Influence of beam **space charge on dynamical aperture** of TWAC storage ring

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Abstract

- **High intensity ion beams** are planned to store in Terra Watt Accumulator (**TWAC**) at ITEP (Moscow).
- The **dynamic aperture** (DA) => a size of the phase-space domain for safe operation with the beam.
- DA is studied numerically:
  - the space charge forces (the “frozen core” model);
  - perturbations of the guiding magnetic field
  - particles with non-zero momentum deviations.
- An economical DA-algorithm (GSI, Micromap-code)-> for MADX-code via its macro-scripts.
- The results => a significant dependence of DA on intensity (especially for particles $\Delta p/p \neq 0$)

Dynamic aperture

- Circular accelerator –
  bending dipoles (D) + focusing quadrupoles (Q).

- High-order multipoles in D & Q =>
  => the non-linear beam-dynamics
  => losses of particles with large amplitudes of betatron oscillations.
- => The motion is stable only inside of
  a restricted phase space area (a stability domain).

- A size of the stability domain is related to DA.
- An accurate estimate of the DA is important ! [1, 2].

- The DA is defined usually as the ***radius of the largest circle inscribed inside*** the domain of stable initial conditions (in units of Courant-Snyder invariant) .

[1] W.Scandale, EPAC'92 , pp. 264, (1992).
[2] E.Todesco & M.Giovannozzi, Phys.Rev. E-53, No.4, pp.4067 (1996).

The space-charge distribution

- Linear approximation:
  constant beam density =>
  => the SC-field results in
  linear tune-shifts

- Non-linear approximation:
  realistic beam density (e.g., Gaussian) =>
  non-linear SC-fields affect on
  the beam trajectories and DA
  (especially with non-linear perturbations of
  the guiding magnetic field).

These distributions are available for MAD-codes (MAD-8 & MAD-X) !!!

History & Actuality

- DA of TWAC studied without account of the s. c. effects
  (A. Bolshakov, P. Zenkevich, Preprint ITEP, 2000, No. 26.)

- Estimates of SC influence on TWAC's DA
  for $\varepsilon_x = \varepsilon_y$ and $\Delta p/p=0$
  (A.Bolshakov & P.Zenkevich, Atomn.Energiya, v.91, 4, p.294, 2001)

- The assumptions ($\varepsilon_x = \varepsilon_y$ and $\Delta p/p=0$) are too rough !!!
  (G.Franchetti, "DA for FAIR" in GSI-2005-1. p. 55)

Our goal –
more detailed study the DA dependence on
beam current and momentum deviation
at non-linear SC effects &
the guiding magnetic field non-linearity.

Non-linearity of Magnetic Field

- Magnetic field nonlinearity for TWAC in Ref.[5].
- The magnetic structure - 8 periods;
- One period 11 C-blocks (manufactured at the factory) & one E-block (at ITEP workshop).
- C-blocks have small fluctuations of nonlinearity.
- E-blocks have significant fluctuations.

The magnetic field is expressed by Taylor series

$$\Delta B_y = \rho B_0 \sum \frac{c_n}{n!} x_n$$

Block name	c_1	c_2	c_3	c_4	c_5	c_6
C-blocks	0.32	-0.07	-9.4	-0.08	-0.07	-0.07
E_1	0.31	1.11	81.9	-1.84	-1.11	1.21
E_2	0.31	1.16	88.2	-1.67	-1.14	1.03
E_3	0.32	0.71	-9.50	-1.32	0.11	0.18
E_4	0.32	-0.29	-29.5	0.12	0.17	-1.16
E_5	0.32	0.52	-24.6	-0.52	0.55	-1.21
E_6	0.31	0.70	136.1	-1.80	-2.34	2.417
E_7	0.32	0.92	68.8	-1.99	-0.98	1.34
E_8	0.31	0.89	105.7	-1.88	-1.41	1.47

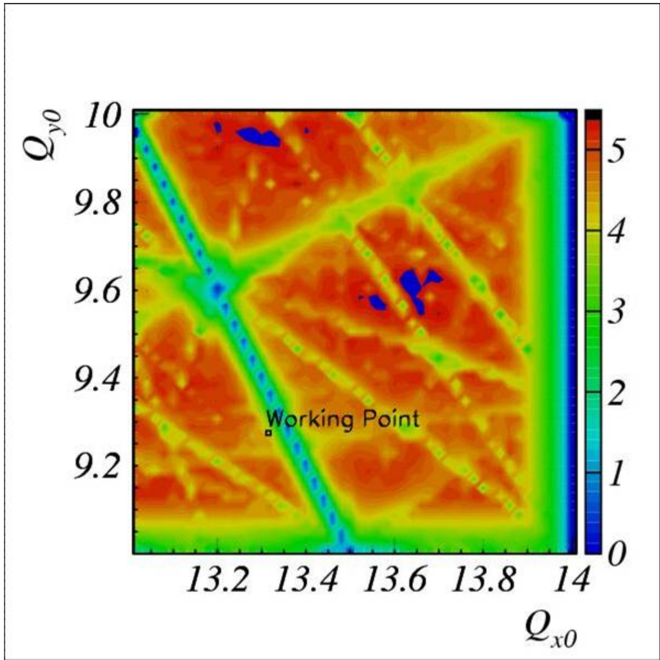
[5] A. Bolshakov, P. Zenkevich, Preprint ITEP, 2000, No. 26.

The values of c_n (n=1…6) for all blocks are given in Table 1 using the units [m^{-2}], [m^{-3}], [m^{-4}], [$m^{-5}\times10^4$], [$m^{-6}\times10^6$], [$m^{-7}\times10^8$], respectively.

DA calculation algorithm

- Beam dynamics for coasting beam in 4D
- To define the DA numerically, one must scan all possible combinations of all 4 coordinates.
  => Direct evaluation of DA is very CPU time consuming.

•=> An economical algorithm for the DA calculations used for SIS-100 with the Micromap-code (G.Franchetti, GSI)
=> for MADX-code via its macro-scripts.



Why MADX ?

ITEP-CERN collaboration in 2004-2006

- MAD => code for the beam dynamcics in accelerators.
- MAD-X is the successor of MAD-8 (frozen in 2002).
- MAD-X has a modular organisation =>
  Development Team:
  Custodian (F.Schmidt) + Module Keepers

- “PTC-TRACK module” is a main feature of MAD-X developed by V.Kapin (ITEP) & F.Schmidt (CERN)

- MAD-X Home Page: “http://mad.home.cern.ch/mad/”

“MADX+S.C.” project

- Similar methods had been already implemented in other beam dynamics codes. (see references, below.)

- “frozen s.c. field” approximation
- thin elements (SC-kicks) :
  linear (MATRIX), nonlinear BEAMBEAM (Gaussian)
- With MADX any number kicks !

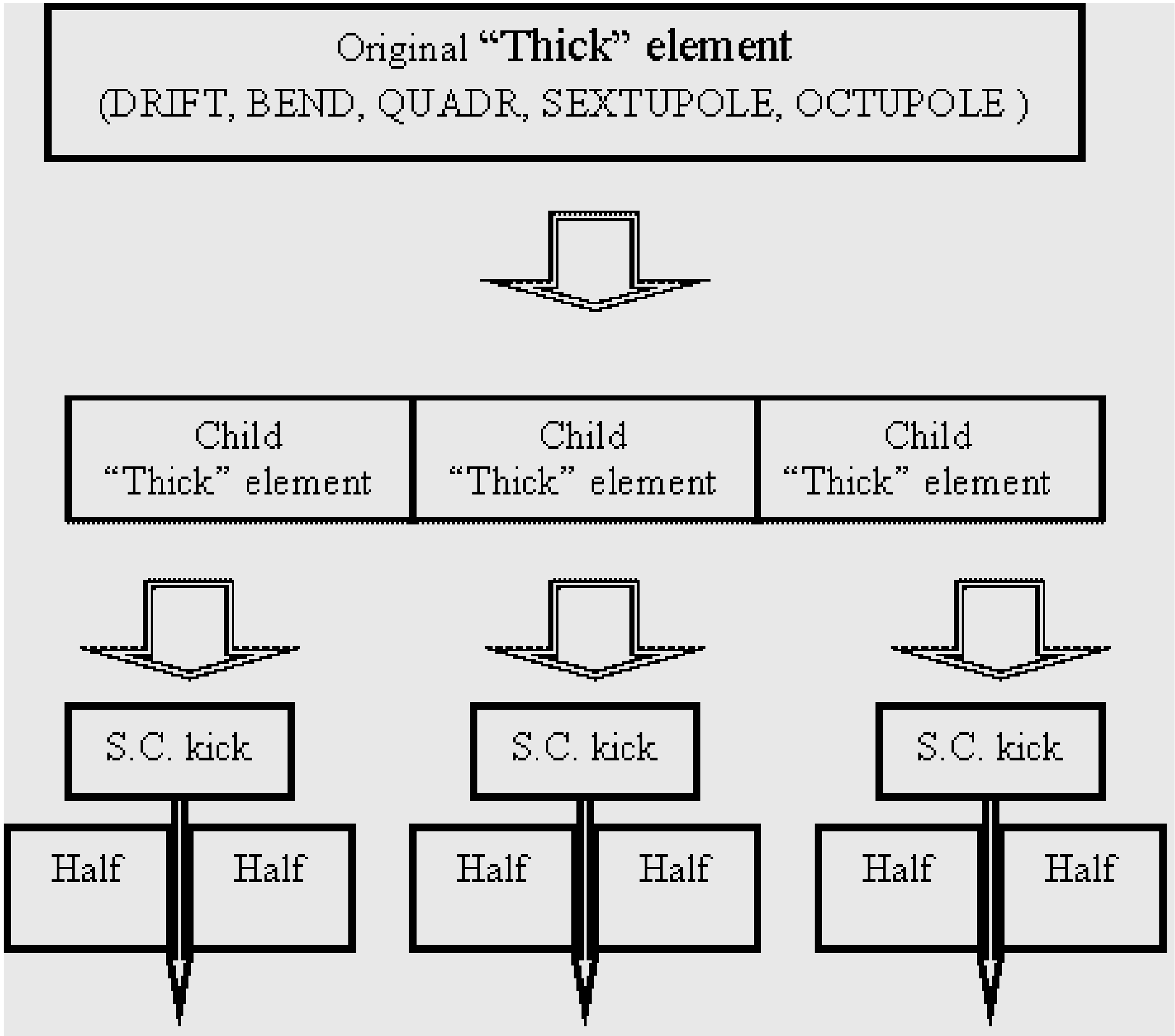
- Our task is a step-by-step adaptation some of them to MADX, which is presently one of the most advanced code for nonlinear beam dynamics simulations without space-charge.

M. Furman, 1987 PAC, pp. 1034-1036. - FRANKENSPOT

Y. Alexahin, 2007 PAC, report code THPAN105. -

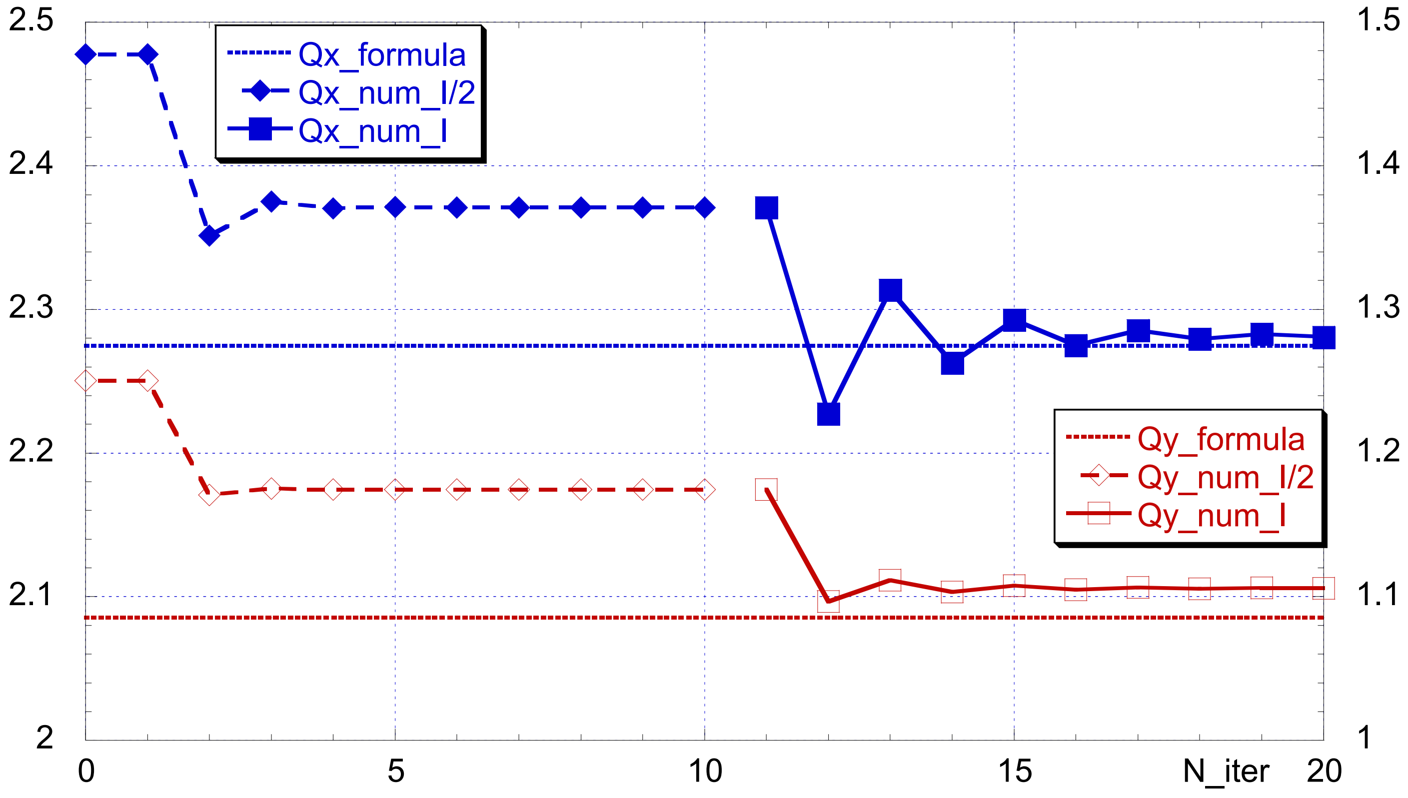
(MAD8 – only 200 BEAMBEAM !)

The 2nd order ray tracing integrator for a number of S-C kicks



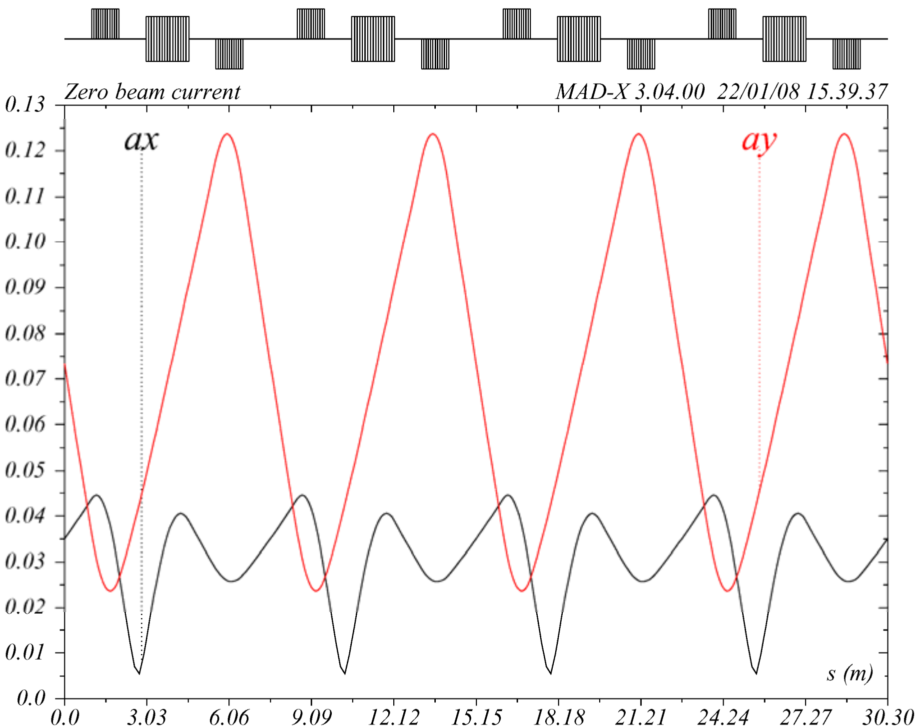
Iterations to find beam sizes at non-zero beam current

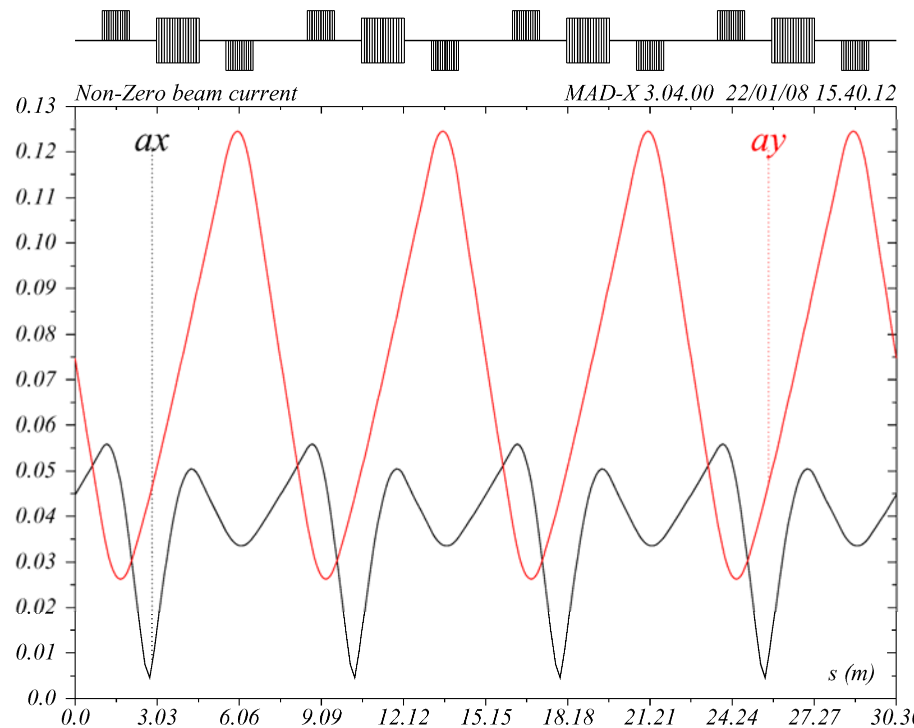
- No Equilibrium solution $Q_{x,y}$=int => (TWISS=ERR)
- Tune value oscillates around a final value;
  => near Q=n iterations with steps for the beam current



Iterations in two steps
Straight lines shows analytical values according to the Laslett's formula for tune shifts.

Resulting beam sizes for a simple 4-bend FODO structure





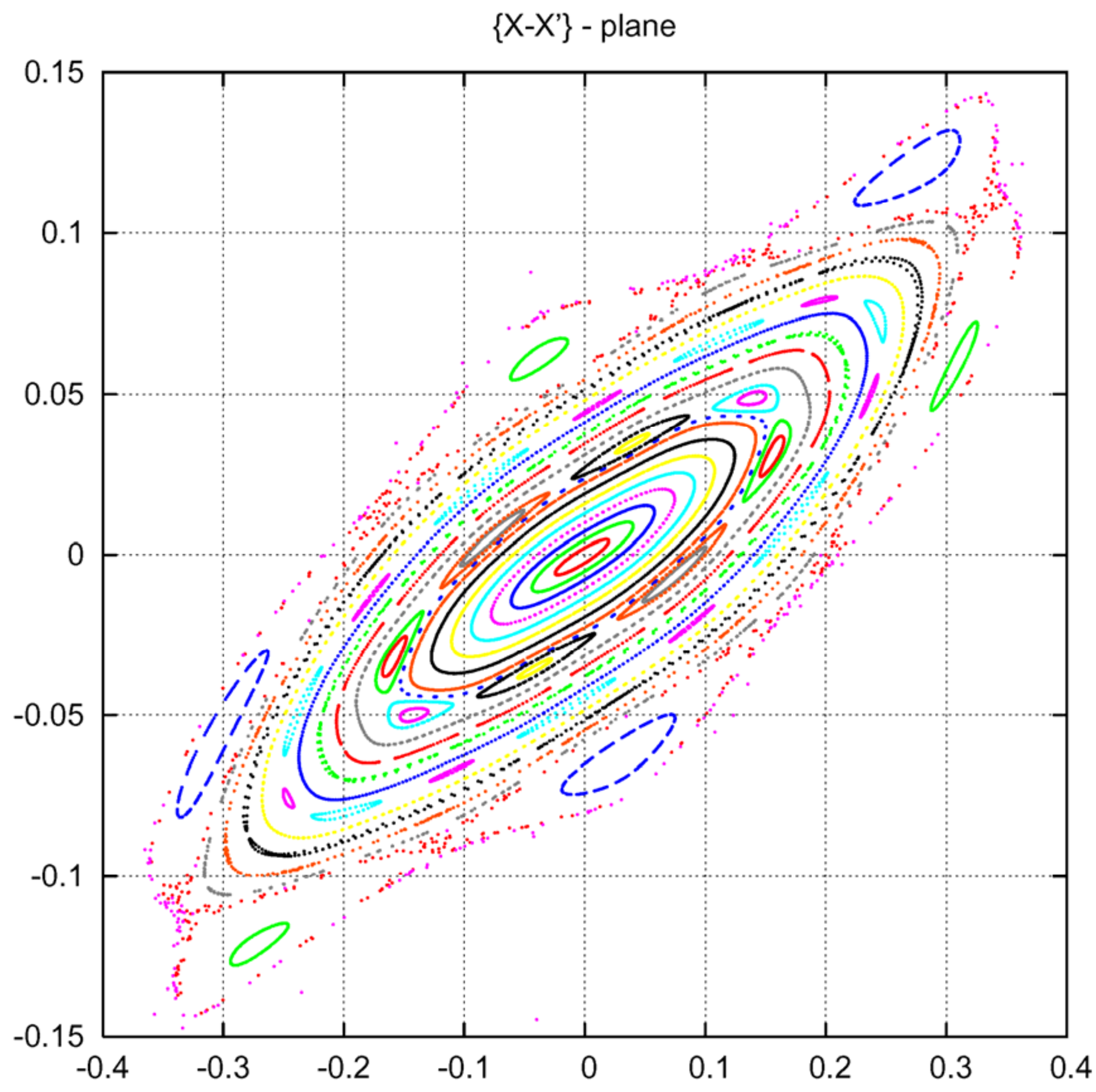
Zero beam current

Non-zero beam current

Tracking with many BB
Example for a simple linear lattice

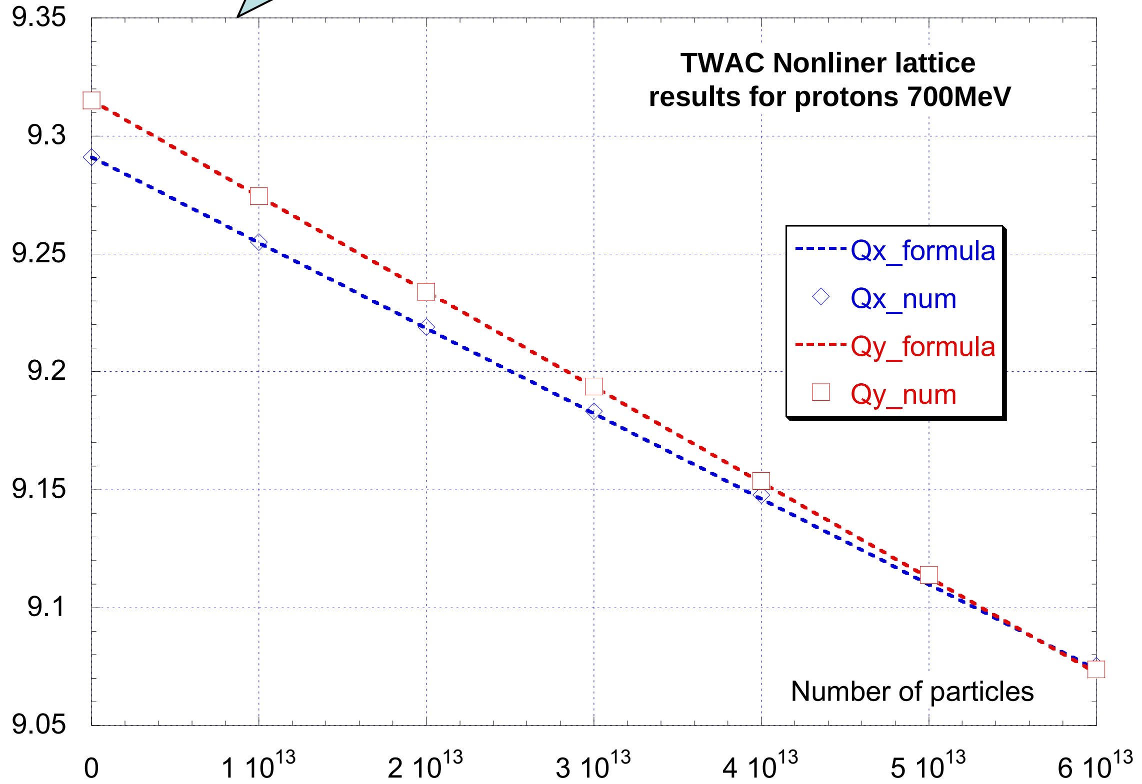
- S.C kicks by BB-elements for non-linear tracking;
  (C.O. shifts are included;
  a total number BB-elements is not limited);

- Since BB is not included into MADX-PTC yet, thin-lens tracking with MADX (similar to MAD8) with lattice conversion by MAKETHIN command



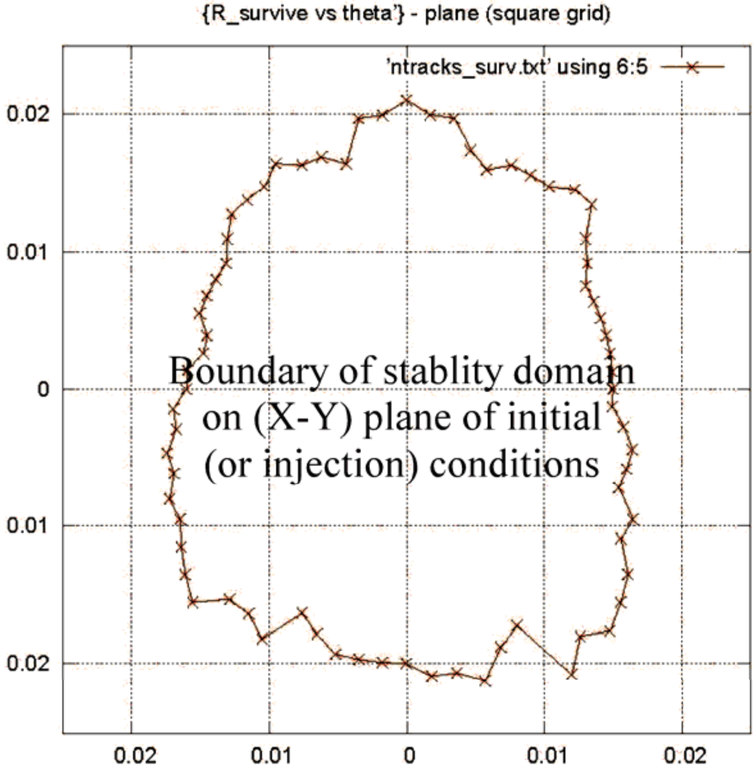
Application to real ring – ITEP's TWAC

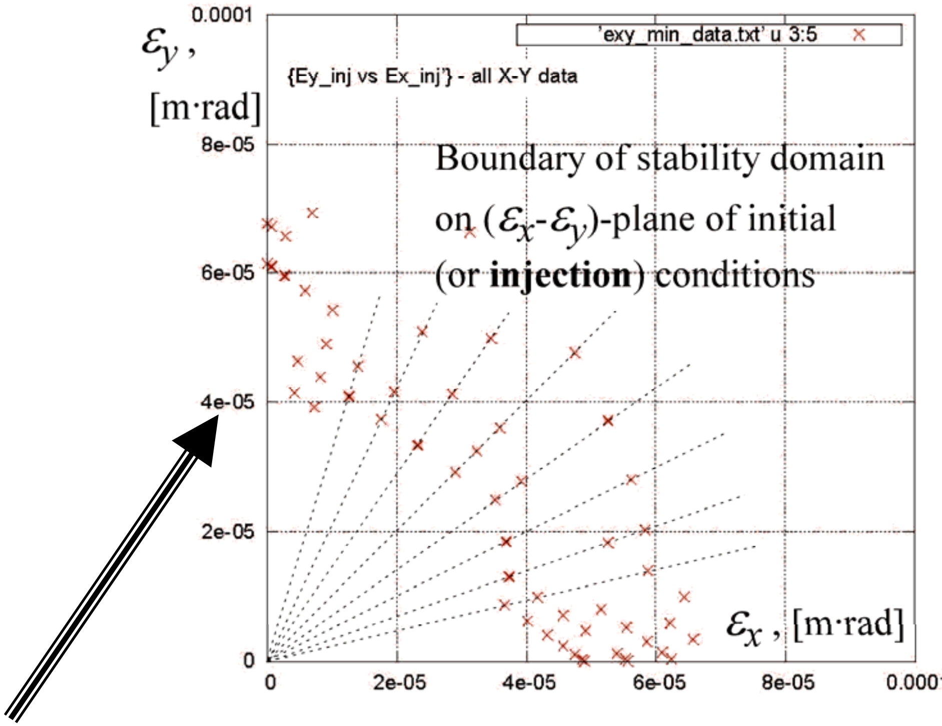
Dependence of analytical (Laslett) and computed (MADX) betatron tunes on intensity.



Economical Algorithm for DA-calculations Example for SIS300 (2007 at GSI)

- Define boundary in (X,Y) - plane at px=py=0
- Calculate Courant-Snyder invariants (e) for every trajectory on boundary at every turn

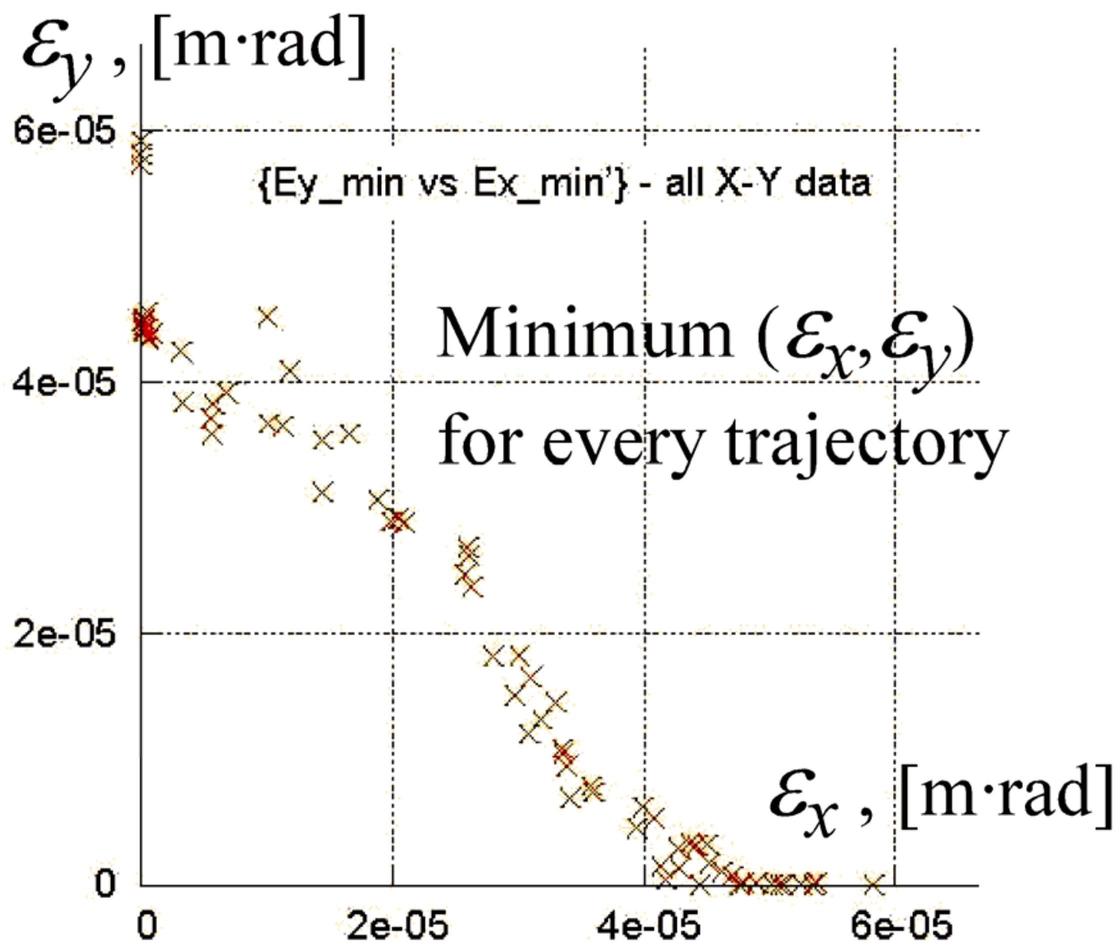




4E-05 m·rad => 40 mm·mrad

Algorithm for DA-calculations (continued-1)

- Find minimum values for every trajectory:
  ex, ey, ex+ey

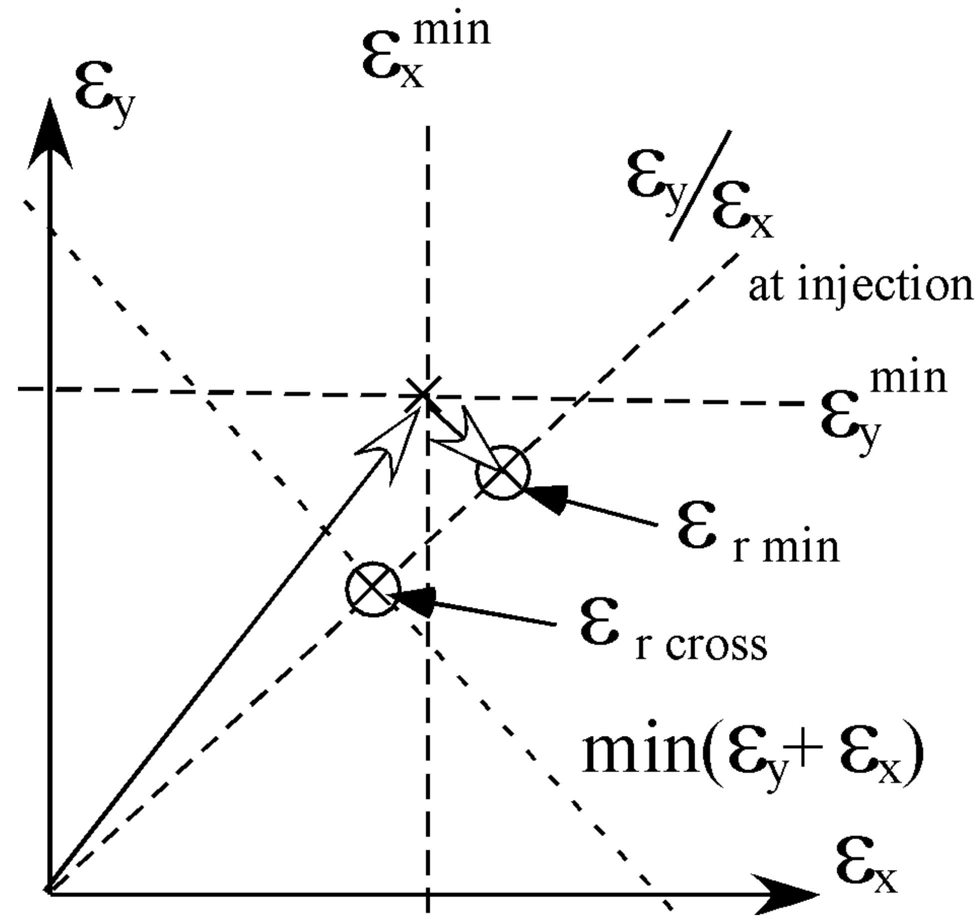


4E-05 m·rad => 40 mm·mrad

DA-Algorithm (continued-2)

Two possible sorting:

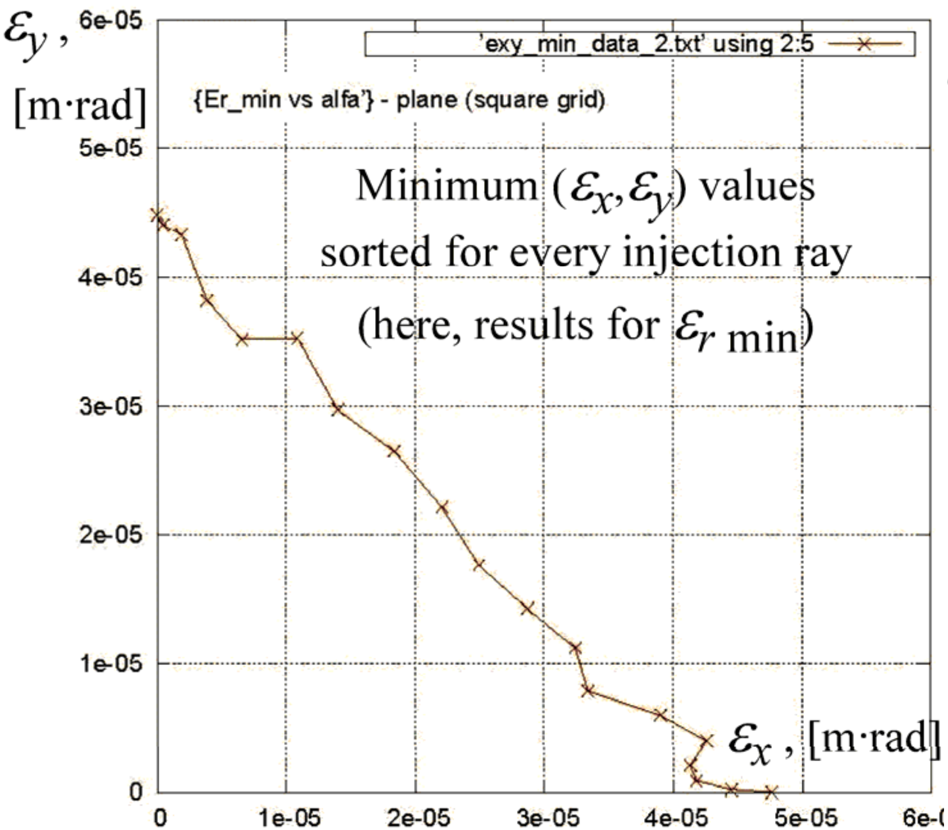
- As minimum of minimum for ex, ey, then projected to injection ray
- As cross between injection ray and minimum of sums ex+ey

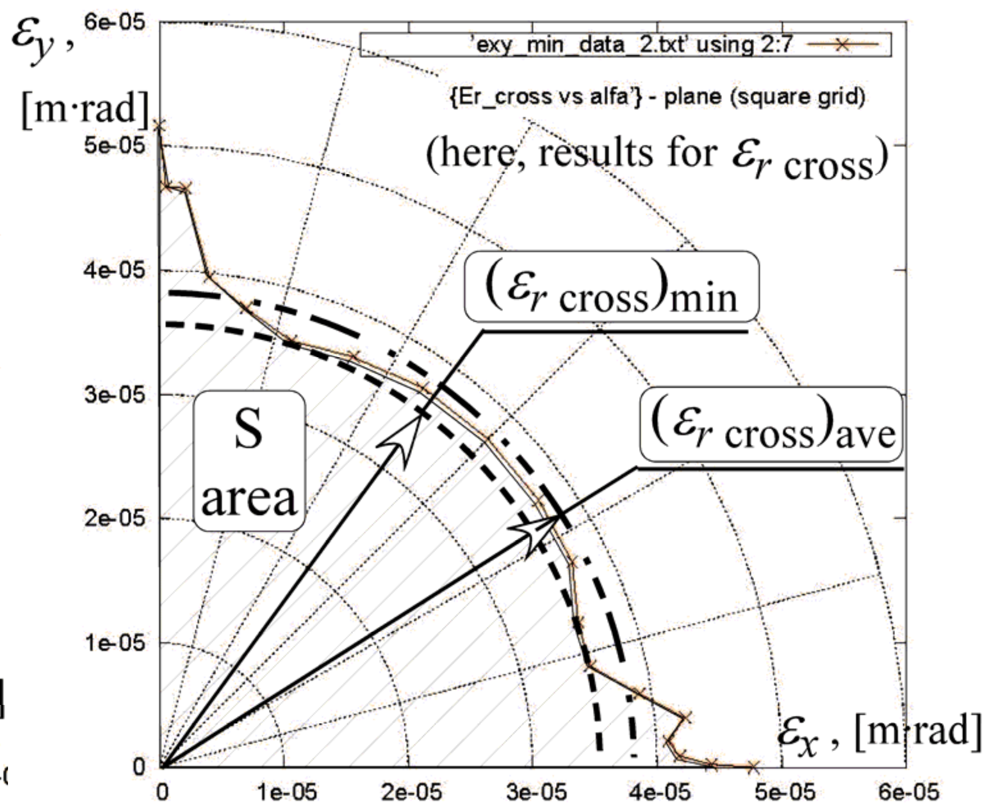


DA-Algorithm (continued-3)

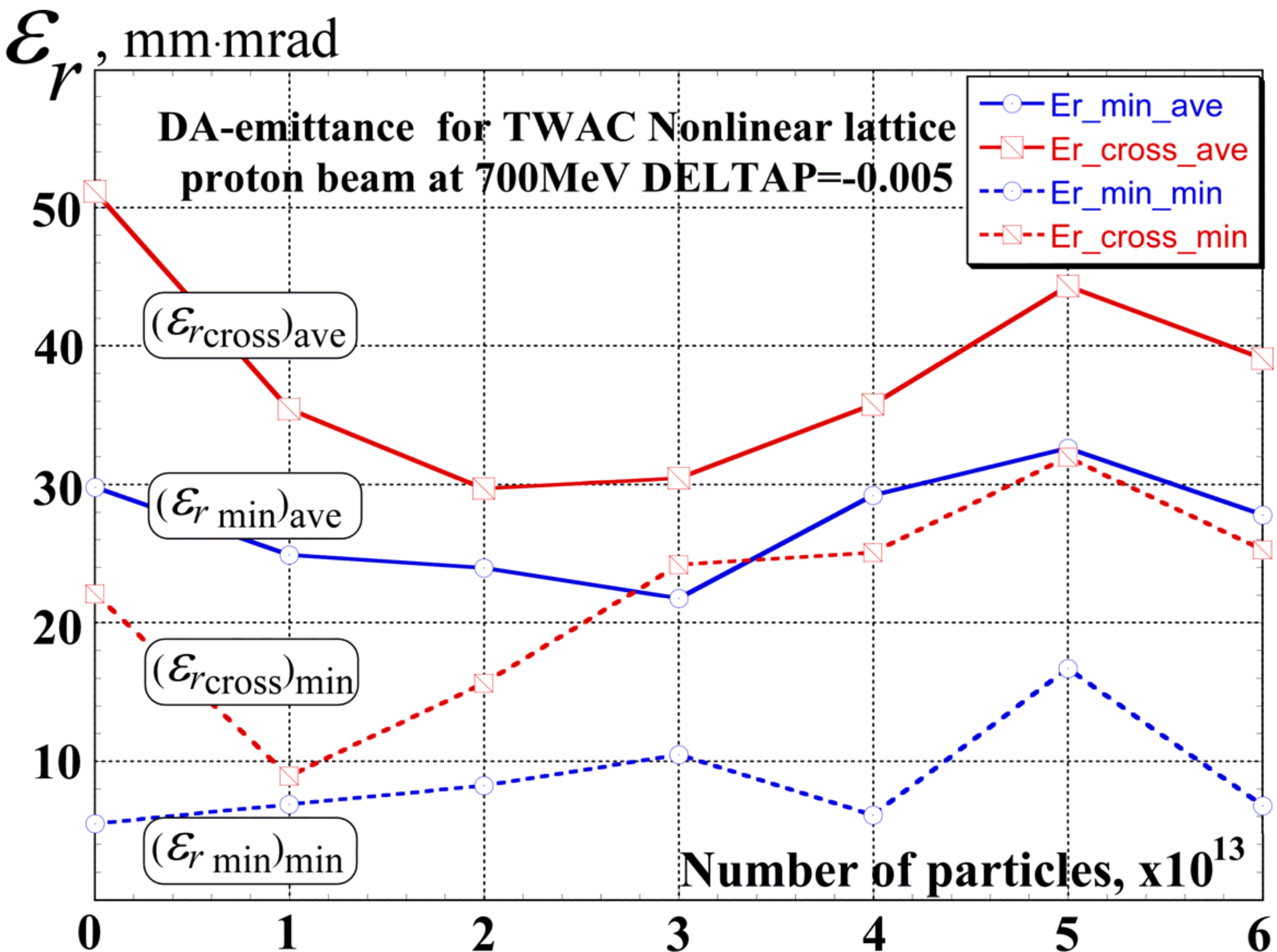
- For every scheme the average radius (via the area) and minimum radius are calculated:

$$(\varepsilon_{r\,\min})_{\min} \quad (\varepsilon_{r\,\min})_{\text{ave}} \quad (\varepsilon_{r\,\text{cross}})_{\min} \quad (\varepsilon_{r\,\text{cross}})_{\text{ave}}$$

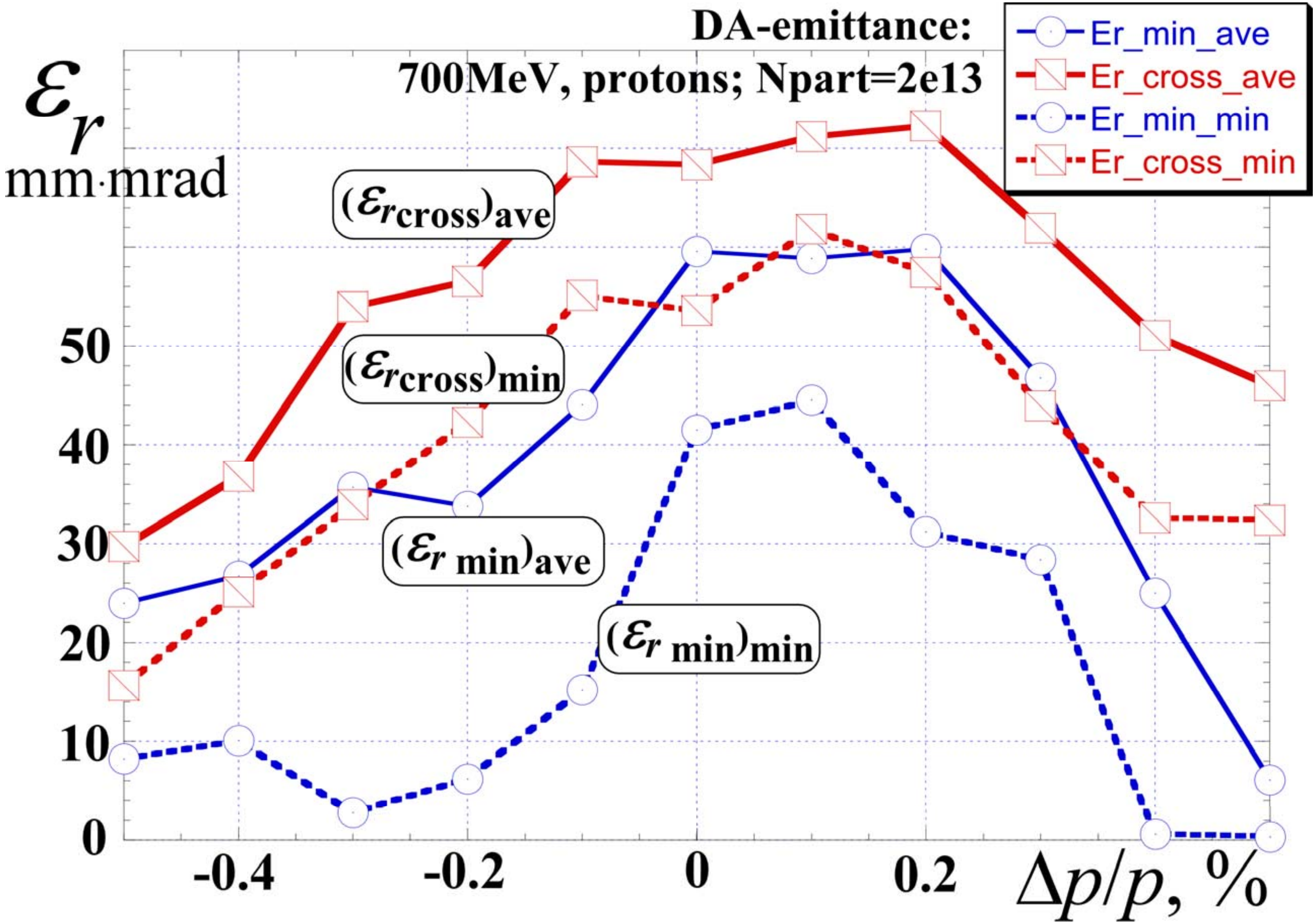




DA vs. *N*p at $\Delta p/p$=-0.5%.



DA vs. $\Delta p/p$ at Np=2x10^13



Acknowlegements

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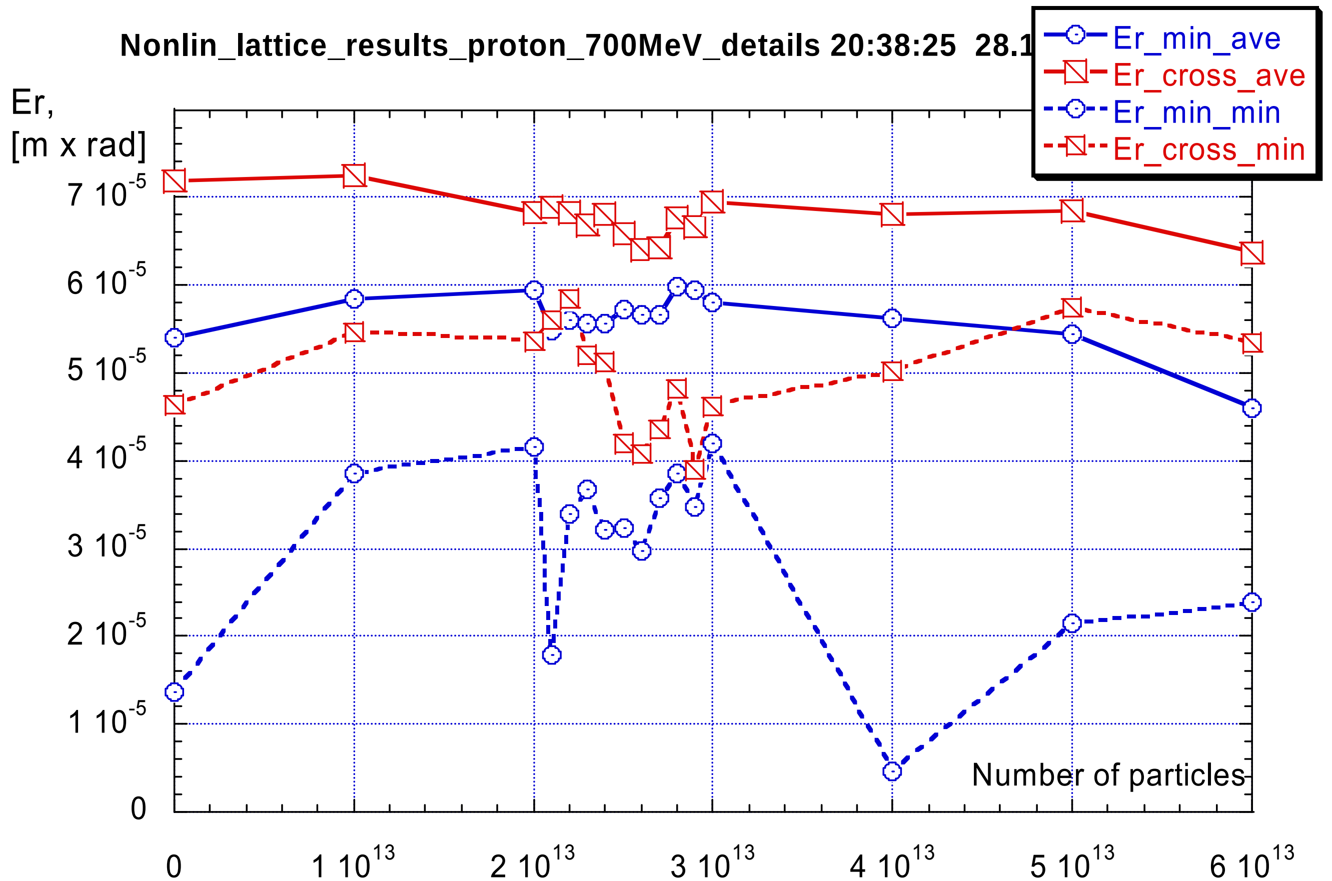
- Dr. F. Schmidt (CERN) for kind help with MAD-X-code implementations

END !

DA vs. Np at $\Delta p/p$=0



DA vs. Np at $\Delta p/p$=-0.5%.