Magnetooptic Structure for Synchrotrons with Negative Momentum Compaction Factor

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In what cases the structure with such properties is necessary to us?

- To exclude the transition energy crossing
- To have the higher collective instability threshold
- To match in longitudinal plane the different accelerators
- To adjust the different local slip factor for the optimized stochastic cooling
- To avoid the sextupoles in the synchrotron light sources

The requirements to the absolute value of slip factor:

I. For many collective instabilities the threshold is proportional the slip factor

$$\eta = 1/\gamma_{tr}^2 - 1/\gamma^2$$

For instance, from the Landau damping theory the stability requires a minimum spread in incoherent frequencies for longitudinal motion

$$\left(\frac{\delta\omega}{\omega}\right)^2 = \eta^2\left(\frac{\sigma}{p}\right)^2 \geq \frac{eI_{peak}|\eta|}{2\pi m_0 c^2 \gamma\beta^2}\left|\frac{Z_L(n)}{n}\right|$$

So, the absolute value of slip factor module is desirable to have as large as possible.

II. The transition energy crossing $\gamma=\gamma_{tr}$ ($\eta=0$) has to be excluded, since the longitudinal stability disappears.

The requirements to the absolute value of slip factor:

III. The longitudinal beam size is determined by the ratio

$$\Delta\phi_{\max} = \pm W \sqrt{\frac{2\pi h|\eta|\Omega_{rev}}{eVp_s R\cos\phi_s}}$$

and **the absolute value of slip factor** can be used as additional factor for the matching between two accelerators or/and control of beam sizes during acceleration.

The requirements to the sign of slip factor:

IY. Many investigations devoted to the beam stability declare that the beam is more stable below the transition energy

$$\gamma < \gamma_{tr} \quad \rightarrow \quad \eta < 0$$

Besides, in the synchrotron light sources the natural chromaticity accords to the transverse stability criteria for the negative slip factor.

So, if the lattice has the imaginary gamma transition

$$\eta = 1/(iG_{tr})^2 - 1/\gamma^2 < 0$$

all requirements can be fulfilled!!!

History of lattice with imaginary gamma-transition

- **in 1955 Vladimirsky and Tarasov** suggested method to get the imaginary γ_{tr} and did it by increasing number of "compensating magnets" with a reversed field but the same gradients, as would be called for in a design with no compensating magnets and where is slightly more than the tune.
- **In 1958 Courant and Snyder** quantitatively described this idea of the negative momentum compaction factor.

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- Later many authors tried to realize this idea of imaginary transition energy in different lattices:

-In 1972 Lee Teng suggested the modular method;

-In 1974 Bruck developed the regular focusing structure with the "missing" magnet cell in Saturne II;

-In 1983 Franczak, Blasche, Reich excited superperiodically the quadrupoles for the SIS-18;

-In 1985 Gupta, Botman, Craddock at an initial design stage of the TRIUMF KF used missing magnet;

History of lattice with imaginary gamma-transition

-In 1989 **Senichev, Golubeva, Iliev** suggested the “resonant” lattice for Moscow Kaon Factory;

-In 1992 **Ng, Trbojevic, Lee** applied the modular method of Lee Teng for MB (FNAL);

-In 1992 **U.Wienands, N.Golubeva, A.Iliev, Yu.Senichev, R.Servranckx** addopted the “resonant” lattice for Kaon Factory (TRIUMF);

-In 1993 **E. Courant , A. Garen and U. Wienands** took the “resonant” lattice for LEB (SSC);

-In 1995 **Y. Senichev** wrote the “resonant” lattice theory and applied it for Main Ring (JPARC)

-In 2000 **H. Schönauer, Yu. Senichev** et al., The “resonant” lattice for Proton driver for a Neutrino Factory (CERN)

-In 2007 **Y.Senichev** et al., The “resonant” lattice for Super-Conducting option of HESR (FAIR)

-In 2008 The “resonant” lattice is one of the candidate for PS2 (CERN)

Regular and Irregular lattices

Momentum Compaction factor (MCF):

$$\alpha = \frac{1}{2\pi}\int_0^C \frac{D(\vartheta)}{\rho(\vartheta)}d\vartheta$$

where the dispersion D(θ) is:

$$\mathrm{D}'' + \mathrm{K}(\vartheta)\mathrm{D} = \frac{1}{\rho(\vartheta)}$$

If in the optics with eigen frequency **ν** the curvature **σ(θ)** is modulated with frequency **ω**

$$\sigma(\vartheta) = 1/\rho(\vartheta) \sim Be^{i\omega\vartheta} + 1/\overline{R}$$

the dispersion solution and Momentum Compaction Factor are:

$$D(\vartheta) \sim Ae^{i\nu\vartheta} + \frac{B}{\nu^2 - \omega^2}e^{i\omega\vartheta} + \overline{D} \qquad \alpha = \frac{\overline{D}}{\overline{R}} + \frac{\overline{\widetilde{D}(\vartheta)\cdot\widetilde{r}(\vartheta)}}{\overline{R}}$$

Regular lattice

In conventional regular FODO lattice **$\omega >> \nu$**.

Therefore the dispersion oscillates with eigen frequency (tune) **ν**:

$$D(\vartheta) \approx A e^{i\nu\vartheta} + \overline{D}$$

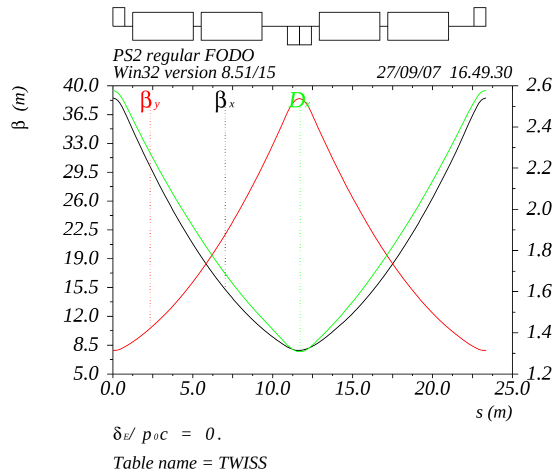
Then Momentum Compaction Factor (MCF) is determined by average values ratio:

$$\alpha = \frac{\langle D(\vartheta) \rangle}{\langle \rho(\vartheta) \rangle} = \frac{\overline{D}}{\overline{R}} \approx \frac{1}{\nu^2}$$

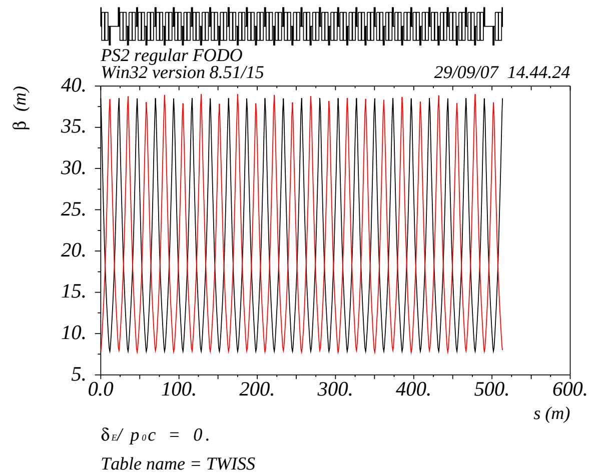
and the maximum energy of accelerator without the transition energy crossing is determined by $\gamma_{max} \approx \nu$ or for the $\pi/2$ phase advance FODO lattice $\gamma_{max} \approx Ncell/4$

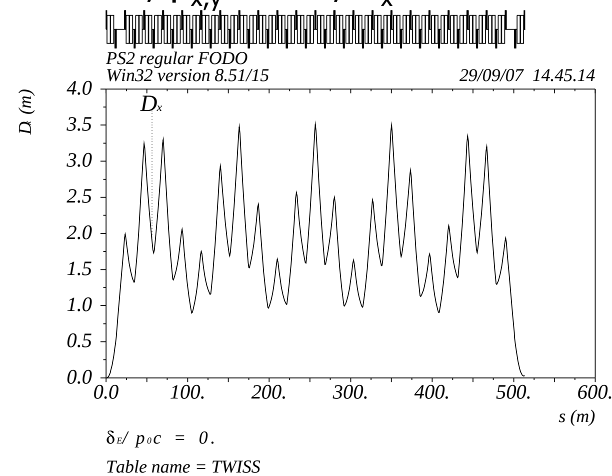
Regular lattice for PS2 (3-50 GeV) based on FODO cells with real transition energy $\gamma_{tr} \sim 10$ (parameters taken from PS2 report by Wolfgang Bartman)

FODO cell with magnet length L=3.79m and drift 1.6m



Arc based on FODO lattice with tune per cell ~84^0
The total length (22 cells) 513.5 m; $\beta_{x,y}$~39 m, D_x~3.5m





To make higher γ_{tr} than 50 the total number of FODO cells has to be increased up to 110 per arc

Conclusion: The only possible solution is the imaginary gamma transition with the wide control of its absolute value

Irregular lattice with curvature modulation (missing magnet lattice)

In case of eigen frequency **ν** is enough close to the curvature oscillation with the superperiodicity frequency S= **ν** +δ, the dispersion oscillates with the forced frequency **ω** =S:

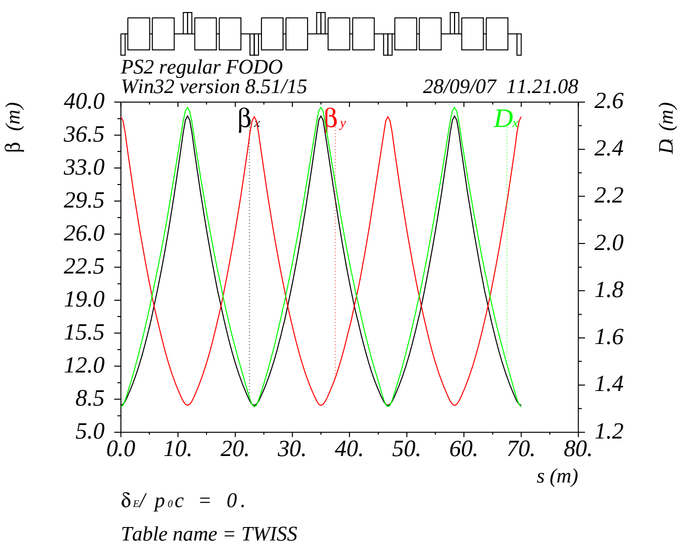
$$D(\vartheta) \sim \frac{B}{\nu^2 - S^2} e^{i\vartheta S} + \overline{D}$$

In irregular structure MCF depends on the curvature modulation *B* and detuning δ= S- ν<< ν :

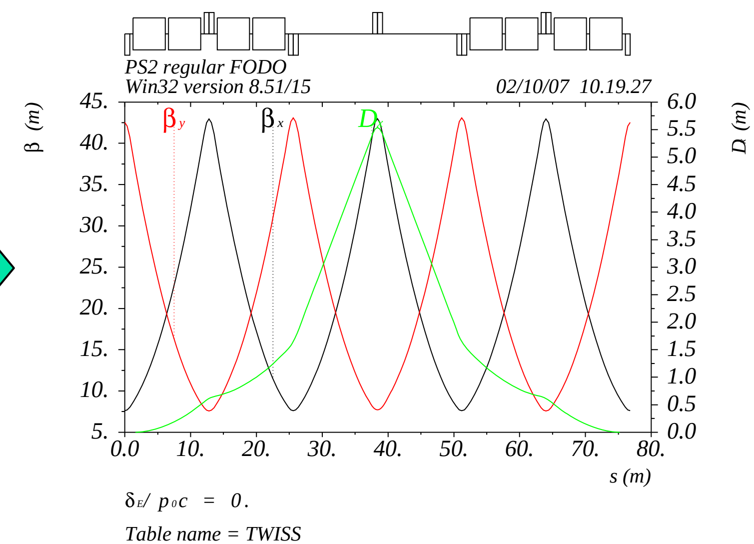
$$\alpha \approx \frac{1}{\nu^2} - \frac{B^2}{2\delta\nu}$$

Irregular PS2 lattice with curvature modulation ("missing" magnet lattice)

3 regular FODO cells with total length
3 x 23.21 m=69.63
L_{mag}=3.7 m

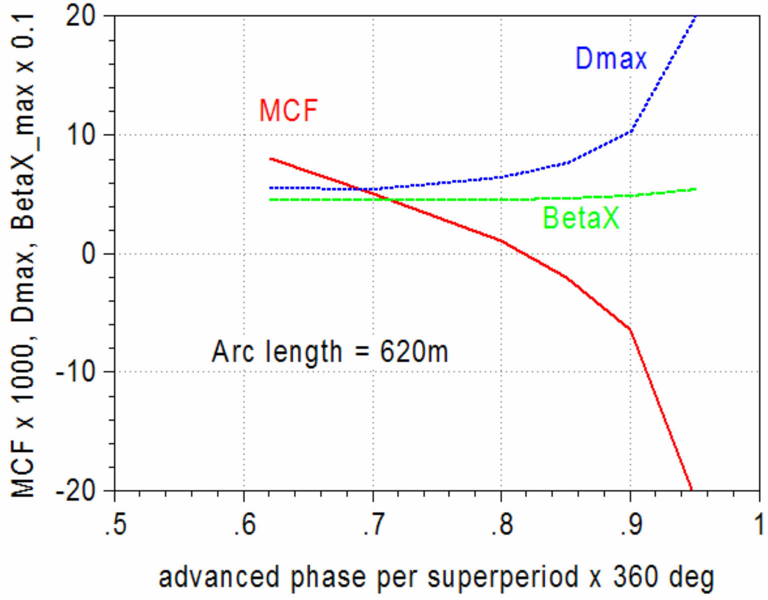


3 irregular FODO missing magnet cells
with total length 76.8 m
L_{mag}=4.9 m



Zero momentum compaction factor in the "missing" magnet lattice

In arc length~620 m MCF<0 at ν>0.82
In arc length~600 m MCF<0 at ν>0.875
In arc length~580 m MCF<0 is not reached



Conclusion

"Missing magnet" lattices has

advantages:

- practically does not perturbs β-functions;

disadvantages:

- requires the large phase advance value,
- significantly increases the arc length.

Results of "Resonant" lattice theory:

From the article: Yu. Senichev, A "resonant" lattice for a synchrotron with a low or negative compaction factor, KEK Preprint 9740, 1997 and JETP, v. 132, n. 5, p.1127

The solution of equation

$$\frac{d^2 D}{ds^2} + \left[K(s) + \varepsilon\, k(s)\right] D = \frac{1}{\rho(s)}$$

with modulation of gradient and curvature:

$$\varepsilon\, k(\phi) = \sum_{k=0}^{\infty} g_k \cos k\phi ; \quad \frac{1}{\rho(\phi)} = \frac{1}{\overline{R}} \left(1 + \sum_{n=1}^{\infty} r_n \cos n\phi \right)$$

gives the expression for MCF:

$$\alpha_s = \frac{1}{\nu^2} \left\{ 1 + \frac{1}{4 \cdot (1 - kS/\nu)} \cdot \left[\left(\frac{\overline{R}}{\nu} \right)^2 \frac{g_k}{[1 - (1 - kS/\nu)^2]} - r_k \right]^2 \right\}$$

1. Negative momentum compaction factor with minimum circumference and control of gamma transition in a wide region

The lattice has the remarkable feature:

The gradient and the curvature modulation amplify each by other if they have opposite signs,

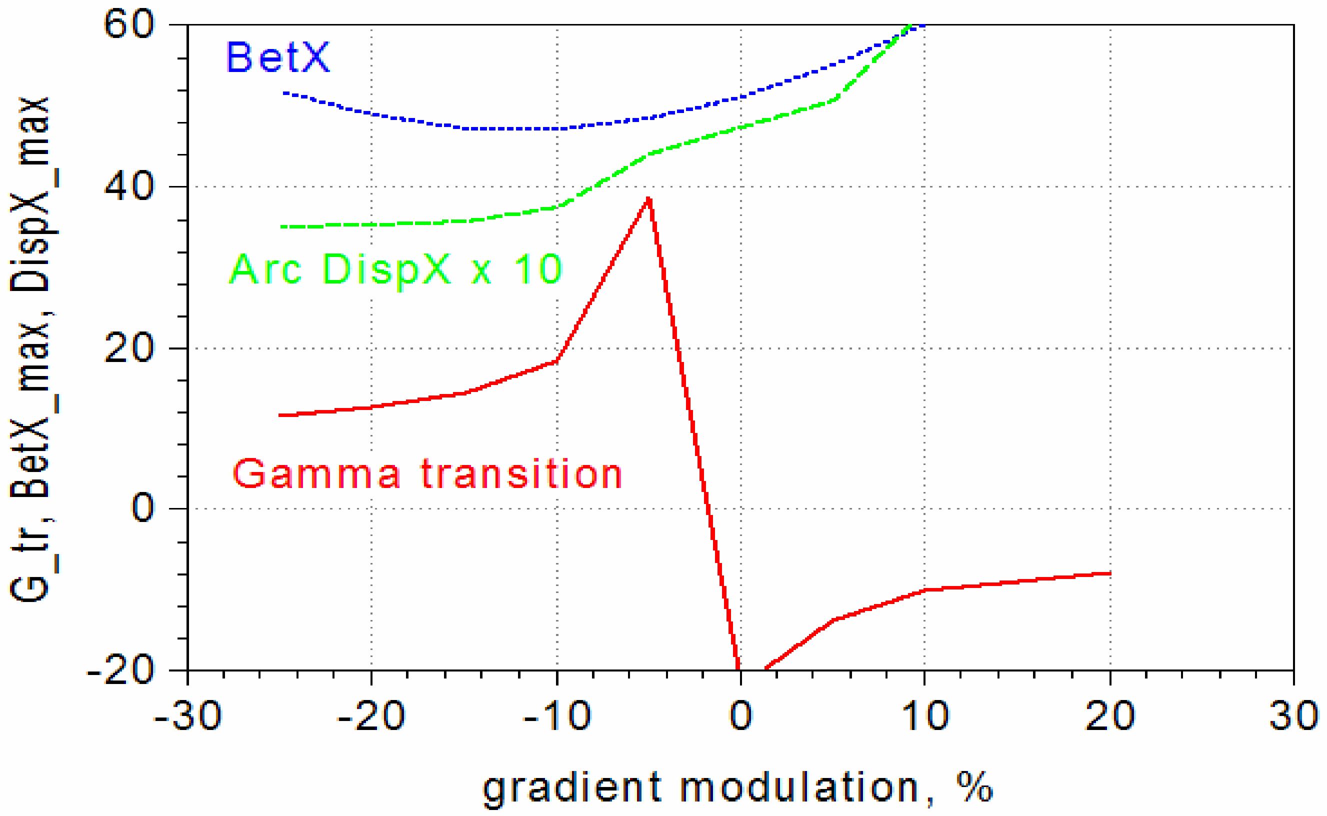
$$g_k \cdot r_k < 0$$

The ratio between them is desirable to have:

$$|r_k| \le \left(\frac{\overline{R}}{\nu}\right)^2 \left|\frac{g_k}{1-(1-kS)^2}\right|$$ and $$\frac{1}{4(kS/\nu-1)} \cdot \left(\frac{g_k}{[1-(1-kS/\nu)^2]} - r_k\right)^2 \approx 2$$

On the contrary they can compensate each other when they have the same sign.

Then gamma transition varies in a wide region from $\gamma_{tr} \sim \nu_x$ to $\gamma_{tr} \sim i\nu_x$ with quadrupole strength variation only!!!



2. Dispersion-free straight section without special suppressor;
3. Low sensitivity to multipole errors and sufficiently large dynamic aperture

First condition:

To provide a **dispersion-free straight section**, the arc consisting of S_{arc} superperiods must have a 2π integer phase advance.

Second condition:

In order to drive the momentum compaction factor, the horizontal betatron tune ν_{arc} must be less than the resonant harmonic of perturbation kS_{arc} , and the difference between them has to be of a minimum integer value. We take $\nu_{arc} - kS_{arc} = -1$

.

Third condition:

The arc superperiodicity S_{arc} has to be even and ν_{arc} is odd.

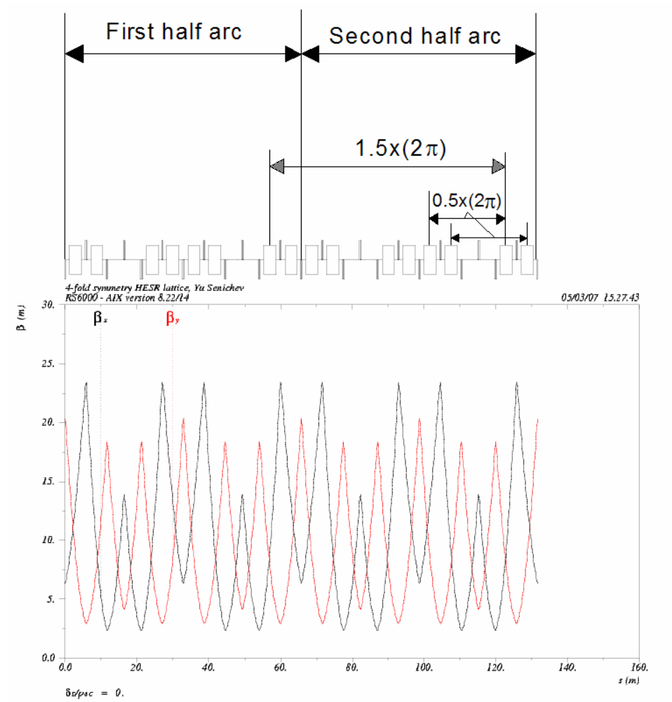
Compensation of sextupole non-linearity

- In that case the phase advance between any two cells located in the different half arcs and separated by $\frac{S_{arc}}{2}$ number of

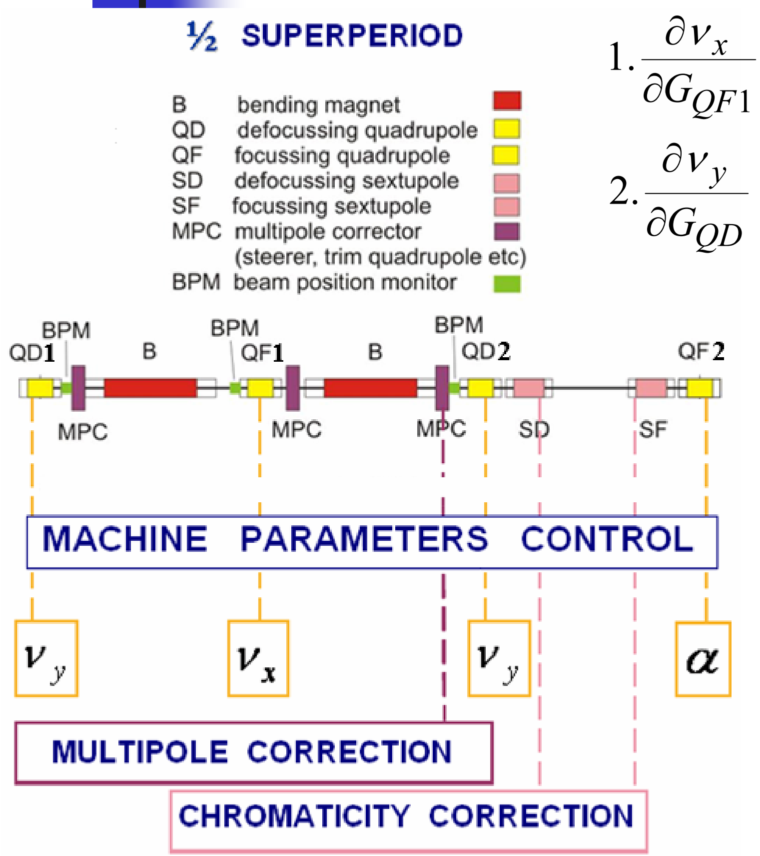
  superperiods is then equal to

  $$\frac{\nu_{arc}}{S_{arc}} \cdot \frac{S_{arc}}{2} = \frac{\nu_{arc}}{2} = \pi + 2\pi n$$.
- the total multipole of third order is canceled:

$$M_3^{total} = \sum_{n=0}^{N} S_{x,xy} \beta_x^{l/2} \beta_y^{m/2} \exp in(l\mu_x + m\mu_y) = 0$$



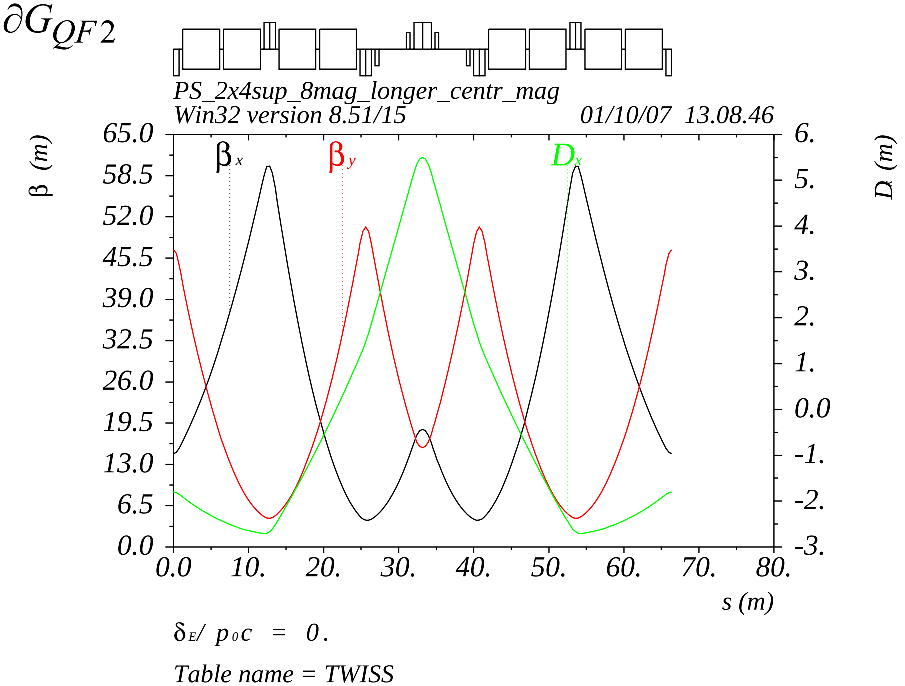
4. Minimum families of focusing and defocusing quadrupoles and separated adjustment of gamma transition, horizontal and vertical tunes



$$1.\ \frac{\partial \nu_x}{\partial G_{QF1}} > \frac{\partial \nu_x}{\partial G_{QF2}} >> \frac{\partial \nu_x}{\partial G_{QD}}$$

$$2.\ \frac{\partial \nu_y}{\partial G_{QD}} >> \frac{\partial \nu_y}{\partial G_{QF1}} \approx \frac{\partial \nu_y}{\partial G_{QF2}}$$

$$3.\ \frac{\partial \alpha}{\partial G_{QF2}} >> \frac{\partial \alpha}{\partial G_{QF1}} \approx \frac{\partial \alpha}{\partial G_{QD1}}$$



5. Convenient sextupole chromaticity correction scheme

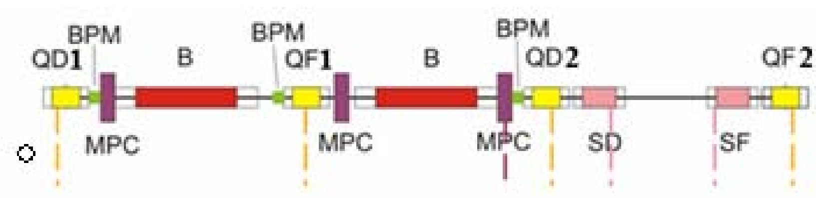
Total chromaticity

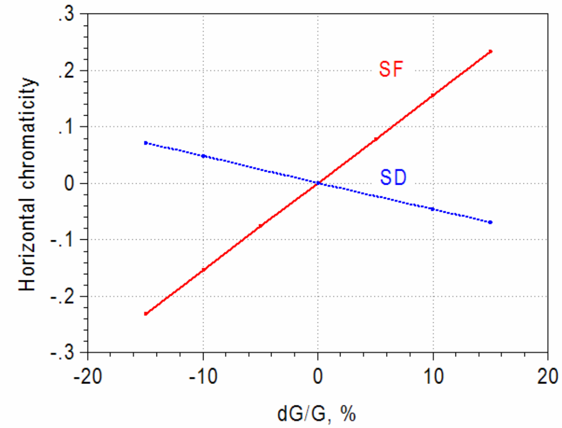
$$\frac{\partial \nu_{x,y}}{\partial \delta} = -\frac{1}{4\pi}\int_0^C \beta_{x,y}(s)K_{x,y}(s)ds$$

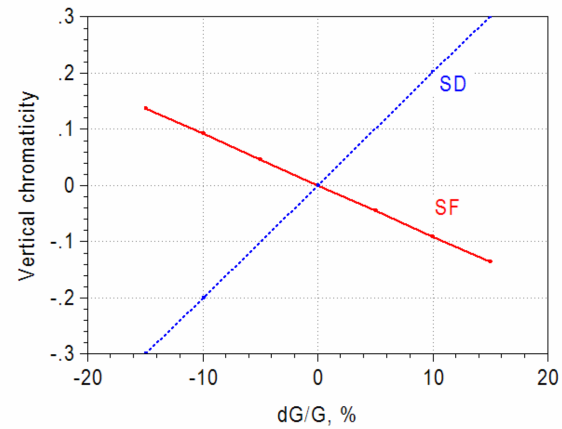
- Sextupole compensation

$$\frac{\partial \nu_{x,y}}{\partial \delta} = \pm\frac{1}{4\pi}\int_0^C \beta_{x,y}(s)\cdot D(s)\cdot S(s)ds$$

Half-superperiod







6. Independent optics parameters of arcs and straight sections

- Tune arc does not depends on the transition energy and is kept constant;
- Special insertion on the straight section allows to match the $\beta_{x,y}$-functions between arcs and straight sections;
- Dispersion-function on the straight sections always equal zero;
- All high order non-linearities are compensated inside each arc.

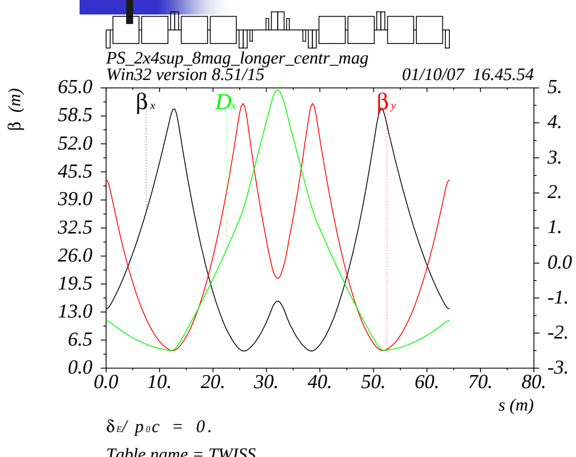
The “golden” ratio between S_{arc} and ν_{arc}

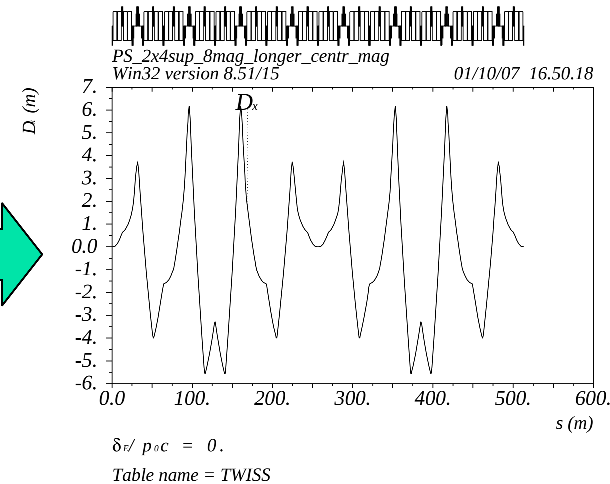
To fulfill all mentioned conditions we have to have the strictly fixed sets of $\mathbf{S_{arc}}$ and $\boldsymbol{\nu_{arc}}$:

4:3;   6:5; 8:6; 8:7,…. and so on.

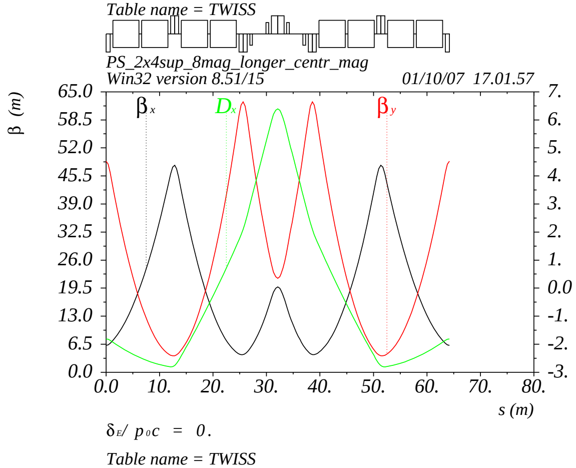
4:3 + 4:3

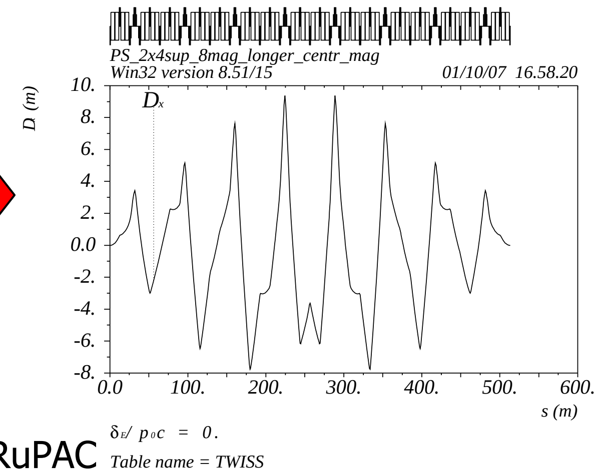
8 superperiodical lattices (for PS2) with ν_{arc}=6 and ν_{arc}=7





ν_{arc}=6:





ν_{arc}=7:

The second order non-linearity

- After some canonical transformation we can get the second order approach of Hamiltonian in the next view:

$$H(J_x, \vartheta_x, \theta_x, I_y, \vartheta_y, \theta_y) =$$
$$\nu_x J_x + \nu_y J_y + \sum g(M, N, n_1, n_2, p) J_x^{M/2} J_y^{N/2} \exp i\left(n_1 \vartheta_x + n_2 \vartheta_y - p\theta\right)$$

- Now let us suppose that we are some where around of the third order resonance:

$$3\overline{\nu}_x = p_0,$$
$$\overline{\nu}_x = \nu_x + \Delta$$

- the Hamiltonian takes a view

$$H_1(J, \psi, \theta) = \nu_x J_x + \nu_y J_y + \frac{1}{2} J_x^{3/2} \left\{ h_{3030 p_0} \exp i(3\psi_x - p_0\theta) + c.c. \right\} +$$
$$\zeta_x J_x^2 + \zeta_{xy} J_x J_y + \zeta_y J_x^2$$

The higher order resonance excitation and non-linear tune shifts

- the coefficients $\zeta_x, \zeta_y, \zeta_{xy}$ are the non-linear tune shifts:

$$\zeta_x = \zeta_x^{sex} + \zeta_x^{oct}$$
$$\zeta_{xy} = \zeta_{xy}^{sex} + \zeta_{xy}^{oct}$$
$$\zeta_y = \zeta_y^{sex} + \zeta_y^{oct}$$

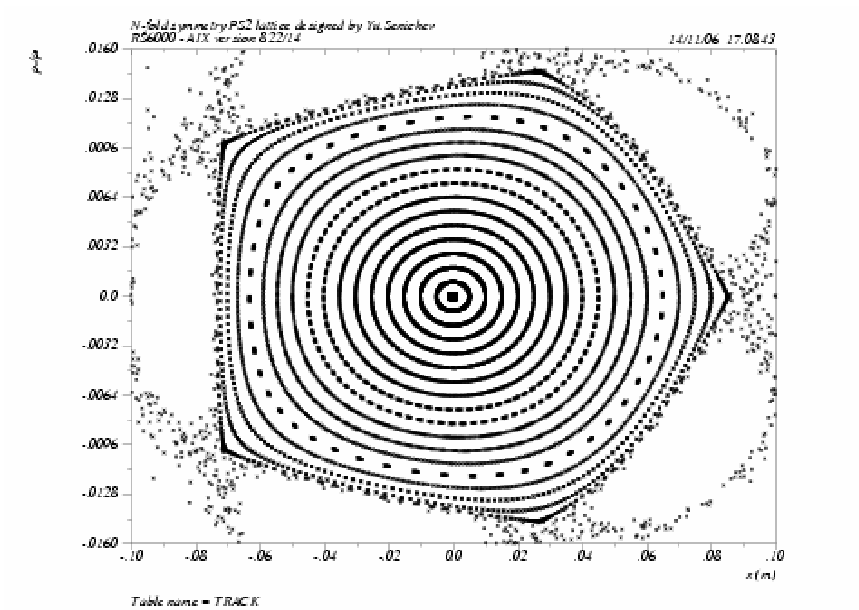
- as example

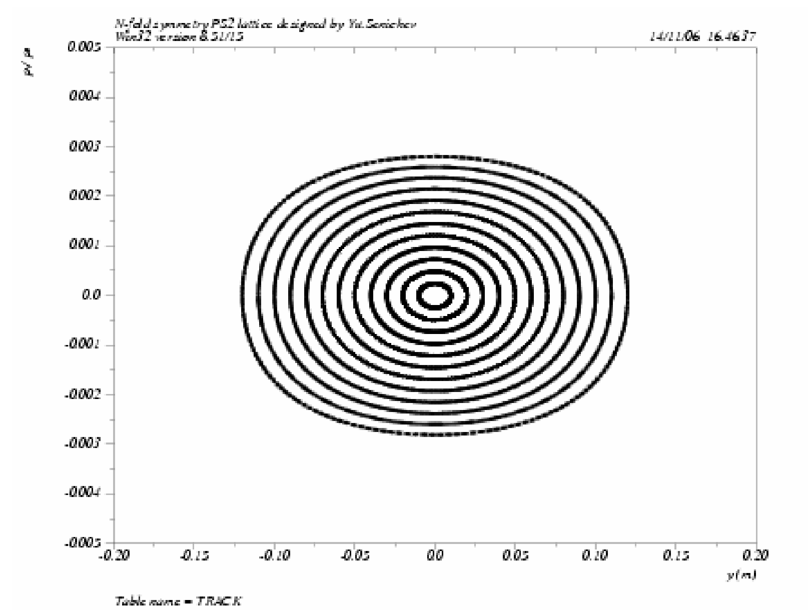
$$\zeta_x^{sex} = -\frac{3}{4}\left[\sum_{p=-\infty}^{\infty}\frac{\left|h_{3010p}\right|^2}{\nu_x - p} + \sum_{\substack{p=-\infty \\ p\neq p_0}}^{\infty}\frac{3\left|h_{3030p}\right|^2}{3\nu_x - p}\right]$$

$$\zeta_x^{oct} = \frac{1}{32\pi\Delta^2}\int_0^{2\pi}\beta_x^2 O_x R d\theta$$

Dynamic aperture after chromaticity compensation (for PS2)

- We calculated the dynamic aperture by the numerical tracking for one of options using MAD. It is ~Hor.=600 mm mrad and Ver.=400 mm mrad





Pro and Con for two types of lattices: "resonant" and regular FODO (for PS2)

	Resonant lattice with ρ and gradient modulation		Regular FODO lattice with suppressors	
	Advantages	disadvantages	advantages	disadvantages
Crossing $W_{transit}$	No			Yes, at $\gamma \sim 10$
Variability and controll of $W_{transit}$	Yes			No
Necessity of dispers. suppressor	No			Yes
Decouping between arc and str. section	Yes			No
Free space on arcs	~16 x 3 m			2 x 8 m
Sextupole comp. on arc	Yes			No

Pro and Con for two types of lattices: "resonant" and regular FODO (PS2)

	Resonant lattice with ρ and gradient modulation		Regular FODO lattice with suppressors	
	Advantages	disadvantages	advantages	disadvantages
Sensitivity to high multipoles	Low			High
Sextupoles on str. section	Yes			No
Quadr. families number		3	2	
Max dispersion		~6÷10 m, depends on var.	~3.5 m	
Max $\beta_{x,y}$ function		48÷70/40÷70 depends on var.	40/40	
3√βxεrms+(DxΔp/p)**2 at εrms=0.68; Δp/prms=1x10^{-3}	~40÷50 mm, depends on var.		~45 mm	

Thus, the “Resonant” structure has the features:

1. Ability to achieve the negative momentum compaction factor with minimum circumference and control of gamma transition in a wide region;

2. Dispersion-free straight section without special suppressor;

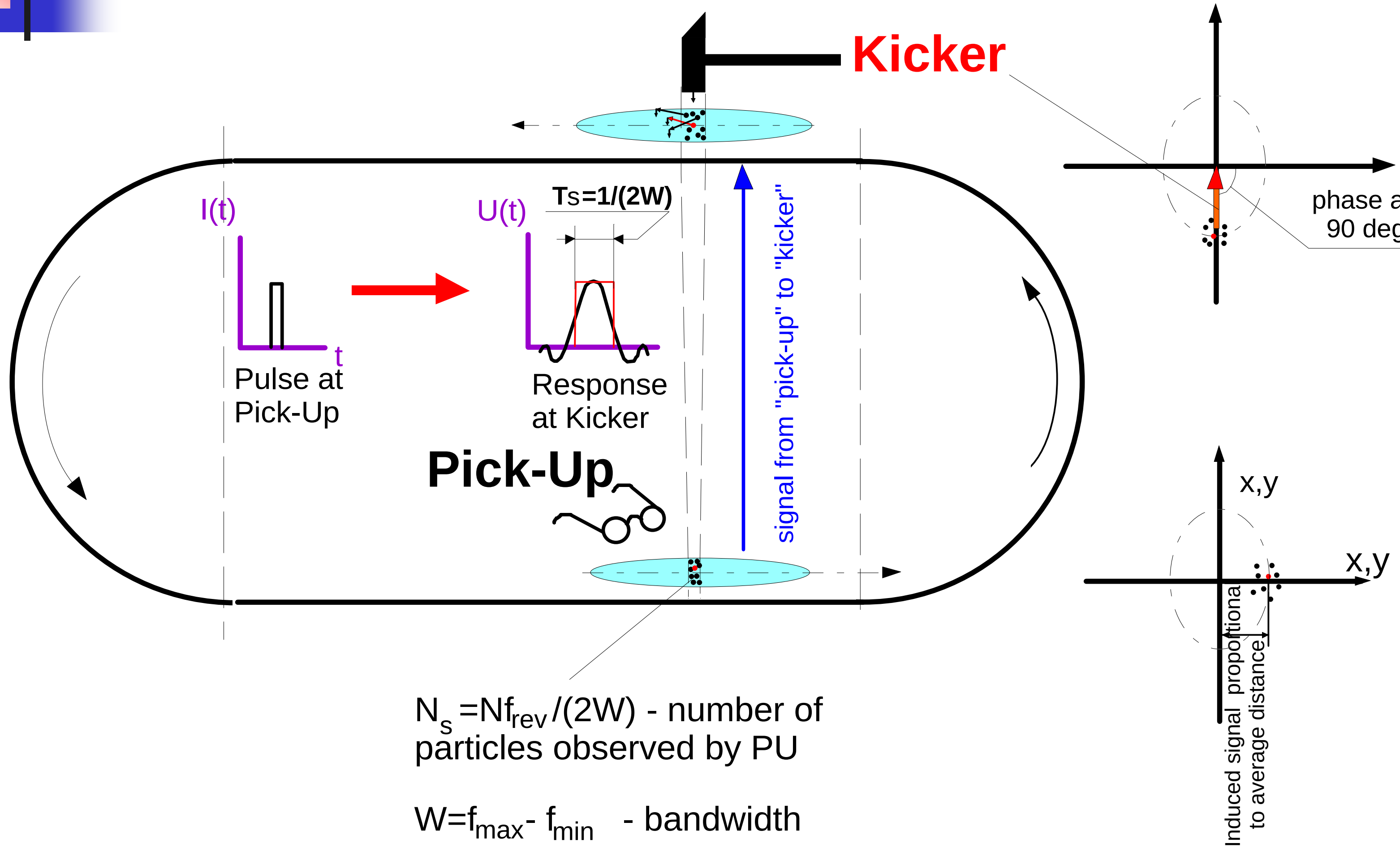
3. Low sensitivity to multipole errors and sufficiently large dynamic aperture.

4. Minimum families of focusing and defocusing quadrupoles and separated adjustment of gamma transition, horizontal and vertical tunes;

5. Convenient sextupole chromaticity correction scheme;

6. Independent optics parameters of arcs and straight sections

Stochastic cooling principe and requirements to the optics



$N_s = N f_{rev}/(2W)$ - number of particles observed by PU

$W = f_{max} - f_{min}$ - bandwidth

Real and Imaginary arcs for Stochastic Cooling:

- The momentum compaction factor in imaginary and real arcs takes the meaning:

$$\alpha_{kp} = -\frac{1}{4\nu_{arc}^2} \qquad \alpha_{pk} = \frac{1}{4\nu_{arc}^2}$$

- and slip factors:

$$\eta_{pk} = \frac{1}{\gamma^2} - \frac{1}{4\nu_{arc}^2}$$

$$\eta_{kp} = \frac{1}{\gamma^2} + \frac{1}{4\nu_{arc}^2}$$

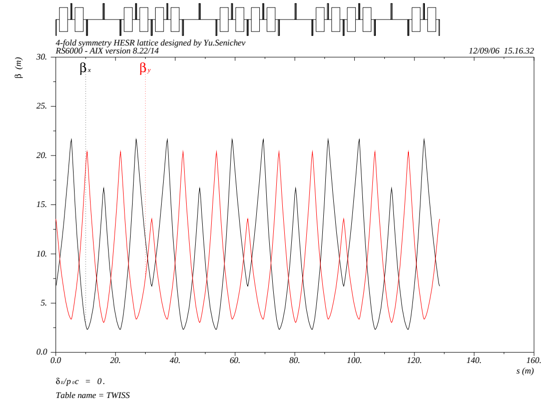
In case $\gamma \approx 2\nu_{arc}$:

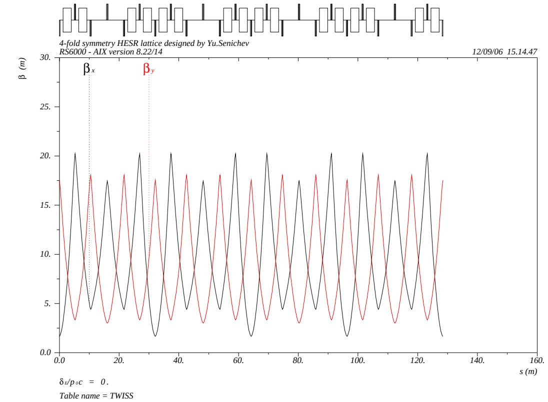
the real arc is isochronous $\eta_{pk} \approx 0$

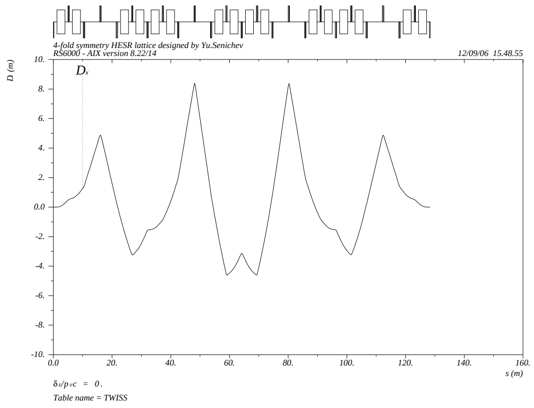
the imaginary arc has a slip factor $\eta_{kp} \approx 1/2\nu_{arc}^2$

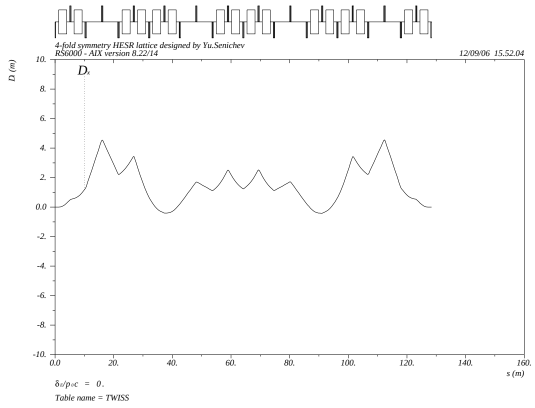
Twiss parameters of the real and imaginary arcs of SC option for HESR (FAIR)

- The β-function and dispersion on the imaginary, the real 4-fold symmetry arcs









What can we do for Synchrotron Light Source Optics?

- Almost all **Synchrotron Light Sources** work higher of the transition energy, therefore chromaticity must be $\xi>0$
- Since the horizontal emittance depends upon the horizontal dispersion function,

  as $\varepsilon_x \propto \langle H \rangle_{dipole}$, where $H = \gamma_x \eta_x^2 + 2\alpha_x \eta_x \eta_x' + \beta_x \eta_x'^2$ to get $\varepsilon_{x,\,min}$

  the dispersion →minimum value

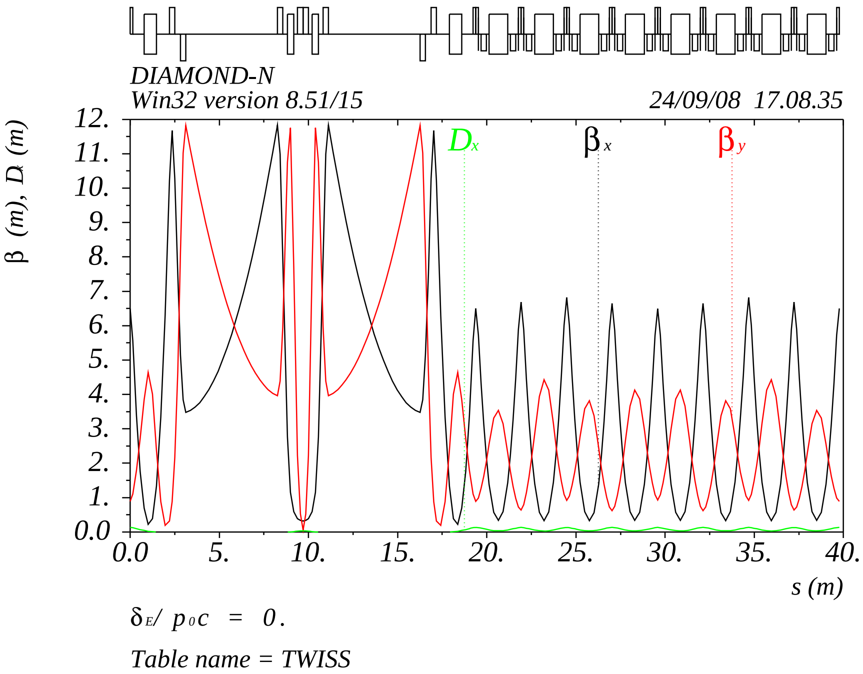
- Stronger sextupoles are required → the dramatic decreasing of DA
- There are two methods:

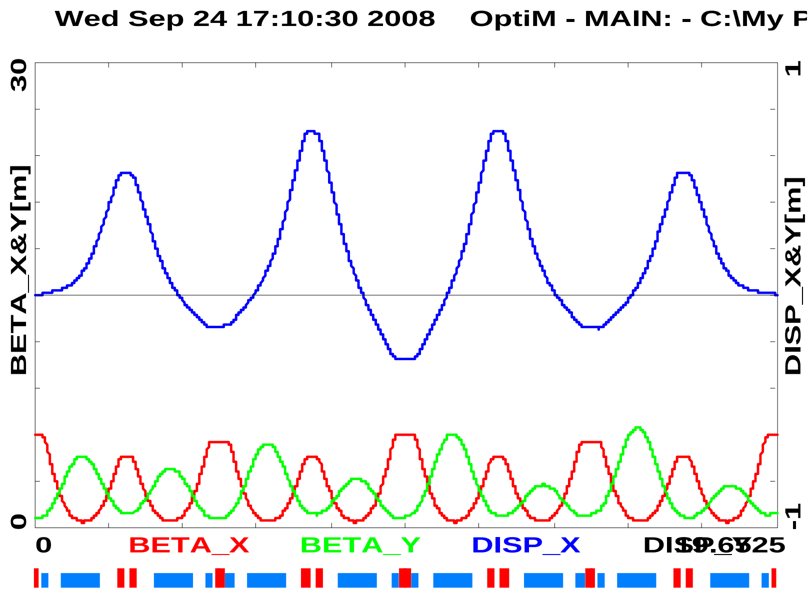
\- Sextupoles have to be compensated

\- Lattice w/o sextupole with imaginary γ_{tr}

SLS Lattices:

- with sextupoles N-bend achromat with $\alpha>0$ w/o sextupoles with $\alpha<0$





Sextupole compensation in SLS optic

- under strong influence of (k_x+k_y)-th Integer resonance

$$H(I_x, I_y, \varphi_x, \varphi_y) = \frac{\left(k_x^2 + k_y^2\right)^{1/2}}{k_x}\Delta_x I_x + \frac{\left(k_x^2 + k_y^2\right)^{1/2}}{k_y}\Delta_y I_y +$$

$$2\left\langle h_{k_x,k_y,p}\right\rangle I_x^{k_x/2} I_y^{k_y/2}\cos\left(k_x\varphi_x + k_y\varphi_y\right) +$$

$$\zeta_x I_x^2 + \zeta_y I_y^2 + \zeta_{xy} I_x I_y$$

- For 3-d integer resonance the influence of the non-linearity in specified by the discriminant in the expression:

$$I_x^{1/2} = -\frac{3h_{30p}\cos 3\vartheta_x}{8\zeta_x} \pm \frac{1}{4\zeta_x}\sqrt{\frac{9}{4}h_{30p} - 8\zeta_x\left(\Delta + \zeta_{xy} I_y\right)}$$

Nekhoroshev's criterium:

the non-linearity in both planes have to have the same sign and $4\zeta_x\zeta_y \geq \zeta_{xy}^2$

- The lattices with $\zeta_x >> h_{30p}$ have to be classified as a special lattice, since it is a case, when the value of h_{30p} is effectively suppressed, but the non-linearity remain to be under control and strong.

- If the sign of the detuning Δ coincides with the sign of the tune shift ζ_x, the discriminant is negative and the system has only one centre at $I_x = 0$
- The quasi-isochronism condition by Nekhoroshev is fulfilled, when

$$k_x\left(2\zeta_x I_x^r + \zeta_{xy} I_y^r\right) + k_y\left(2\zeta_y I_y^r + \zeta_{xy} I_x^r\right) = 0$$

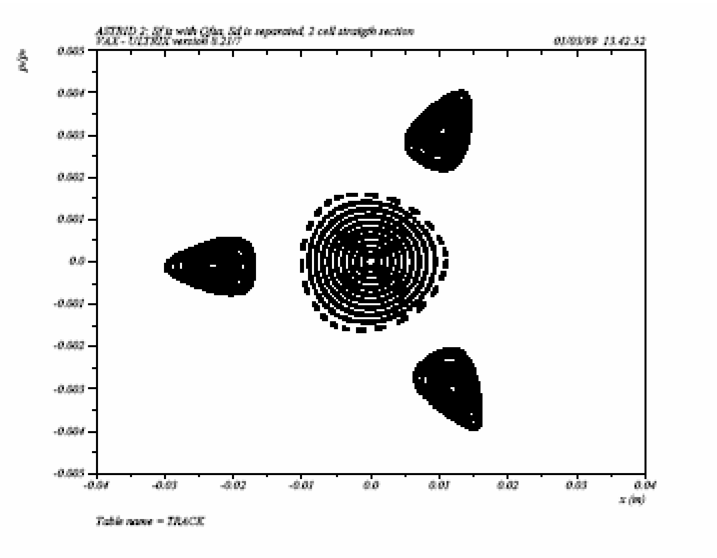
$$\zeta_x k_x^2 + \zeta_{xy} k_x k_y + \zeta_y k_y^2 = 0$$

→ Convex or concave resonant surface with maximum stable region

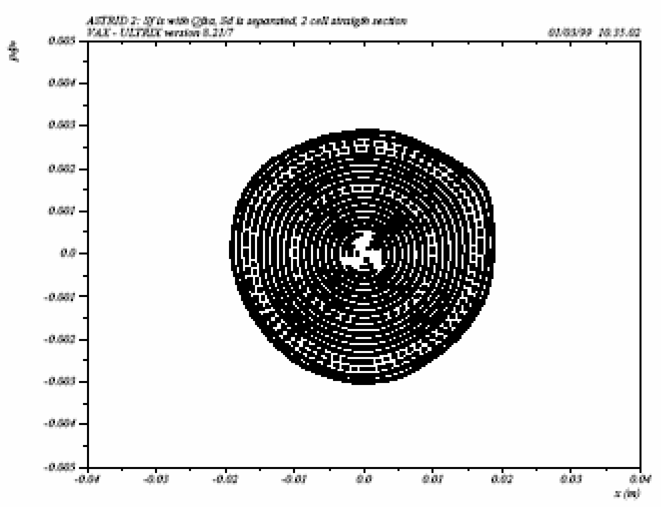
Dynamic apperture tracking

- negative and positive detune

ζ>0; Δ<0



ζ>0; Δ>0



Conclusion
"Resonant" lattice was developed with features:

- ability to achieve the negative momentum compaction factor using the resonantly correlated curvature and gradient modulations;
- gamma transition variation in a wide region from $\gamma_{tr}=\nu_x$ to $\gamma_{tr}=i\nu_x$ with quadrupole strength variation only;
- integer odd 2π phase advance per arc with even number of superperiod and dispersion-free straight section;
- independent optics parameters of arcs and straight sections;
- two families of focusing and one of defocusing quadrupoles;
- separated adjustment of gamma transition, horizontal and vertical tunes;
- convenient chromaticity correction method using sextupoles;
- first-order self-compensating scheme of multipoles and as consequence low sensitivity to multipole errors and a large dynamic aperture

Conclusion
"Resonant" lattice can be used:

- In the heavy ion and proton synchrotron lattice without the transition energy crossing
- In the lattice with high efficiency of stochastic cooling
- In the Synchrotron Light Source lattices w/o sextupoles or with selfcompensated sextupoles