ANALYTICAL STUDY OF BEAM EQUILIBRIUM IN HIGH ENERGY STORAGE RINGFOR NON-MAGNETIZED ELECTRON COOLING *

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Abstract

In high energy storage rings (for example, in proposed antiproton High Energy Storage Ring (HESR), which is part of the international accelerator complex FAIR), it is intended to use electron cooling for the compensation of beam heating due to beam interaction with the internal pellet target. For beams with high emittance (such emittance is necessary in order to get sufficient beam-target overlap) magnetized electron cooling does not work. Therefore we consider an analytical theory of beam equilibrium in presence of internal target using formulae derived in [1] for non-magnetized model of electron cooling. As a result we find the analytical expressions for momentum spread and the electron current as function of the beam emittance and beam energy. This theory is applied for estimation of HESR parameters.

Introduction

Circular accelerators and storage rings are used for the physical experiments with the internal pellet target, which heats the beam in transverse and longitudinal direction. For compensation of beam heating the electron or stochastic cooling can be applied. The beam equilibrium parameters in the proposed antiproton ring (HESR) were considered by use of the numerical [2] and analytic methods [3] with account of the following processes: 1) transverse Coulomb scattering and energy straggling in the internal target; 2) electron cooling; 3) intra-beam scattering (IBS) inside the p-bar beam.

One of main physical goals in HESR is “monochromatic” experiments with small momentum spread ($\sigma_p \sim 10^{-4}$). For achievement of so small momentum spread it was proposed to use “magnetized” electron cooling with high magnetic field in the cooling section. Therefore in paper [3] for analysis of the equilibrium conditions it was used Parchomchuk’s model of magnetized cooling [4].

However, last efforts on the pellet target design have shown that this target has large angular divergence of the pellet beam. Therefore for improvement of the crossing it is desirable to use the antiproton beam with high transverse emittance. For such emittance the Parchomchuk’s model of magnetized cooling is non-applicable since the transverse velocity of the p-bar beam in the cooling section is much higher then Parchomchuk’s “effective velocity”. Taking into account these considerations we apply to solving of the problem a theory of “non-magnetized” cooling for “flattened” velocity distribution developed in [1]. The theory is utilized for calculation of the equilibrium conditions of the beam in HESR.

We introduce the following simplifying assumptions: 1) the ion beam has Gaussian distribution in with equal rms sizes on both transverse degrees of freedom; 2) the electron beam in the cooling section has the circular cross-section with radius a and uniform density and Gaussian distribution of velocities; 3) the target is uniform with width equivalent to the “averaged” width of the pellet target; 4) the beam is relativistic ($\gamma \gg 1$): the last assumption allows us to use simplified model of IBS.

List of Symbols

In the beam rest frame (BRF) all parameters are marked by the sign $*$, Z_i, A_i are the ion charge and atomic numbers number, m_e, m_p the electron and proton masses, r_e, r_i classical radii electron and ion ($r_{e,i} = (Z_{e,i})^2 e^2 / A_{e,i} m_{e,p} c^2$, $E_{p,e}$ are, correspondingly, the proton and electron rest energy, E_s=15 MeV is the characterizing parameter for multiple Coulomb scattering in the target, β,γ are relativistic parameters in in the laboratory frame (LF), $\vec{v}^*$ and v^* the ion velocity and its modulus in BRF, n_e^* and n_e are the electron charge density in BRF and in LF, L_c^{cool} and L_c^{ibs} the Coulomb logarithms for non-magnetized cooling and IBS, $\eta = L_C/C$, where L_C is the length of the cooling section, C is the ring length, I_e, I_i the electron and ion beam current measured in A, N number of ions per ring, a the electron beam radius (the beam is assumed to be uniform), T_{ex}^* and T_{ez}^* are, correspondingly, transverse and longitudinal temperature of the electron cooling beam in BRF, β_{cool} - beta-function in the cooling section, ϵ the rms emittance of the ion beam in units m·rad, σ_p the r. m. spread of the ion beam on relative momentum (Δp/p), β_t beta-function at the target, ρx the target length in g/cm^2, X_r the radiation length of the target material (for hydrogen X_r=58 g/cm^2).

Non-magnetized electron cooling

Let us assume that the horizontal and vertical rms velocity spreads of the antiproton and the electron beams in the cooling section are equal: $\sigma_{px}^* = \sigma_{py}^*, 0_{ex}^* = \sigma_{ey}^*$. The rms longitudinal velocity spreads are σ_{pz}^* and σ_{ez}^*, respectively. Then, according to [1], we obtain the following equations for evolution of these parameters:

$$\begin{cases} \frac{1}{(\sigma_{px}^*)^2}\frac{d}{dt^*}(\sigma_{px}^*)^2 = \frac{B^*}{((\sigma_{ex}^*)^2+(\sigma_{px}^*)^2)^{3/2}}\mathrm{X}(\alpha) \\ \frac{1}{(\sigma_{pz}^*)^2}\frac{d}{dt^*}(\sigma_{pz}^*)^2 = \frac{B^*}{[(\sigma_{ex}^*)^2+(\sigma_{px}^*)^2]\sqrt{(\sigma_{ez}^*)^2+(\sigma_{pz}^*)^2}}\Upsilon(\alpha) \end{cases} \quad (1)$$

Here parameter $\alpha = \frac{(\sigma_{ez}^*)^2+(\sigma_{pz}^*)^2}{(\sigma_{ex}^*)^2+(\sigma_{px}^*)^2}$. The functions $\mathrm{X}(\alpha)$ and $\Upsilon(\alpha)$ are defined by

$$\begin{cases} \mathrm{X}(\alpha) = \frac{\arccos(\sqrt{\alpha}) - \sqrt{\alpha(1-\alpha)}}{2(1-\alpha)^{3/2}} \\ \Upsilon(\alpha) = \frac{1 - \sqrt{\frac{\alpha}{1-\alpha}}\arccos(\sqrt{\alpha})}{1-\alpha} \end{cases} \quad (2)$$

For high ion emittances $\sigma_{ex}^*, \sigma_{ez}^*, \sigma_{pz}^* \ll \sigma_{px}^*$, and $\alpha \ll 1$. If $\alpha \to 0$, then $\mathrm{X}(\alpha) \to \pi/4$, $\Upsilon(\alpha) \to 1$. The coefficient $B^* = \sqrt{2/\pi}4\pi m_e (r_e)^2 c^4 n_e^* f L_c^{cool}$. Plots of the functions $\mathrm{X}(\alpha)$ and $\Upsilon(\alpha)$ are given at Fig. 1. After transfer to LF we obtain:

$$\begin{cases} \frac{1}{\epsilon}\frac{d\epsilon}{dt} = B\frac{1}{[(\beta\gamma)^2\epsilon/\epsilon_{eff}+1]^{\frac{3}{2}}}\mathrm{X}(\alpha) = \frac{1}{\tau_\perp^{cool}} \\ \frac{1}{(\sigma_p)^2}\frac{d}{dt}(\sigma_p)^2 = B\frac{1}{[(\beta\gamma)^2\epsilon/\epsilon_{eff}+1]^{3/2}}\frac{\Upsilon(\alpha)}{\sqrt{\alpha}} = \frac{1}{\tau_\parallel^{cool}} \end{cases} \quad (3)$$

Here $\epsilon_{eff} = 4\beta_{cool}\frac{T_{ex}}{E_e}$; in LF parameter $\alpha = \frac{T_{ez}/T_{ex} + (\beta\sigma_p/\theta_{ex}^{eff})^2}{1+(\beta\gamma)^2\epsilon/\epsilon_{eff}}$ $(\theta_{ex}^{eff} = \sqrt{2T_{ex}^*/E_e})$. In numerical calculations it is useful to express the electron density through the electron current I_e using the relation $n_e = \frac{I_e}{\beta c e 4\pi a^2}$; then we obtain:

$$B = c\sqrt{\frac{2}{\pi}}f\frac{r_i L_c^{cool}}{(\theta_{ex}^{eff})^3\beta\gamma^2 a^2}\frac{I_e}{I_{Alf}} \quad (4)$$

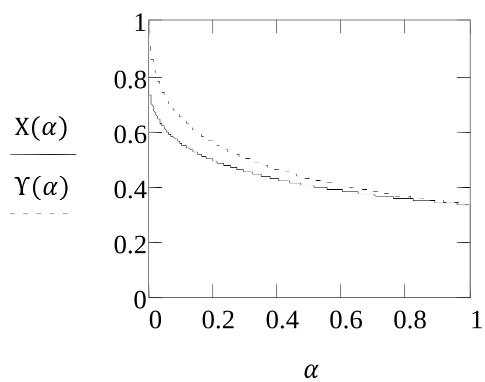

Fig.1. Dependence of the functions $\mathrm{X}(\alpha)$ and $\Upsilon(\alpha)$ versus the dimensionless parameter α.

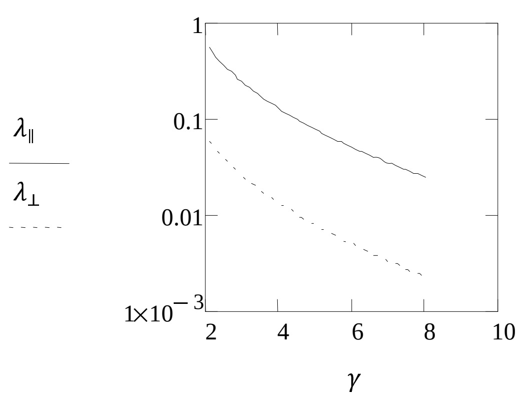

Fig.2. Dependence of cooling rates $\lambda_\parallel$ and $\lambda_\perp$ (in logarithmic scale) versus the relativistic parameter γ for $\epsilon = 10^{-6} m \cdot rad$ and $\sigma_p = \langle\frac{\Delta p}{p}\rangle = 10^{-4}$.

In Eq.(4) Alfven current $I_{Alf} = 1.7 \cdot 10^4$ A. For high emittance and small momentum spread $\alpha \ll 1$; taking into account that $\Upsilon(0) = 1$ and $\mathrm{X}(0) = \pi/4$ we find:

$$\frac{1}{\tau_\perp^{cool}} = \frac{\pi}{4}\frac{1}{\tau_{cool}^0}, \frac{1}{\tau_\parallel^{cool}} = \frac{1}{\tau_{cool}^0}\frac{1}{\sqrt{\alpha}} \quad (5)$$

$$\frac{1}{\tau_{cool}^0} = B\frac{1}{[(\beta\gamma)^2\epsilon/\epsilon_{eff}+1]^{\frac{3}{2}}} \quad (6)$$

Let us consider a numerical example for HESR: $\beta_{cool} = 100$ m, transverse electron temperature $T_{ex}^* = 0.2$ eV, longitudinal electron temperature $T_{ez} = 0.001$ eV, ion beam emittance $\epsilon = 10^{-6}$ m. Under these conditions we obtain $\epsilon_{eff} = 1.6 \cdot 10^{-6}$ m, $\theta_{ex}^{eff} = 8.86 \cdot 10^{-4}$, $\alpha(\sigma_p, \gamma) = \frac{0.005 + 1.274 \cdot 10^6(\beta\sigma_p)^2}{1+0.625(\beta\gamma)^2}$. A dependence of cooling rates versus γ for HESR ($\epsilon = 10^{-6}$ and $\sigma_p = 10^{-4}$) is plotted at Fig. 2.

Target

Transverse heating

The emittance growth rate due to Coulomb scattering on the target is defined by

$$\frac{d\epsilon}{dt} = A_\perp = \beta_t\left(\frac{E_s}{E_p\beta^2\gamma}\right)^2\frac{\rho x}{X_r}\nu_0 \quad (7)$$

Here the target width ρx (measured in g/cm^2) is the target density ρ multiplied the target *x* thickness, X_r is the radiation length (for hydrogen X_r =58 g/cm^2), E_s=15 MeV, $\nu_0 = \frac{\beta c}{C}$ is the number of particle crossings per second. If the target length $\Delta\tilde{t}$ is given in cm^{-2}, then $\rho x = 2 \cdot \Delta\tilde{t}/N_A$ (here $N_A = 6 \cdot 10^{23}$ is Avogadro number). For a target thickness areal density $4 \cdot 10^{15}$cm^{-2} the target width $\rho x = 1.3 \cdot 10^{-8}$ g/cm^2

Longitudinal heating

The maximum energy of the delta-electrons reads $E_{max} = \frac{2E_e\beta^2\gamma^2}{1+2\gamma\frac{E_e}{E_p}+(\frac{E_e}{E_p})^2}$ and the average energy losses per one target crossing are given by $\xi_0 = 0.1534\frac{Z^2}{2\beta^2}\rho x$[MeV]. Let us assume that average energy losses are compensated (for example, by use of induction coil). Then the growth rate of the squared energy deviations per sec is described by the following expression

$$\langle(\Delta E)^2\rangle = \xi_0 E_{max}\left(1 - \frac{\beta^2}{2}\right) \quad (8)$$

From kinematics we have $\frac{\Delta p}{p} = \frac{\Delta E}{E_0\beta^2\gamma}$ γ, and then we obtain the final result:

$$\frac{d(\sigma_p^2)}{dt} = A_\parallel = \frac{\xi_0 E_{max}(1-\frac{\beta^2}{2})}{\beta^4\gamma^2}\nu_0 \quad (9)$$

IBS heating

If the transverse ion beam temperature is much larger than the longitudinal one ($\frac{\epsilon}{2\langle\beta_x\rangle} \gg (\frac{\sigma_p}{\gamma})^2$), than the IBS heating rate of the longitudinal momentum is defined by the following approximate equation [3]:

$$\frac{d(\sigma_p{}^2)}{dt} = \frac{\Lambda_\parallel^{ibs}}{\epsilon^{3/2}} \qquad (10)$$

This expression is valid for a coasting relativistic beam ($\gamma^2 \gg 1$). Here $\Lambda_\parallel^{ibs} = \frac{\sqrt{\pi} c r_i{}^2 N L_c^{ibs}}{4\gamma^3\beta^3\sqrt{\langle\beta_x\rangle} C}$., where the averaged beta-function $\langle\beta_x\rangle = R/\nu$. The emittance growth rate due to IBS

$$(\frac{d\epsilon}{dt})_{IBS} = \gamma^2 \mathrm{K} \frac{\Lambda_\parallel^{ibs}}{\epsilon^{3/2}} \qquad (10)$$

Here constant $\mathrm{K} = \frac{1}{2}\frac{\mathrm{R}}{\mathrm{\nu}}\left\lfloor\frac{(D^2+\tilde{D}^2)}{\langle\beta_x\rangle^2}\right\rfloor$, where sign $\lfloor \quad \rfloor$ means averaging over the ring, D is the ring dispersion function and $\tilde{D} = \beta_x \acute{D} + \alpha_x D$, (here α_x is the Twiss parameter of the lattice).

Equilibrium conditions

Taking into account all effects (electron cooling, target heating and IBS) we can write the following differential equations for the evolution of the beam parameters:

$$\begin{cases} \frac{d\epsilon}{dt} = -\frac{\pi}{4}\frac{\epsilon}{\tau_{cool}^0} + A_\perp + \gamma^2 \mathrm{K}\frac{\Lambda_\parallel^{ibs}}{\epsilon^{3/2}} \\ \frac{d(\sigma_p)^2}{dt} = -\frac{(\sigma_p)^2}{\tau_{cool}^0}\frac{1}{\sqrt{\alpha}} + A_\parallel + \frac{\Lambda_\parallel^{ibs}}{\epsilon^{3/2}} \end{cases} \qquad (11)$$

For equilibrium conditions the derivatives in the left hand side are equal to zero; substituting α we obtain:

$$\begin{cases} \frac{\pi}{4}\frac{\epsilon}{\tau_{cool}^0} = A_\perp + \gamma^2 \mathrm{K}\frac{\Lambda_\parallel^{ibs}}{\epsilon^{3/2}} \\ \frac{(\sigma_p)^2}{\tau_{cool}^0} = \sqrt{\frac{T_{ez}/T_{ex} + (\beta\sigma_p/\theta_{ex}^{eff})^2}{1+(\beta\gamma)^2\epsilon/\epsilon_{eff}}}\,[A_\parallel + \frac{\Lambda_\parallel^{ibs}}{\epsilon^{3/2}}] \end{cases} \qquad (12)$$

Dividing the second equation by the first one we find:

$$(\sigma_p)^2 = \mu(\epsilon)\sqrt{T_{ez}/T_{ex} + (\beta\sigma_p/\theta_{ex}^{eff})^2} \qquad (13)$$

where $\mu(\epsilon) = \frac{\pi}{4}\frac{\epsilon}{\sqrt{1+(\beta\gamma)^2\epsilon/\epsilon_{eff}}}\frac{A_\parallel + \frac{\Lambda_\parallel^{ibs}}{\epsilon^{3/2}}}{A_\perp + \gamma^2\mathrm{K}\frac{\Lambda_\parallel^{ibs}}{\epsilon^{3/2}}}$. Eq. (13) can be reduced to the quadratic equation by substituting the variable $x = (\sigma_p)^2/\mu(\epsilon)$. Solving this quadratic equation relative the x we obtain the final solution for the momentum spread:

$$\sigma_p = \sqrt{\mu(\epsilon)\frac{b+\sqrt{b^2+4T_{ez}/T_{ex}}}{2}} \qquad (14)$$

where $b = (\frac{\beta}{\theta_{ex}^{eff}})^2\mu(\epsilon)$. As can be seen the equilibrium momentum does not depend on the electron cooling rate, which can be found in the first expression of Eq. (14). Using Eq. (7) we obtain that the necessary electron current is given by

$$I_e = I_{Alf}\frac{(\theta_{ex}^{eff})^3\beta\gamma^2 a^2}{c r_i f L_c^{cool}}\frac{4}{\pi\epsilon}\sqrt{\frac{\pi}{2}}(A_\perp + \gamma^2\mathrm{K}\frac{\Lambda_\parallel^{ibs}}{\epsilon^{3/2}})\ [(\beta\gamma)^2\epsilon/\epsilon_{eff}+1]^{\frac{3}{2}} (15)$$

Application to HESR

The list of parameters is summarized in Table 1. The parameters correspond to the one of the last options of the HESR lattice designed by Yu. Senichev [5].

Table 1. Parameters of the ring, ECS and target

Ring circumference (m)	570
Betatron tune ($\nu_x = \nu_y$)	9.3
Number of ions per ring	10^{11}
Coulomb logarithm for ECS(L_c^{cool})	5
Coulomb logarithm for IBS (L_c^{ibs})	20
Transverse beam emittance (m*rad)	10^{-6}
Target thickness (cm^{-2})	$4 \cdot 10^{15}$
Beta-function in the target (m)	1
Beta-function in the ECS (m)	100
Radius of the electron beam in ECS (mm)	15
Length of the cooling section (m)	24
Longitudinal temperature of the electron beam (eV)	0.001
Transverse temperature of the electron beam (eV)	0.2
Parameter K (m)	1.334

The dependencies $\sigma_p(\gamma)$ and $I_e(\gamma)$, corresponding to these parameters, are plotted, correspondingly, on Figs. 3, 4.

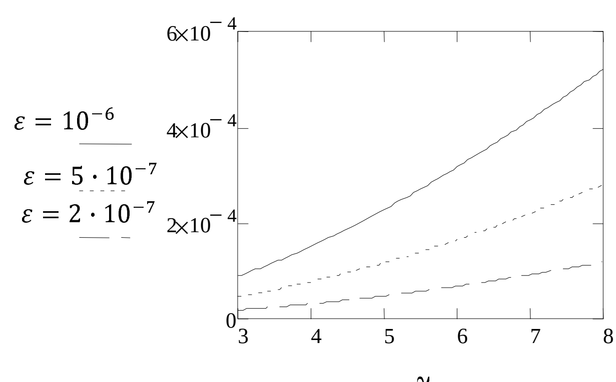

Fig.3. Dependence of r.m.s. momentum spread versus relativistic factor γ.

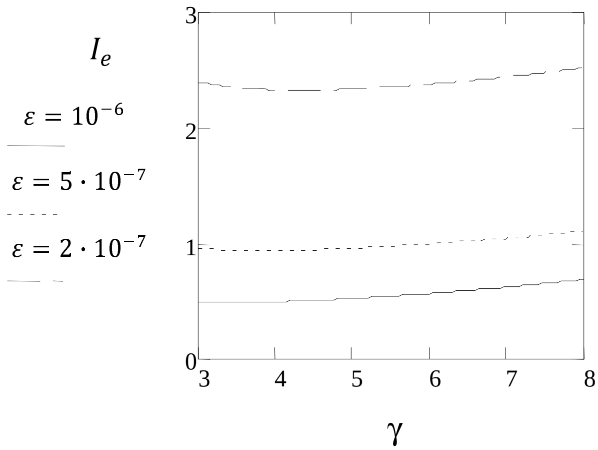

Fig. 4. Dependence of the electron current versus relativistic factor γ .

Conclusion

Let us formulate the main results:

- In the in HESR the non-magnetized cooling gives too large momentum spread for the beams with high emittance.
- The beam momentum spread can be decreased by decrease of the required emittance (at present at FZ-IKP, Juelich the it is developed new pellet target with low angular divergence). However decrease of the emittance results in enlargement of the cooling electron current.
- Let us underline that the results weakly depend on the beam intensity since the IBS is small due to large beam emittance.
- In this operation mode (large emittance and non-magnetized cooling) the magnetic field in the ECS should be designed taking into account the considerations of the electron beam transport and providing stability of the dipole oscillations of coupled electron and ion beams (see [6,7]).
- In conclusion we would like to underline that this remark does not pretend to be the final solution; it's only a contribution in this long discussion.
- These results, of course, can be used for analysis of dynamics in other similar storage ring.

References

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