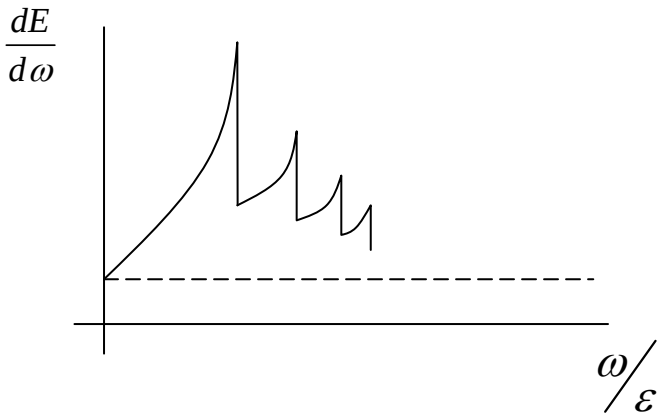
$$l_c = 2\gamma^2 / \omega$$

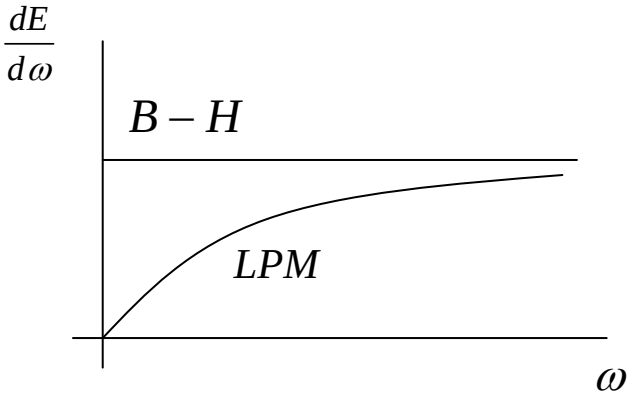
crystal



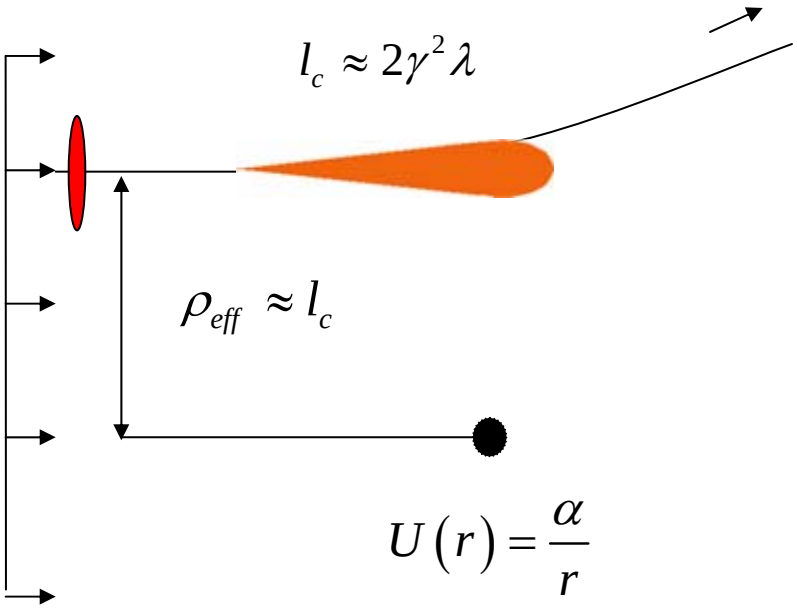
amorphous media





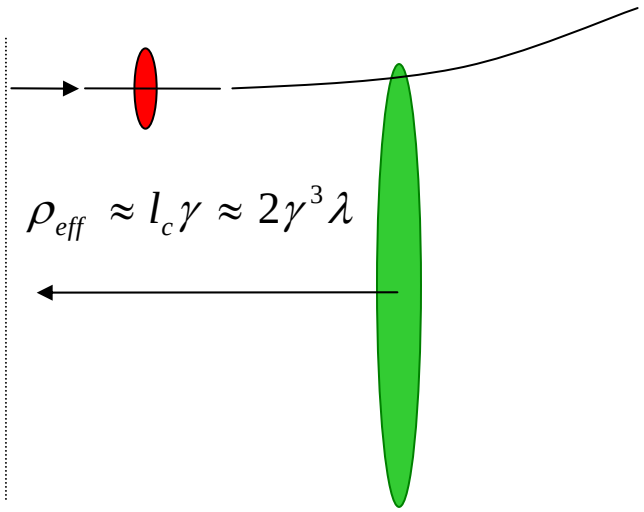


Radiation in Coulomb field of relativistic particle



$$U(r) = \frac{\alpha}{r}$$

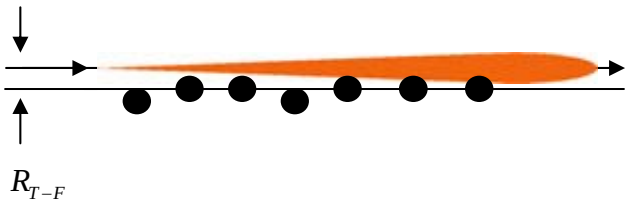
!!!



$$U(r) = \frac{\alpha}{\sqrt{(z+vt)^2 + \rho^2/\gamma^2}}$$

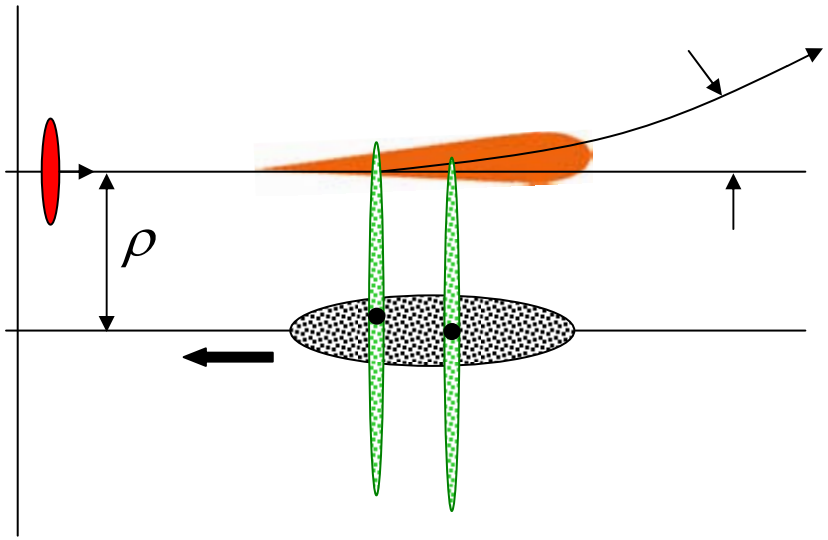
Coherent radiation in crystal and at electron collision with a short bunch

crystal atomic string



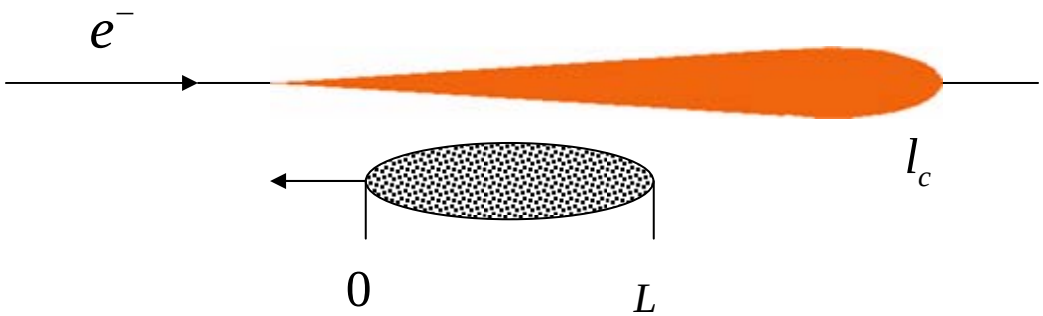
bunch

$$\vartheta_N = \frac{2Ne^2}{\varepsilon\rho}$$

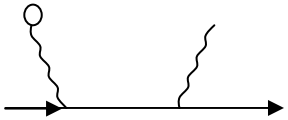


N. Shul'ga, D. Tyutyunnik, JEPT Lett. 78(2003)700., Proc. of SPIE, v. 5974(2005)60.

Coherent radiation at electron-beam collision



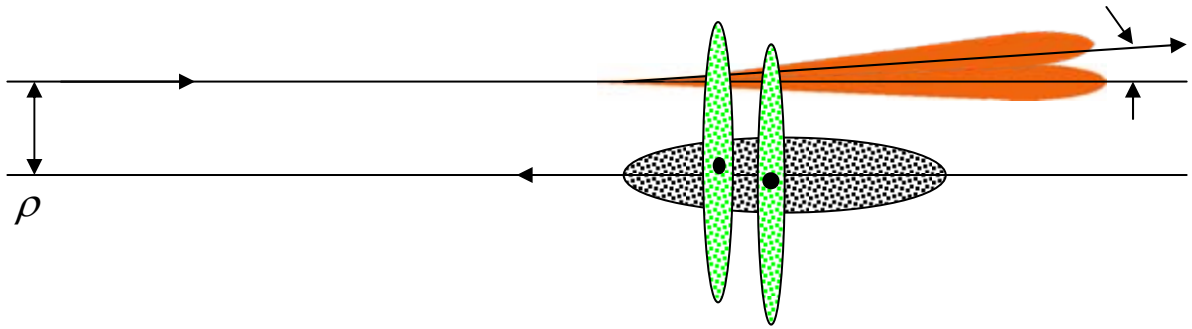
$$\frac{dE_B}{d\omega} \approx N_B^2$$



I. Ginzburg, G. Kotkin, S. Polityko, V. Serbo
Phys. Lett. B286 (1992) 395
Phys. Rev. E51 (1995) 2493

$$\frac{N_B e^2}{\hbar c} << 1$$

Suppression of coherent radiation (analog of LPM-effect)



$$\vartheta_N = \frac{2Ne^2}{\varepsilon\rho}$$

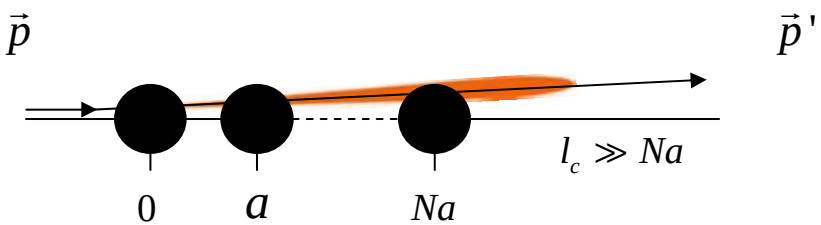
$$\frac{dE_N}{d\omega} = \frac{2e^2}{\pi}\left[\frac{2\xi^2+1}{\xi\sqrt{\xi^2+1}}\ln\left(\xi+\sqrt{\xi^2+1}\right)-1\right], \qquad \xi = \frac{\gamma\vartheta_N}{2} \qquad , \qquad l_c \gg L$$

$$\frac{dE_N}{d\omega} \approx \begin{cases} N^2\dfrac{e^6}{m^2\rho^2} & \gamma\vartheta_N \ll 1 \\ 4e^2\ln\left(\dfrac{Ne^2}{m\rho}\right) & \gamma\vartheta_N \gg 1 \end{cases}$$

ε=5 Gev, L=0.1 cm, ρ=0.01 cm, N=10^{10}, $\omega_c = 4\gamma^2/L \approx 50kev,$ $\gamma\vartheta_N \approx 1$

N. Shul'ga, D. Tyutyunnik. JETP Lett. 78 (2003) 700.
NiM B227 (2005) 152

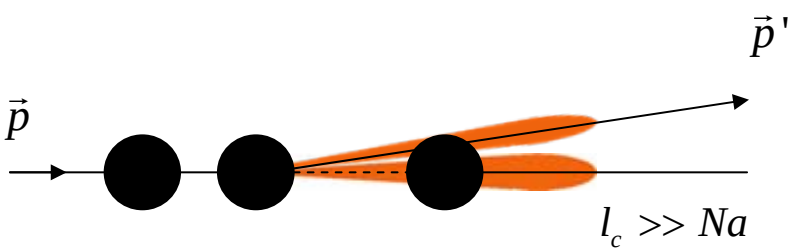
Coherent radiation in a thin crystal



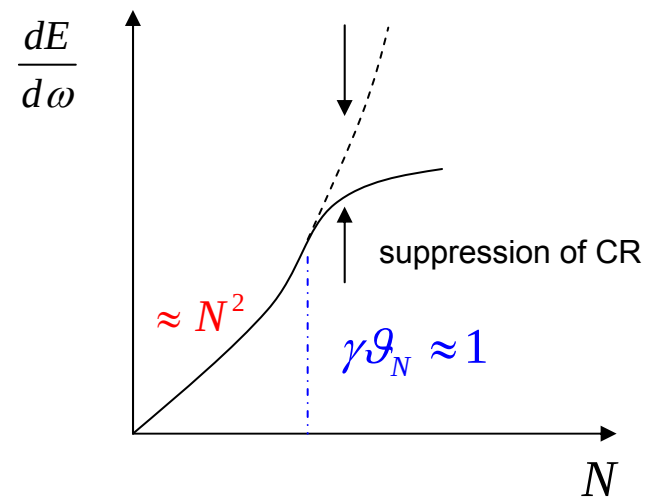
$\gamma\vartheta_N << 1$

$$\vartheta_N = N\frac{2Ze^2}{\varepsilon\rho}, \qquad \rho \le R$$

$$\frac{dE}{d\omega} \approx \frac{2e^2}{3\pi}\begin{cases}\gamma^2\vartheta_N^2, & \gamma\vartheta_N << 1\\ 6\ln\left(\gamma\vartheta_N\right), & \gamma\vartheta_N >> 1\end{cases}$$

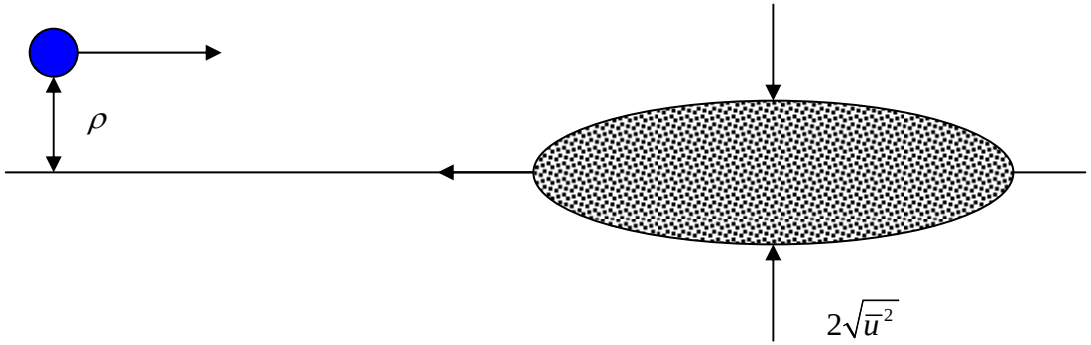


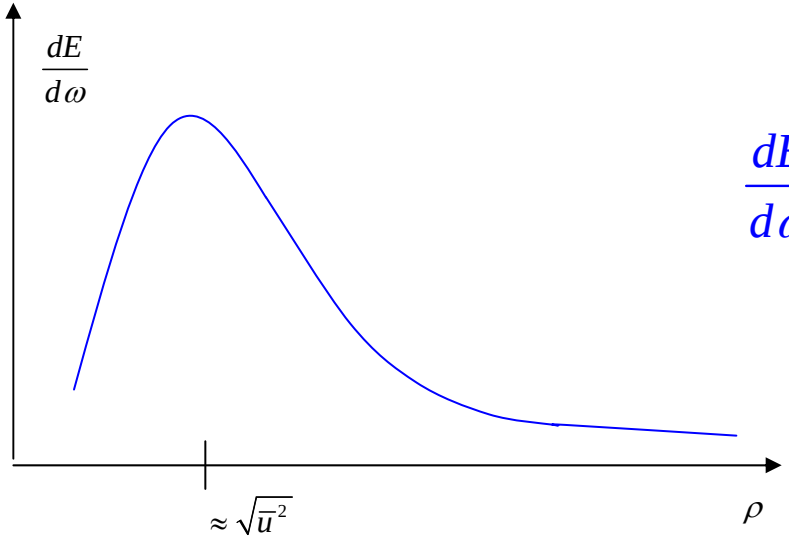
$\gamma\vartheta_N >> 1$



A. Akhiezer, N. Shul'ga et al. Sov. J. Part. Nucl. 10 (1979)

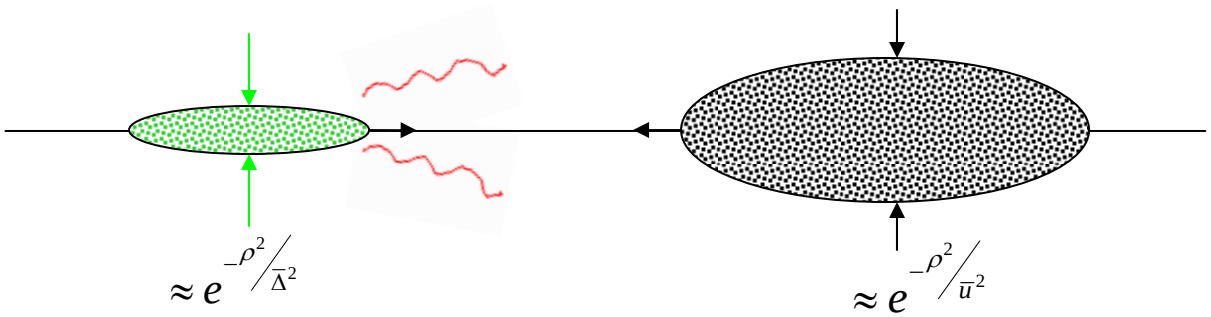
Beam-beam monitoring





$$\frac{dE}{d\omega} = \frac{2e^2}{3\pi}\left(\gamma\vartheta_N\right)^2 = N^2 \frac{32e^6}{3\pi m^2 \rho^2} e^{-\rho^2/\overline{u}^2} sh^2\left(\frac{\rho^2}{2\overline{u}^2}\right)$$

Beam-size dependence



$$\gamma\vartheta_N << 1$$

$$\left\langle \frac{dE}{d\omega} \right\rangle = \int d^2\rho \frac{1}{\pi\bar{\Delta}^2} e^{-\rho^2/\bar{\Delta}^2} \frac{dE(\rho)}{d\omega}$$

$$\left\langle \frac{dE}{d\omega} \right\rangle = \frac{32N^2e^6}{3\pi^2m^2\bar{\Delta}^2} \int \frac{d^2\rho}{\rho^2} e^{-\left(\frac{\rho^2}{\bar{\Delta}^2}+\frac{\rho^2}{\bar{u}^2}\right)} sh^2\left(\rho^2/2\bar{u}^2\right)$$

$$\left\langle \frac{dE}{d\omega} \right\rangle = \frac{8N^2e^6}{3\pi m^2\bar{\Delta}^2} \begin{cases} \left(2\bar{\Delta}^2/\bar{u}^2\right)^2 & \bar{\Delta}^2 << \bar{u}^2 \\ \ln\left(\bar{\Delta}^2/\bar{u}^2\right) & \bar{\Delta}^2 >> \bar{u}^2 \end{cases}$$

Conclusions

Beam-beam collision and CR
Beam-crystal collision and CR } analogy

Born approximation $Ne^2/\hbar c << 1$

Semiclassical approximation $Ne^2/\hbar c >> 1$ $N \approx 10^{10}$

coherent radiation $\gamma\vartheta_N << 1$

suppression of CR $\gamma\vartheta_N >> 1$ $N \approx 10^{10}$

$l_c \approx 2\gamma^2/\omega$
$\rho_{eff} \approx \gamma^3/\omega$ } long-distance effect

Beam-beam monitoring
Beams-sizes dependence of CR } applications

Radiation of Charge Distribution at $\vec{r}(t+T)=\vec{r}(t)+\vec{v}T$

$$\frac{dE}{d\omega do}=\frac{e^2}{4\pi^2}\left|\vec{k}\times\vec{I}\right|^2$$

$$\vec{J}(\vec{r},t)=e\vec{\upsilon}(t)\,\rho\left(\vec{r}-\vec{r}(t)\right)$$

$$\vec{I}=\int dt\, e^{i\left(\omega t-\vec{k}\vec{r}(t)\right)}\vec{\upsilon}(t)\int d^3k\, e^{-i\vec{k}\vec{r}}\rho(\vec{r})$$

$$\vec{r}(t+T)=r(t)+\vec{\upsilon}T$$

$$\vec{I}=\frac{2\pi}{T}\sum_n\delta\left(\omega-\vec{k}\vec{\upsilon}-\omega_n\right)\int_0^T dt\, e^{i\left(\omega t-\vec{k}\vec{r}(t)\right)}\vec{\upsilon}(t)\int d^3r\, e^{-i\vec{k}\vec{r}}\rho(\vec{r})$$

$$\omega_n=\frac{2\pi}{T}n$$

Classically Radiationless Motion of Oscillating Charge

$$\vec{r}(t+T)=\vec{r}(t)$$

1. Spherically symmetrical charge distribution (G.Schott 1933)

$$\rho(\vec{r})=\frac{1}{4\pi R}\delta(r-R) \qquad \rho_{\vec{k}}=\int d^3r\, e^{-i\vec{k}\vec{r}}\rho(\vec{r})=\frac{1}{\omega_n R}\sin(\omega_n R)$$

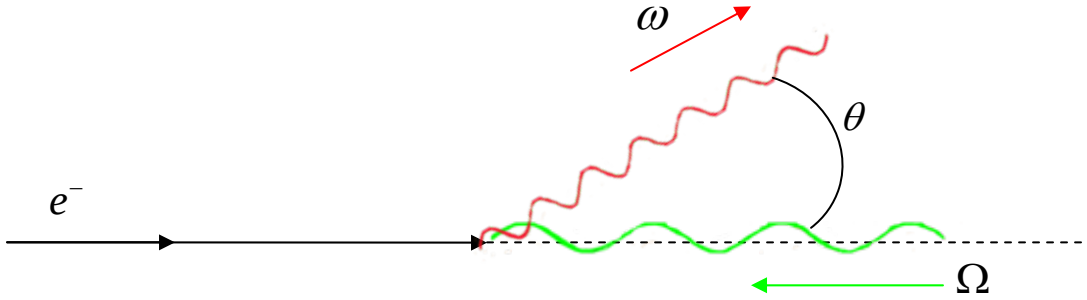
$$\omega_n=\frac{2\pi}{T}n$$

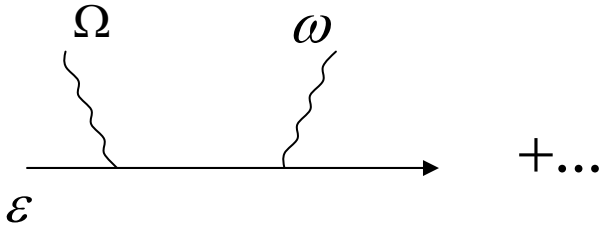
The radiation is absent for $\frac{2\pi}{T}R=kn\pi$

2. $\rho(\vec{r})=\rho(r-R)$, nonspherically symmetrical charge distribution, …

G.Goedecke, Phys.Rev 1964
A.Devaney, E.Wolf, J.Math.Phys. 1974

Backward Compton Scattering (Quantum Theory)

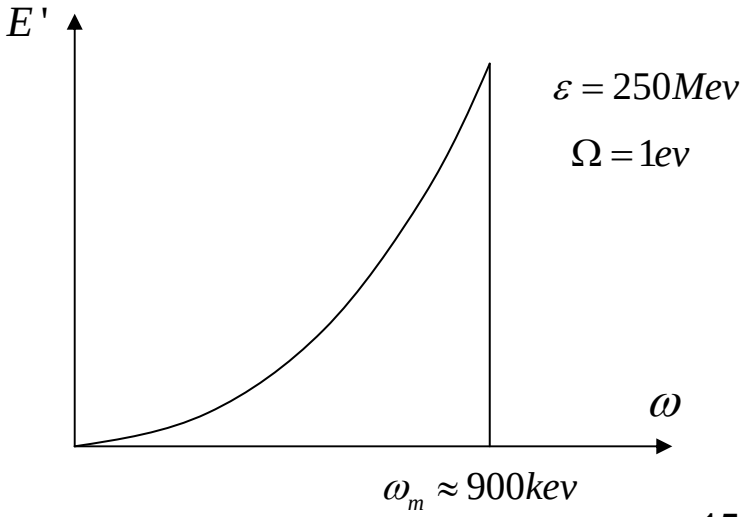




$$\frac{dE}{d\omega do} = T\frac{a^2 r_0^2}{2\Omega}\frac{\varepsilon'}{\varepsilon}\frac{\omega}{\omega_m}\left(\frac{\varepsilon}{\varepsilon'}+\frac{\varepsilon'}{\varepsilon}-4\frac{\omega}{\omega_m}\left(1-\frac{\omega}{\omega_m}\right)\right)$$

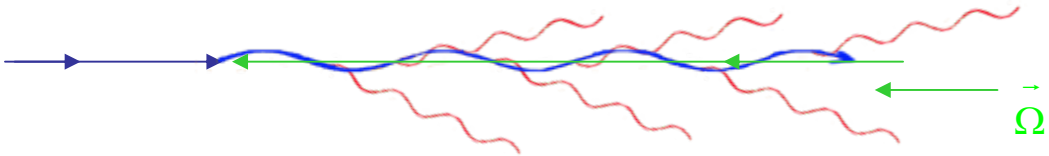
$$\omega_m = \frac{4\varepsilon\varepsilon'\Omega}{m^2}$$

$$\varepsilon' = \varepsilon - \hbar\omega$$



Backward Compton Scattering = Undulator Radiation

$$\frac{dE}{d\omega do} = \frac{e^2}{4\pi^2}\left|\vec{k}\times\int_{-\infty}^{+\infty}\vec{v}(t)e^{i(\omega t-\vec{k}\vec{r}(t))}dt\right|^2$$



$$\gamma\vartheta_e << 1$$

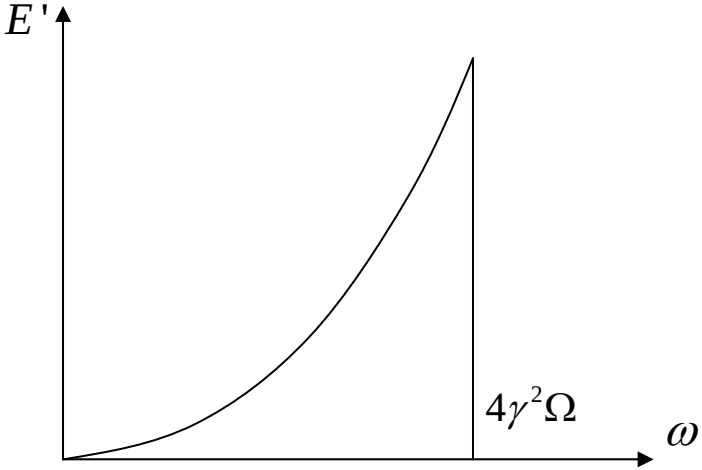
$$\frac{dE}{d\omega do} = \frac{e^2}{4\pi^2}\frac{\omega^2}{q^2}\left[1-4\frac{\delta}{q}\left(1-\frac{\delta}{q}\right)\cos^2\varphi\right]\left|\vec{W}(q)\right|^2$$

$$\frac{dE}{d\omega} = T\frac{A^2 r_0^2}{\Omega}\frac{\omega}{\omega_m}\left[1-2\frac{\omega}{\omega_m}\left(1-\frac{\omega}{\omega_m}\right)\right]$$

$$q = \omega - \vec{k}\vec{v} \geq \delta = \omega/2\gamma^2$$

$$\vec{W}(q) = \int \dot{\vec{v}}_\perp(t)e^{iqt}dt$$

$$\dot{\vec{v}}_\perp(t) = \frac{2e}{\varepsilon}\mathrm{Re}\,\vec{A}e^{i(\Omega-\vec{\Omega}\vec{v}_0)t}$$



N. Shul'ga, D. Tyutyunnik. Phys. Lett. A326 (2004) 287.

Suppression of Radiation at Undulator Motion

$$\vec{r}(t+T)=\vec{r}(t)+\vec{\upsilon}T, \qquad \upsilon_\perp \ll \upsilon$$

Пластинка: $$\rho_{\vec{k}}=\int d^3k\, e^{i\left(\vec{k}_\perp\vec{\rho}+k_z z\right)}\rho(\vec{r})=\frac{1}{L_x L_y L_z}(2\pi)^2\delta(k_\perp)2\frac{\sin\left(k_z L_z/2\right)}{k_z}$$

$$\omega-\vec{k}\vec{\upsilon}=\Omega\cdot n \qquad \omega_n=\frac{4\pi\gamma^2}{T}n$$

Отсутствие излучения: $$\frac{k_z L_z}{2}=\frac{2\pi\gamma^2}{T}L_z=\pi \qquad L_z=\frac{1}{2\gamma^2}T$$

Подавление излучения при $\omega\sim\omega_n$:

$$k_\perp L_\perp\sim\omega_n\theta_\gamma L_\perp\sim\frac{4\pi\gamma}{T}L_\perp >\sim 1 \qquad L_\perp >\sim\frac{T}{4\pi\gamma}$$

KEK Preprint 2005-60
physics/0509016
CARE/ELAND Document-2005-013
CLIC Note 639
LAL 05-94
September 2005
A / H

Conceptual Design of a Polarised Positron Source Based on Laser Compton Scattering

- A Proposal Submitted to Snowmass 2005 -

Sakae Araki [1], Yasuo Higashi [1], Yousuke Honda [1], Yoshimasa Kurihara [1], Masao Kuriki [1], Toshiyuki Okugi [1], Tsunehiko Omori [1], Takashi Taniguchi [1], Nobuhiro Terunuma [1], Junji Urakawa [1], X. Artru [2], M. Chevallier [2], V. Strakhovenko [3], Eugene Bulyak [4], Peter Gladkikh [4], Klaus Mönig [5], Robert Chehab [6], Alessandro Variola [6], Fabian Zomer [6], Susanna Guiducci [7], Pantaleo Raimondi [7], Frank Zimmermann [8], Kazuyuki Sakaue [9], Tachishige Hirose [9], Masakazu Washio [9], Noboru Sasao [10], Hirokazu Yokoyama [10], Masafumi Fukuda [11], Koichiro Hirano [11], Mikio Takano [11], Tohru Takahashi [12], Hiroki Sato [12], Akira Tsunemi [13], Jie Gao [14] and Viktor Soskov [15]

[1] *KEK, Ibaraki, Japan*
[2] *IPN, Lyon, France*
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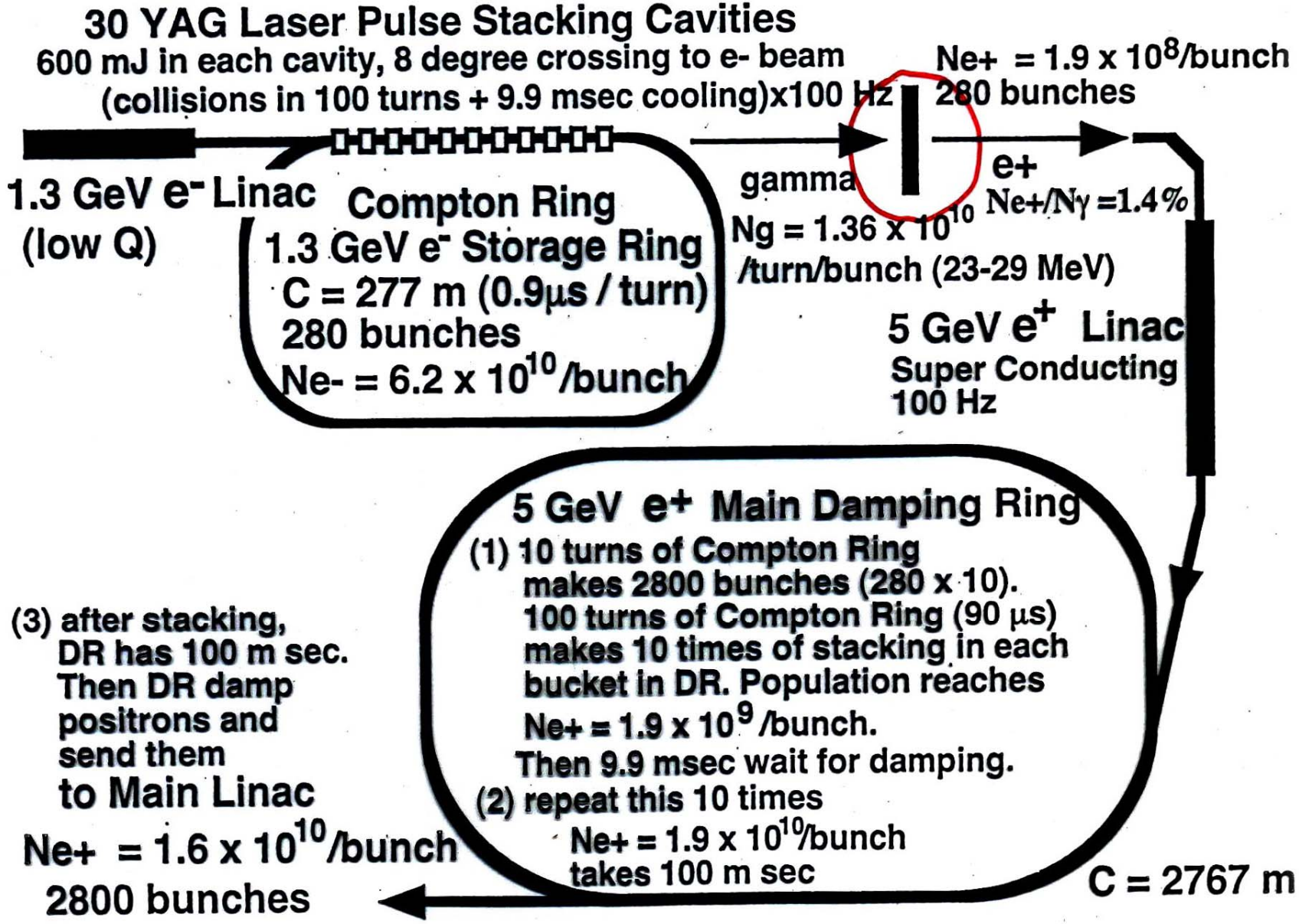
[4] *NSC KIPT, Kharkov, Ukraine*
[5] *DESY, Zeuthen, Germany & LAL, Orsay, France*
[6] *LAL, Orsay, France*
[7] *INFN, Frascati, Italy*
[8] *CERN, Geneva, Switzerland*
[9] *Waseda University, Tokyo, Japan*
[10] *Kyoto University, Kyoto, Japan*
[11] *NIRS, Chiba, Japan*
[12] *Hiroshima University, Hiroshima, Japan*
[13] *Sumitomo Heavy Industries Ltd., Tokyo, Japan*
[14] *IHEP, Beijing, China*
[15] *IHEP, P.N. Lebedev Physical Institute, Russian Academy of Sciences, Moscow*

High Energy Accelerator Research Organization

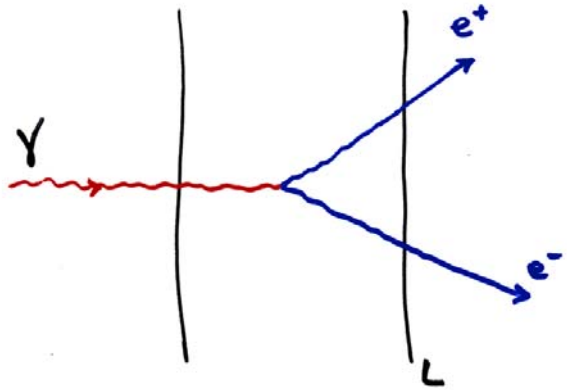
Positron Source Problem

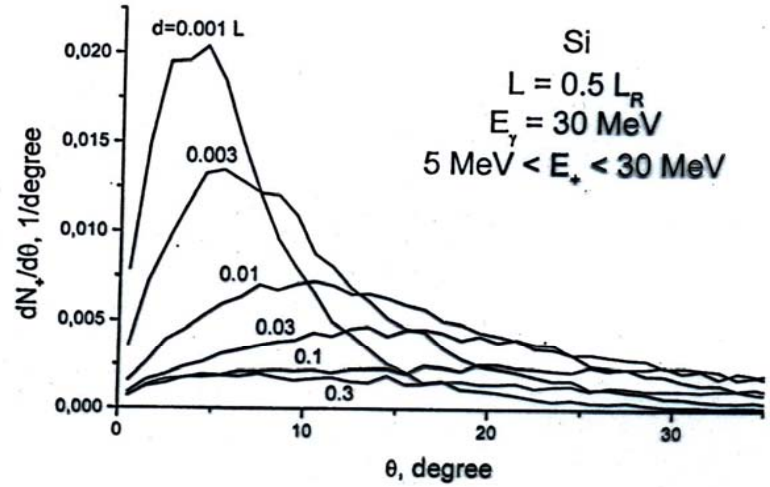


Proposal of a Polarised Positron Source Based on Laser Compton Scattering, August 2005
Figure 14: Schematic diagram of the total system in the case of YAG laser.

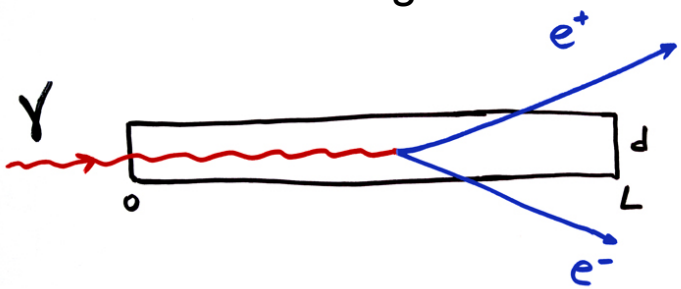
Positron Source from Pencil-like Target

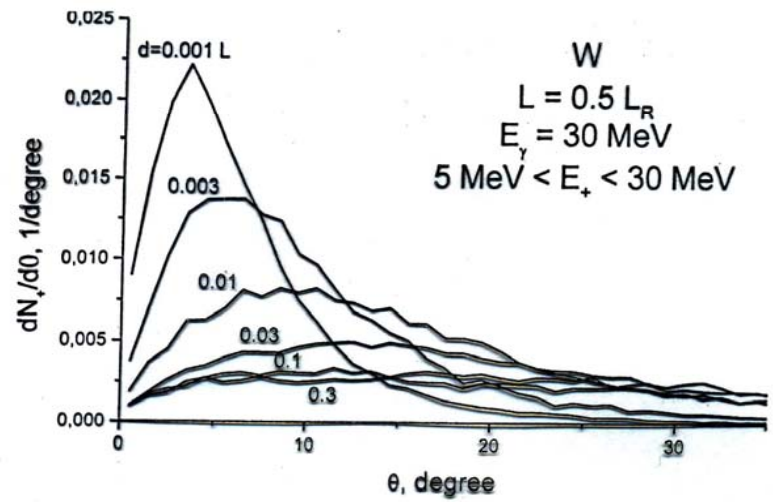
N.Shul'ga, V.Lapko Phys.Lett.A v.359, p.8-9 (2006)





Pencil-like target





A.Mikhailichenko (2003)