RF-GAP MODEL PRESENTATION

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Abstract

To design the linear accelerators and to make the particle tracking simulations a rf-gap model for the accelerating cells has to be implemented. The standard rectangular rf-gap field presentation, as well the trapezium and cos-like waveform field models were analyzed and compared.

INTRODUCTION

For the accelerator design (mainly for linear accelerators [1]) an influence of the accelerating elements on the transverse and longitudinal particle dynamics is an important part to create a rf-gap model. Further the general rf-gap mathematical description is based on the paper [1], where the longitudinal particle dynamics is characterized by:

- an average electric field on the axis of the accelerating period

$$E_0 = \frac{1}{L}\int_L E_g(z)dz \quad , \tag{1}$$

where L is an acceleration period length; $E_g(z)$ is a waveform factor of the axis electric field, z is the longitudinal coordinate;

- the time flight coefficient

$$T_0 = \frac{1}{E_0 L}\int_L E_g(z)\cos\frac{2\pi z}{L}dz \quad ; \tag{2}$$

- the synchronous particle rf-phase at the gap center φ_s.

The standard model for the transverse rf-gap action uses the matrix presentation [1,2], consisting from the thin lens matrices for the edge rf-gap electric field and a matrix for the rf-gap central part (as a rule the drift space matrix).

Note, the further results were got for the accelerating elements of the ion linear accelerators, where a particle energy increase is much less then the particle energy.

TRANSVERSE PRESENTATION

Supposing the azimuthally symmetric gap electromagnetic field and neglecting by the particle displacement along the rf-gap, the particle trajectory refractions in the input and output gap halves will be [1]

$$\left(\Delta\frac{dr}{dz}\right)_{in} = -\frac{er}{2m_0}\int_{-L^-}^{0}\frac{1}{\gamma(z)v^2(z)}\left(\frac{\partial E_z}{\partial z}+\frac{v(z)}{c^2}\frac{\partial E_z}{\partial t}\right)dz \tag{3}$$

$$\left(\Delta\frac{dr}{dz}\right)_{out} = -\frac{er}{2m_0}\int_{0}^{L^+}\frac{1}{\gamma(z)v^2(z)}\left(\frac{\partial E_z}{\partial z}+\frac{v(z)}{c^2}\frac{\partial E_z}{\partial t}\right)dz \ ,$$

where $E_z \equiv E_z(z,t) = E_g(z)\cos\left(\omega t(z)\right)$; ω is the angular rf-field frequency; r is the transverse particle displacement; e and m_0 are the charge and mass of the accelerating particles; c is the light speed; v_z is the longitudinal particle velocity; $\gamma(z) = \left(1-\beta^2(z)\right)^{-1/2}$ is the particle relativistic factor; $\beta(z) = v(z)/c$; L^- and L^+ are the first and second rf-gap halves. Further the longitudinal coordinate of the gap center will be defined as zero. The gap length is $L^- + L^+ = L$.

Trapezium rf-Gap Field Waveform

The rf-field waveform is described by

$$E_g(z) = \begin{cases} E_g^T\,; & |z| \le z_2 \\ \dfrac{E_g^T}{a}\cdot\left[(g+a)/2 - |z|\right]; & z_2 < |z| < z_1 \\ 0\,; & |z| \ge z_1 \end{cases} \tag{4}$$

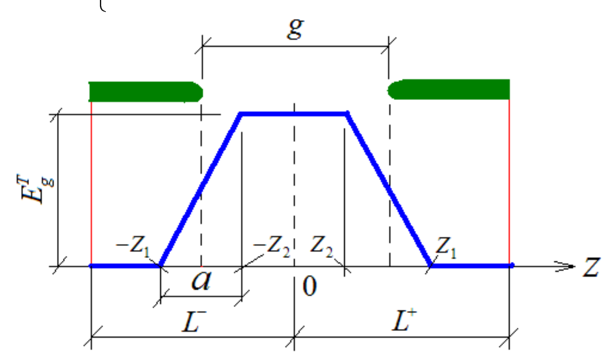

Figure 1: Trapezium rf-field waveform

The waveform (4) and some important parameters are presented schematically in fig.1.

Substituting (4) to (3), the followed expressions may be obtained

$$\left(\Delta\frac{dr}{dz}\right)_{in} = -\frac{erE_g^T}{2m_0c^2\overline{(\gamma\beta^2)}_{in}}\cdot\left(\beta_s^{\,2}\cos\varphi_s + \frac{K_x}{\gamma_s^2}\cos\varphi_{in}\right) \tag{5}$$

$$\left(\Delta\frac{dr}{dz}\right)_{out} = \frac{erE_g^T}{2m_0c^2\overline{(\gamma\beta^2)}_{out}}\cdot\left(\beta_s^{\,2}\cos\varphi_s + \frac{K_x}{\gamma_s^2}\cos\varphi_{out}\right) \ ,$$

where β_s, γ_s and φ_s are the synchronous particle parameters at the gap center; $\overline{\left(\gamma\beta^2\right)}_{in}$ and $\overline{\left(\gamma\beta^2\right)}_{out}$ are the average over L^- and L^+ coefficients respectively; $\varphi_{in} \sim (\varphi_s - \pi/L)$; $\varphi_{out} \sim (\varphi_s + \pi/L)$; $K_x = \sin x / x$; $x = \pi a / L$. The thin lenses (5) are placed in the points $|z| = (g-a)/2$.

Rectangular rf-Gap Field Waveform

The traditional representation of the rf-gap waveform is a rectangular model:

$$E_g(z) = \begin{cases} E_g^R & |z| \le g/2 \\ 0 & |z| > g/2 \end{cases} \ . \tag{6}$$

From (5) by assuming $a \to 0$ it follows

$$\left(\Delta\frac{dr}{dz}\right)_{in} = -\frac{erE_g^T}{2m_0c^2\overline{(\gamma\beta^2)}_{in}}\cdot\left(\beta_s^2\cos\varphi_s + \frac{1}{\gamma_s^2}\cos\varphi_{in}\right) \quad (7)$$

$$\left(\Delta\frac{dr}{dz}\right)_{out} = \frac{erE_g^T}{2m_0c^2\overline{(\gamma\beta^2)}_{out}}\cdot\left(\beta_s^2\cos\varphi_s + \frac{1}{\gamma_s^2}\cos\varphi_{out}\right) .$$

Cos-like rf-Gap Field Waveform

The cos-like rf-field waveform is described by

$$E_g(z) = \begin{cases} 0\,; & |z| \ge z_1 \\ E_g^C/2\cdot\left[1-\sin\left(\pi\dfrac{z-g/2}{a}\right)\right]; & z_2 < z < z_1 \\ E_g^C/2\cdot\left[1+\sin\left(\pi\dfrac{z+g/2}{a}\right)\right]; & -z_1 < z < -z_2 \\ E_g^C\,; & |z| \le z_2\,. \end{cases} \quad (8)$$

The thin lens refractions (3) will be

$$\left(\Delta\frac{dr}{dz}\right)_{in} = -\frac{erE_g^S}{2m_0c^2\overline{(\gamma\beta^2)}_{in}}\cdot\left(\beta_s^2\cos\varphi_s + \frac{K_y}{\gamma_s^2}\cos\varphi_{in}\right)$$

$$\left(\Delta\frac{dr}{dz}\right)_{out} = \frac{erE_g^S}{2m_0c^2\overline{(\gamma\beta^2)}_{out}}\cdot\left(\beta_s^2\cos\varphi_s + \frac{K_y}{\gamma_s^2}\cos\varphi_{out}\right), \quad (9)$$

where $K_y = \cos(\pi y/2)\big/(1-y^2)$ and $y = 2a/L$.

LONGITUDINAL PRESENTATION

Assume that there are some reliable estimations for the integral parameters (1) and (2) (further they will be denoted E_0^r and T_0^r) as well as for a gap coefficient $\alpha = g/L$. Also there is the law to change the synchronous particle phase φ_s. The longitudinal phase space coordinates $(p(t) = \beta(t)\gamma(t)\,;\, z)$ and an independent time variable t will be used. The synchronous particle transverse momentum is equal zero. The longitudinal dynamics of the synchronous particle [1] is governed by

$$\frac{dp(t)}{dt} = \frac{e}{m_0c}E_g(z)\cos(\omega t + \varphi_0)$$

$$\frac{dz}{dt} = c\frac{p_s(t)}{\sqrt{1+p_s^2(t)}} \quad , \quad (10)$$

where φ_0 is a rf-field phase of the synchronous particle at the gap entrance. The rf-field model is designed to conserve both the average electric field and time flight coefficient the same as for the real rf-field (E_0^r and T_0^r).

Trapezium rf-Gap Field Waveform

Applying the definitions and results of the previous section the followed expressions may be derived:

$$E_0^T = E_g^T\cdot\frac{g}{L} = E_g^T\cdot\alpha\,,\quad T_0^T = \frac{\sin(\pi g/L)}{\pi g/L}\cdot\frac{\sin(\pi a/L)}{\pi a/L}\,. \quad (11)$$

Therefore

$$E_g^T = E_0^T/\alpha \equiv E_0^r/\alpha \quad . \quad (12)$$

To define the parameters a it needs to solve equation

$$(\sin\varsigma)/\varsigma = k \quad , \quad (13)$$

where $k = T_0^r/T_0^*$; $T_0^* = \sin(\pi\alpha)/(\pi\alpha)$; $\varsigma = \pi a/L$. (14)
As a rule the practical values are $k<1$ and $0<\varsigma<\pi/2$. However from (13) it follows the lowest limit to use the proposed algorithm:

$$k\frac{\pi}{2} \ge 1 \quad\Rightarrow\quad k \ge \frac{2}{\pi} \sim 0.64 \quad . \quad (15)$$

To overcome the restriction (15) it was introduced the effective parameters

$$g_{eff} > g \quad ; \quad \alpha_{eff} = g_{eff}/L > \alpha \quad . \quad (16)$$

As a result, $T_{0eff}^* = \sin(\pi\alpha_{eff})/(\pi\alpha_{eff}) < T_0^*$ and the parameter k in (14) may be increased up to the desired value, then the parameters a_{eff} and $E_{g\,eff}^T = E_0^r/\alpha_{eff}$ may be recalculated according to (12). To solve the equation (13) the simple tangent method was used. Finally all parameters of the trapezium rf-field waveform (fig.1) will be determined with the requirements

$$E_{0\,eff}^T = E_0^T = E_0^r \quad\text{and}\quad T_{0\,eff}^T = T_0^r\,. \quad (17)$$

Rectangular rf-Gap Field Waveform

Following the approach from the above section and applying the field waveform (6) it may be shown:

$$E_0^R = E_g^R\cdot\frac{g}{L} = E_g^T\cdot\alpha\,,\quad T_0^R = \frac{\sin(\pi g/L)}{\pi g/L} = T_0^* \quad (18)$$

$$E_g^R = E_0^R/\alpha \equiv E_0^r/\alpha \quad . \quad (19)$$

In practice for the real parameters the inequality $k = T_0^r/T_0^* < 1$ is always valid. Therefore it was proposed to use the effective parameters

$$T_{0eff}^R = \sin(\pi\alpha_{eff})/(\pi\alpha_{eff}) = T_0^r\,;\quad E_{g\,eff}^R = E_0^r/\alpha_{eff}\,. \quad (20)$$

As results it is possible to achieve the followed relations:

$$E_0^R = E_{g\,eff}^R\cdot\alpha_{eff} = E_0^r \qquad T_{0\,eff}^R = T_0^r$$

$$E_0^R T_{0\,eff}^R = E_0^r T_0^r \quad . \quad (21)$$

Cos-like rf-Gap Field Waveform

Following the above approach the next relations may be derived:

$$E_0^C = E_g^C\cdot\frac{g}{L} = E_g^C\cdot\alpha\,,\quad T_0^C = \frac{\sin(\pi g/L)}{\pi g/L}\cdot\frac{\cos\frac{\pi}{2}y}{1-y^2} \quad (22)$$

$$E_g^C = E_0^C/\alpha \equiv E_0^r/\alpha \quad . \quad (23)$$

To defined the parameter a it needs to solve the equation

$$\frac{\cos\varsigma}{1-\left(\dfrac{2\varsigma}{\pi}\right)^2} = k \quad , \quad (24)$$

where all parameters are identical to ones in (14).

As a rule the practical values are $k<1$ and $0<\varsigma<\pi/2$. However from (24) it follows the lowest limit to use the proposed algorithm:

$$k \ge \frac{\pi}{4} \sim 0.79 \quad . \quad (25)$$

The requirement (25) is more difficult compared with (15). Therefore the effective parameter approach is more probable for the cos-like rf-field waveform model. The formulas (16) are also valid. Finally all parameters of the waveform will be determined with the requirements

$$E^C_{0\,eff} = E^C_0 = E^r_0 \quad ; \quad T^C_{0\,eff} = T^r_0 \ . \tag{26}$$

PRACTICAL REALIZATION

The above mathematical models were realized to design qualitatively the accelerating elements of the ion linear accelerators. The proposed simulation algorithm consists from the followed stages:

- the proposal of the specific parameters of the accelerating structure : E^r_0, T^r_o, α, the input velocity $\beta(0)$ and rf-phase $\varphi(0)$ of the synchronous particle as well as the rf-phases of the synchronous particle at the gap center φ_s and exit $\varphi(L)$;
- the choice of the preliminary gap geometry and rf-field waveform model, calculation of the model characteristic parameters E_g and a;
- the simulation of the synchronous particle dynamics (10) to get the synchronous particle phases at the gap center φ^*_s and end $\varphi^*(L)$;
- applying the gradient method and varying the L^- and L^+ with conservation of the E^r_0, T^r_o, α to achieve the equalities $\varphi^*_s = \varphi_s$ and $\varphi^*_s(L) = \varphi_s(L)$;
- in dependence from the rf-field waveform model to calculate the focusing properties (3) of the accelerating gap in the thin lens approximation.

The important integral parameter of the accelerating gap is the synchronous particle energy increase [1]:

$$\Delta W_s = eE_0T_0L\cos\varphi_s \ . \tag{27}$$

Excluding the calculated gap length L, all other parameters in (27) are identical for any applied rf-field waveform model. The value L is the result of the computations presented above and therefore depends from the used rf-field waveform model.

To compare the studied models the calculations for Alvarez-type accelerating gap [3] were done. Additionally the simulation of the gap electromagnetic field was carried out by the power program codes [4]. The gap parameters were:

$$\beta_s = 0.08 \ ; \ \alpha = 0.219 \ ; \ D = 1\ \text{cm} \ ; \ f_{rf} = 324\ \text{MHz}$$

$$T^r_0 = 0.92318 \ ; \ E^r_0 = 1.15752\ \text{MV/m} \ ; \ \varphi_s = -35^\circ \ .$$

The graphical results are presented in fig.2, where the solid line is the data received by [4]; the dashed lines are the results for the different rf-field waveform models. The highest rectangle was calculated by using the rectangular model with the gap size g whereas the lower rectangular is for the model with g_{eff}.

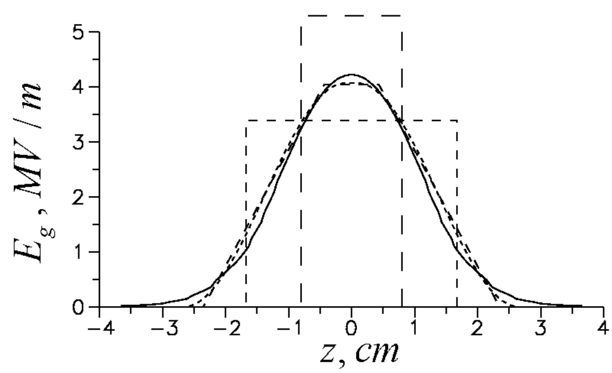

Figure 2: Computational rf-field waveform.

It is evidence the application of the trapezium and cos-like rf-field waveform models is more adequate to the real field distribution. The calculated parameters for the rf-gap are presented in Table 1. The cos-like rf-field waveform model has length the same as for the model with real rf-field distribution, thus the energy increase (27) for these models will be equal.

CONCLUSIONS

The various rf-field waveform models for the accelerating gap have been studied. It was shown that the cos-like rf-field waveform model is more adequate to the real field parameters compared with trapezium and rectangular waveform models. The developed algorithm is used both to design the ion linear accelerators and to study the multi particle dynamics of the accelerated beams.

Table1: Calculation Results for the Alvarez-Type Accelerating Gap

Waveform	L^-, cm	L^+, cm	L, cm	α_{eff}	$E_{g\,eff}$, MV/m	a, cm
Rectangular	2.95005	4.34025	7.2908	0.45903	3.3851	0
Trapezium	3.33495	3.90745	7.2424	0.38452	4.0411	1.9178
Cos-like	3.33730	3.88200	7.2193	0.38212	4.0665	2.5813
Real (by [4])	3.59320	3.62610	7.2193	-	-	-

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