Analytical study of beam equilibrium in high energy storage ring for non-magnetized electron cooling.

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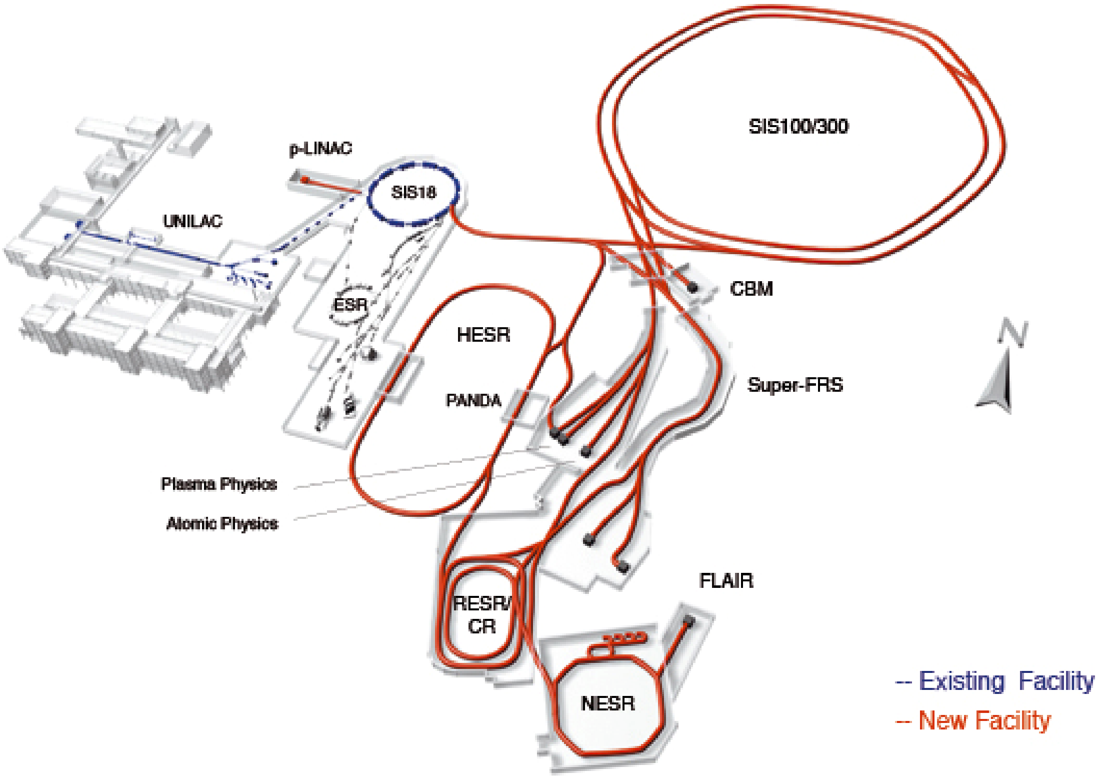
Introduction 1.

- Circular accelerators and storage rings are used for the physical experiments with the internal pellet target, which heats the beam in transverse and longitudinal direction. For compensation of the beam heating the electron or stochastic cooling can be applied (see FNAL).
- In a frame of FAIR project it is developed a design of high energy storage ring (HESR) where the high energy antiprotons (-15 GeV) should be stored . One of main goals is "monochromatic" experiments with small momentum spread ($\sigma_p \sim 10^{-4}$) of the stored beam for examination of thin resonances. To achieve such beam parameters it was proposed to use "magnetized" electron cooling. This situation was studied with very attention in a frame of collaboration GSI-ITEP-JINR-Kiev University.:

  'Advanced beam dynamics for storage rings' (INTAS grant with Ref. Nr. 03-54-5584, 222, 2004-2006 years).
- The beam dynamics in the HESR was considered with an account of the following processes:
  1) **transverse Coulomb scattering** and **energy straggling** in the internal target;
  2) **magnetized** electron cooling;
  3) **intra-beam scattering** (IBS) inside the p-bar beam.
- For numerical modelling we used codes "MOCAC" (ITEP) and "BETACOOL" (JINR). These calculations are time consuming, especially in a case of many free parameters.
- Approximate analytical solution for equilibrium parameters was obtained in paper O. Boine-Frankenheim, R. Hasse , F. Hinerberger, A. Lehrah and P. Zenkevich, "Cooling Equilibrium and beam loss with internal targets in high energy storage rings".
- In this paper it was used ***Parchomchuk's model for magnetized cooling***!

FAIR project



Introduction 2.

- For monochromatic experiments it is proposed to use pellet target. At present time the pellet beam has large angular divergence . Therefore for improvement of “beam-target” crossing it is clear that it is desirable to use the antiproton beam with high transverse emittance.
- However for such emittance the Parchomchuk’s model of magnetized cooling is non-applicable since the transverse velocity of the p-bar beam in the cooling section is much higher then Parchomchuk’s “effective velocity”. Taking into account these considerations we apply to solving of the problem a theory of “non-magnetized” cooling for “flattened” velocity distribution developed by **Shemyakin** for FNAL. Using this theory we find simple analytical solution for a case of high beam emittance and mono-magnetized cooling.

Simplifying assumptions of the model.

- The ion beam has Gaussian distribution with equal rms sizes on both transverse degrees of freedom ($\sigma^*_{px} = \sigma^*_{py}$). Here and in the following text asterisk * marks all parameters related to the beam rest frame (BRF).
- The electron beam in the cooling section has the circular cross-section with radius *a* and uniform density and Gaussian distribution of velocities.
- The target is uniform with width equivalent to the “averaged” width of the pellet target.
- The beam is relativistic ($\gamma \gg 1$): the last assumption allows us to use simplified model of IBS.

Non-magnetized electron cooling 1.

- Let us assume that the horizontal and vertical rms velocity spreads of the antiproton and the electron beams in the cooling section are equal: ($\sigma_{px}^* = \sigma_{py}^*$, $\sigma_{ex}^* = \sigma_{ey}^*$). The rms longitudinal velocity spreads are σ_{pz}^* and σ_{ez}^*, respectively. Then, according to Shemyakin, we obtain the following equations for evolution of these parameters:

$$\begin{cases} \dfrac{1}{(\sigma_{px}^*)^2}\dfrac{d}{dt^*}(\sigma_{px}^*)^2 = \dfrac{B^*}{((\sigma_{ex}^*)^2 + (\sigma_{px}^*)^2)^{3/2}}\mathrm{X}(\alpha) \\ \dfrac{1}{(\sigma_{pz}^*)^2}\dfrac{d}{dt^*}(\sigma_{pz}^*)^2 = \dfrac{B^*}{[(\sigma_{ex}^*)^2 + (\sigma_{px}^*)^2]\sqrt{(\sigma_{ez}^*)^2 + (\sigma_{pz}^*)^2}}\Upsilon(\alpha) \end{cases}$$

- Here parameter $\alpha = \dfrac{(\sigma_{ez}^*)^2 + (\sigma_{pz}^*)^2}{(\sigma_{ex}^*)^2 + (\sigma_{px}^*)^2}$. The functions $\mathrm{X}(\alpha)$ and $\Upsilon(\alpha)$ are defined by

$$\begin{cases} \mathrm{X}(\alpha) = \dfrac{\arccos(\sqrt{\alpha}) - \sqrt{\alpha(1-\alpha)}}{2(1-\alpha)^{3/2}} \\ \Upsilon(\alpha) = \dfrac{1 - \sqrt{\dfrac{\alpha}{1-\alpha}}\arccos(\sqrt{\alpha})}{1-\alpha} \end{cases}$$

- For high ion emittances $\sigma_{ex}^*, \sigma_{ez}^*, \sigma_{pz}^* \ll \sigma_{px}^*$, $\alpha \ll 1$ and $\alpha \to 0$. If $\mathrm{X}(\alpha) \to \pi/4$, then, $\Upsilon(\alpha) \to 1$
- . The coefficient $B^* = \sqrt{2/\pi}4\pi m_e (r_e)^2 c^4 n_e^* f L_c^{cool}$. Plots of the functions $\mathrm{X}(\alpha)$ and $\Upsilon(\alpha)$ are given at Fig. 1. After transfer to LF we obtain:

Non-magnetized electron cooling 2.

- After transfer to LF we obtain:

$$\begin{cases} \dfrac{1}{\epsilon}\dfrac{d\epsilon}{dt} = A\dfrac{1}{[(\beta\gamma)^2\epsilon/\epsilon_{eff}+1]^{\frac{3}{2}}}\mathrm{X}(\alpha) = \dfrac{1}{\tau_{\perp}^{cool}} \\ \dfrac{1}{(\sigma_p)^2}\dfrac{d}{dt}(\sigma_p)^2 = A\dfrac{1}{[(\beta\gamma)^2\epsilon/\epsilon_{eff}+1]^{3/2}}\dfrac{\Upsilon(\alpha)}{\sqrt{\alpha}} = \dfrac{1}{\tau_{\parallel}^{cool}} \end{cases}$$

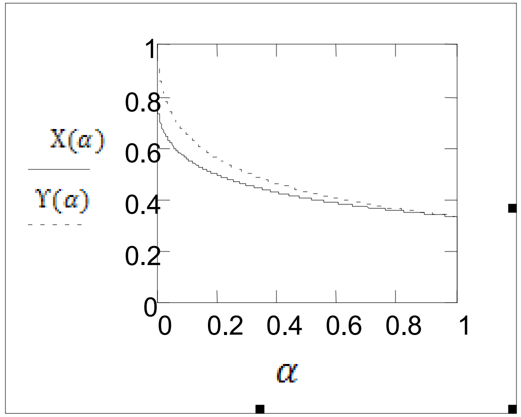
- Here $\epsilon_{eff} = 4\beta_{cool}T_{ex}/E_e$; in LF parameter $\alpha = \dfrac{T_{ez}/T_{ex} + (\beta\sigma_p/\theta_{ex}^{eff})^2}{1+(\beta\gamma)^2\epsilon/\epsilon_{eff}}$. In numerical calculations it is useful to express the electron density through the electron current I_e using the relation $n_e = \dfrac{I_e}{\beta c e 4\pi a^2}$; then we obtain

$$B = c\sqrt{\frac{2}{\pi}}f\frac{r_i L_c^{cool}}{(\theta_{ex}^{eff})^3\beta\gamma^2 a^2}\frac{I_e}{I_{Alf}}$$

- In Eq.(4) Alfven current $I_{Alf} = 1.7\cdot 10^4$ A. For high emittance and small momentum spread $\alpha \ll 1$; taking into account that $\Upsilon(0) = 1$ and $\mathrm{X}(0) = \pi/4$ we find:

$$\frac{1}{\tau_{\perp}^{cool}} = \frac{\pi}{4}\frac{1}{\tau_{cool}^{0}},\ \frac{1}{\tau_{\parallel}^{cool}} = \frac{1}{\tau_{cool}^{0}}\frac{1}{\sqrt{\alpha}}$$

$$\frac{1}{\tau_{cool}^{0}} = B\frac{1}{[(\beta\gamma)^2\epsilon/\epsilon_{eff}+1]^{\frac{3}{2}}}$$



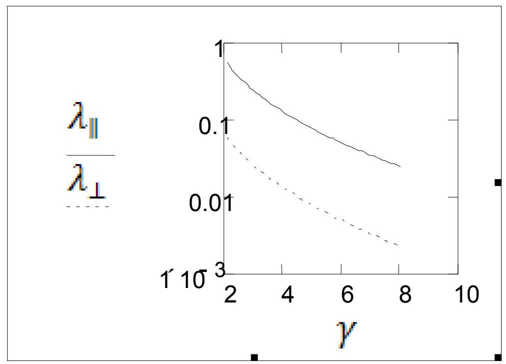


Fig. 1 (upper) and Fig. 2 (down)

Fig.1. Dependence of the functions X(α) and Y(α) versus the dimensionless parameter α.

Fig.2. Dependence of cooling rates $\lambda_{\parallel}$ and $\lambda_{\perp}$ (in logarithmic scale) versus the relativistic parameter γ for $\alpha = 10^{-6}$ m and $\sigma_p = 10^{-4}$

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Non-magnetized electron cooling 3.

- L et us consider a numerical example for HESR, corresponding the following parameters:
  beta-function in cooling section β_{cool} =100m;
  transverse electron temperature T_{ex}^{*} = 0.2 eV;
  longitudinal electron temperature T_{ez} = 0.001 eV;
  ion beam emittance $\epsilon = 10^{-6} m \cdot rad$;
  momentum spread $\sigma_p = \langle \frac{\Delta p}{p} \rangle = 10^{-4}$.

- Under these conditions we obtain:
  $\theta_{ex}^{eff} = 8.86 \cdot 10^{-4}$; $\epsilon_{eff} = 1.6 \cdot 10^{-6}$.
  We see that Parchomchuk's model is not appropriate for given conditions. In LF parameter

$$\alpha(\sigma_p, \gamma) = \frac{0.005 + 1.274 \cdot 10^6 (\beta \sigma_p)^2}{1 + 0.625 (\beta \gamma)^2}$$

- Calculated dependence of decrements on relativistic factor γ is plotted at Fig. 2.

Target

- ***Transverse heating***

  The emittance growth rate due to Coulomb scattering on the target is defined by

$$\frac{d\epsilon}{dt} = A_{\perp} = \beta_t \left(\frac{E_s}{E_p \beta^2 \gamma}\right)^2 \frac{\rho x}{X_r} \nu_0$$

  Here the target width ρx (measured in g/cm²), ρ is the target density multiplied on the target thickness x, X_r is the radiation length (for hydrogen X_r=58 g/cm²), $E_s = 13.6$ MeV, $\nu_0 = (\beta c)/C$ is the number of particle crossings per second. If the target areal density is given in $\Delta \tilde{n}$, then $\rho x = 2\Delta \tilde{n}/N_A$ (here N_A is Avogadro number). For a target thickness areal density $\Delta \tilde{n} = 4 \cdot 10^{15}$ cm^{-2} the target width $\rho x = 1.3 \cdot 10^{-8}$ g/cm².

- The maximum energy of the delta-electrons reads $E_{max} = 2E_e \beta^2 \gamma^2 / [1 + 2\gamma \frac{E_e}{E_p} + \left(\frac{E_e}{E_p}\right)^2]$

  and the average energy losses per one target crossing are given by $\xi_0 = 0.1534 Z^2 \rho x / 2\beta^2$ [MeV]. Let us assume that average energy losses are compensated (for example, by use of induction coil). Then the growth rate of the squared energy deviations per sec is described by the following expression $\langle (\Delta E)^2 \rangle = \xi_0 E_{max} (1 - \beta^2/2)$

- Taking into account kinematics we obtain the final result: $\frac{d(\sigma_p{}^2)}{dt} = A_{\parallel} = \frac{\xi_0 E_{max}(1 - \beta^2/2)}{\beta^4 \gamma^2} \nu_0$

IBS heating

- If the transverse ion beam temperature is much larger than the longitudinal one

$$\frac{\epsilon}{2\langle\beta_x\rangle} \gg (\frac{\sigma_p}{\gamma})^2$$

than the IBS longitudinal heating rate is defined by the following approximate equation :

$$\frac{d({\sigma_p}^2)}{dt} = \frac{\Lambda_{\parallel}^{ibs}}{\epsilon^{3/2}}$$

- This expression is valid for a coasting relativistic beam. Here $\Lambda_{\parallel}^{ibs} = \frac{\sqrt{\pi} c {r_i}^2 N L_c^{ibs}}{4\gamma^3\beta^3\sqrt{\langle\beta_x\rangle} C}$

The emittance growth rate due to IBS:

$$(\frac{d\epsilon}{dt})_{IBS} = \gamma^2 \mathrm{K} \frac{\Lambda_{\parallel}^{ibs}}{\epsilon^{3/2}}$$

Here constant

$$\mathrm{K} = \frac{1}{2}\frac{\mathrm{R}}{\nu}\left[\frac{(D^2+\tilde{D}^2)}{(\beta_x)^2}\right]$$

where sign $[\ \ ]$ means averaging over the ring, *D* is the ring dispersion function, $\tilde{D} = \beta_x \acute{D} + \alpha_x D$ (α_x is the Twiss parameter of the lattice, and $\acute{D}$ is its derivative).

Equilibrium conditions 1.

- Taking into account all mentioned above effects (electron cooling, target heating and IBS) we can write the following differential equations for the evolution of the beam parameters:

$$\begin{cases} \dfrac{d\epsilon}{dt} = -\dfrac{\pi}{4}\dfrac{\epsilon}{\tau_{cool}^{0}} + A_{\perp} + \gamma^{2}\mathrm{K}\dfrac{\Lambda_{\parallel}^{ibs}}{\epsilon^{3/2}} \\ \dfrac{d(\sigma_p)^2}{dt} = -\dfrac{(\sigma_p)^2}{\tau_{cool}^{0}}\dfrac{1}{\sqrt{\alpha}} + A_{\parallel} + \dfrac{\Lambda_{\parallel}^{ibs}}{\epsilon^{3/2}} \end{cases}$$

- For equilibrium conditions the derivatives in the left hand side are equal to zero; substituting we obtain:

$$\begin{cases} \dfrac{\pi}{4}\dfrac{\epsilon}{\tau_{cool}^{0}} = A_{\perp} + \gamma^{2}\mathrm{K}\dfrac{\Lambda_{\parallel}^{ibs}}{\epsilon^{3/2}} \\ \dfrac{(\sigma_p)^2}{\tau_{cool}^{0}} = \sqrt{\dfrac{T_{ez}/T_{ex} + (\beta\sigma_p/\theta_{ex}^{eff})^2}{1+(\beta\gamma)^2\epsilon/\epsilon_{eff}}}\left[A_{\parallel} + \dfrac{\Lambda_{\parallel}^{ibs}}{\epsilon^{3/2}}\right] \end{cases}$$

- Dividing the second equation by the first one we find:

$$(\sigma_p)^2 = \mu(\epsilon)\sqrt{T_{ez}/T_{ex} + (\beta\sigma_p/\theta_{ex}^{eff})^2}$$

Equilibrium conditions 2.

- Eq. (13) can be reduced to the quadratic equation by substituting the dimensionless variable $x = (\sigma_p)^2/\mu(\epsilon)$. Solving this quadratic equation relative the *x* we obtain the final expression for the momentum spread:

$$\sigma_p = \sqrt{\mu(\epsilon)\frac{b + \sqrt{b^2 + 4T_{ez}/T_{ex}}}{2}}$$

- where $$\mu(\epsilon) = \frac{\pi}{4}\frac{\epsilon}{\sqrt{1 + (\beta\gamma)^2\epsilon/\epsilon_{eff}}}\frac{A_\parallel + \Lambda_\parallel^{ibs}/\epsilon^{3/2}}{A_\perp + \gamma^2 K\Lambda_\parallel^{ibs}/\epsilon^{3/2}}$$

$$b = (\beta/\theta_{ex}^{eff})^2\mu(\epsilon)$$

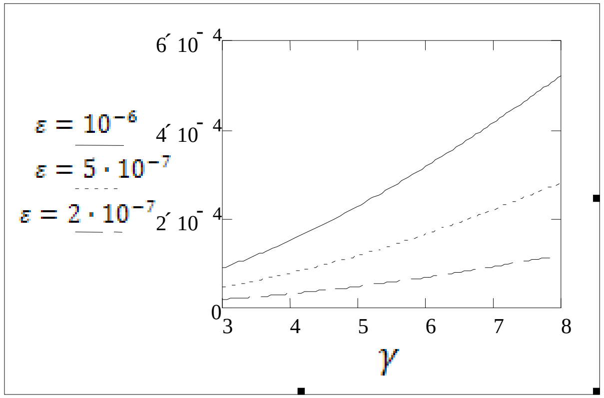
- As can be seen the equilibrium momentum does not depend on the electron cooling rate, which can be found in the first expression of Eq. (14). Using Eq. (7) we obtain that the necessary electron current is given by

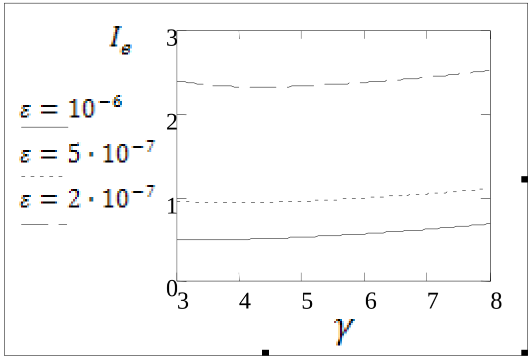
$$I_e = I_{Alf}\frac{(\theta_{ex}^{eff})^3\beta\gamma^2 a^2}{cr_i f L_c^{cool}}\frac{4}{\pi\epsilon}\sqrt{\frac{\pi}{2}}\,(A_\perp + \gamma^2 K\frac{\Lambda_\parallel^{ibs}}{\epsilon^{3/2}})\;[(\beta\gamma)^2\epsilon/\epsilon_{eff} + 1]^{\frac{3}{2}}$$

Ring circumference (m)	570
Betatron tune $\nu_x = \nu_y$)	9.3
Number of ions per ring	10^{11}
Coulomb logarithm for ECS (L_c^{cool})	5
Coulomb logarithm for IBS (L_c^{ibs})	20
Transverse beam emittance (m*rad)	10^{-6}
Target thickness (cm^{-2})	$4 \cdot 10^{15}$
Beta-function in the target (m)	1
Beta-function in the ECS (m)	100
Radius of the electron beam in ECS (mm)	15
Length of the cooling section (m)	24
Longitudinal temperature of the electron beam (eV)	0.001
Transverse temperature of the electron beam (eV)	0.2
Parameter K (m)	1.334

Table 1. Parameters of the ring, ECS and target.

Application to HESR.





Calculated dependencies momentum spread and the electron current versus relativistic factor.

Fig.3. Dependence of r.m.s. momentum spread versus relativistic factor (upper figure).

Fig. 4. Dependence of the electron current versus relativistic factor (low figure) .

Conclusion.

- In the HESR the non-magnetized cooling gives too high momentum spread for the beams with high emittance.
- Momentum spread can be decreased by decrease the required emittance; (at present time at FZ-IKP, Juelich it is developed new pellet target with lower angular divergence). However in this case we need in enhancement of the cooling electron current.
- Let us underline that the results weakly depend on the beam intensity since the IBS is small due to large beam emittance.
- In this operation mode (large emittance and non-magnetized cooling) the magnetic field in the ECS should be designed taking into account the considerations of the electron beam transport and providing stability of the dipole oscillations of coupled electron and ion beams (let us mark that in FNAL the designers have chosen small magnetic field). Use of high magnetic field results in some difficulties in the antiproton beam dynamics).
- In conclusion we would like to say that this report does not pretend to be the final solution; we consider it as some contribution in this long discussion.
- These results, of course , can be used for analysis of the situation in other similar rings.