

# INCOHERENT VERTICAL ION LOSSES AT COOLING STACKING INJECTION

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## Abstract

The efficiency of the cooling-stacking injection depends on two parameters: the cooling-accumulation efficiency and the ion lifetime. The lifetime of newly injected ions is usually smaller than the stack lifetime of high intensity ion beams. The incoherent losses of newly injected ions are related to multiscattering on residual gas atoms and vertical heating caused by ion stack noise. The short lifetime of newly injected ions restricts the efficiency of the cooling stacking injection. The experimental dates, analytical estimations and BETACOL simulations of vertical incoherent ion losses at cooling stacking injection are discussed.

## INTRODUCTION

The maximum stack intensity is limited by the ion lifetime. The lifetime of newly injected ions strongly depends on the stack intensity. At a high intensive ion beam the lifetime of newly injected ions is shorter than the lifetime of stack ions. Below we consider cooling of ions together with vertical diffusion heating produced by the stack noise realized in a regime with a fixed critical stack density. Ion vertical heating and weak cooling of newly injected ions are main causes for a decrease of their lifetime.

## COOLING STACKING INJECTION AT HIMAC AND S-LSR

The maximum  $Ar^{18+}$  ion stack intensity of  $2.5 \cdot 10^9$  ppp was accumulated at HIMAC cooling stacking injection [1-4]. The typical stack gain is 3-5 at the stack intensity of  $(1.5 - 2.5) \cdot 10^9$  ppp and injection intensity of  $(0.3 - 1) \cdot 10^9$  ppp. The HIMAC cooling stacking injection have two peculiarities: the lifetime of newly injected ions is by a factor of 2 smaller than the stack lifetime at high ion intensity; the stack size is by a factor of 2 larger than it follows from equilibrium between intra beam scattering (IBS) and cooling.

The stack lifetime is 6-8 s and the lifetime of injected ions corresponds to 3-4 s at the high stack intensity of  $(1.5 - 2.5) \cdot 10^9$  ppp (Fig.1). A decrease in the lifetime of injected ions is connected with the vertical losses resulting from diffusion of ions with large horizontal betatron amplitudes. The behaviour of the stack equilibrium size at HIMAC indicates diffusion occurring at high ion intensity. At low ion intensity the stack emittance is proportional to  $\epsilon_{st} \propto N_{st}^{2/3}$  [1,2] (Fig.2).

At ion intensity larger than  $10^8$  ppp the stack emittance increases linearly with increasing ion intensity  $\epsilon_{st} \propto N_{st}$  at constant ion density of  $0.9 \cdot 10^7$  ions/mm<sup>2</sup> [1,2] (Fig.2). The stack intensity linearly decreases with decreasing vertical synchrotron aperture at the scraper position of 5-15 mm [4] (Fig.3).

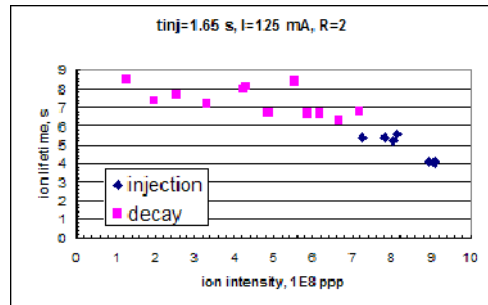


Fig.1 Ion lifetime versus ion intensity [1].

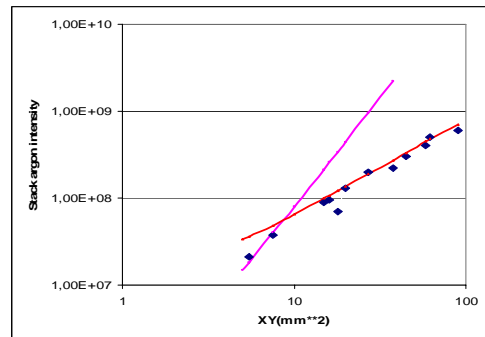


Fig. 2 Stack size versus ion intensity [2].

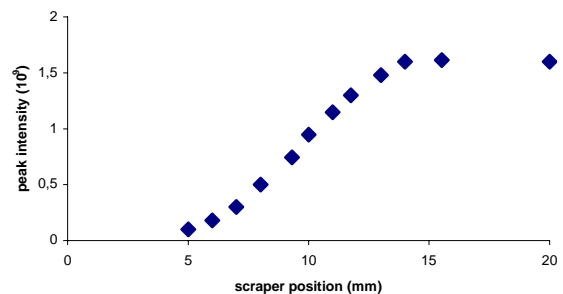


Fig.3 Dependence of HIMAC stack intensity on vertical scraper position [4].

The cooling stacking injection was used at the S-LSR for storage of high intensity proton beams. The maximal intensity of stored protons is about 500  $\mu A$  ( $2 \cdot 10^9$  protons) at the injection current of 100  $\mu A$  [5].

The lifetime of stack protons at this intensity is 2-3 orders of magnitude larger than the lifetime of newly injected protons (Fig.4). The lifetime of newly injected protons is fast reduced because the stack noise produces the incoherent losses in the vertical direction as the stack intensity increases. These losses limited the maximal proton stack intensity at the S-LSR.

The transverse coherent instability also restricts the maximal proton stack intensity.

The feedback system permits suppression of the vertical coherent oscillations and an increase in the stack intensity to 1.2 mA ( $4.5 \cdot 10^9$  protons) [6].

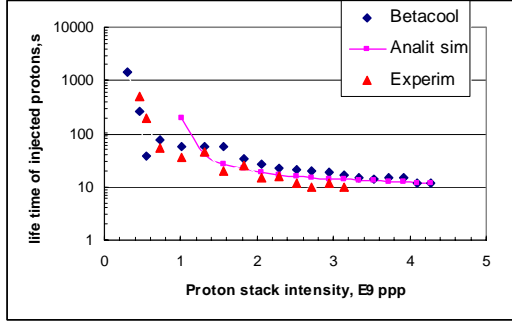


Fig. 4: Dependence of the lifetime of newly injected protons on the stack intensity at the injection intensity of  $4 \cdot 10^8$  ppp,  $T_{inj}=10$  s,  $I_e=100$  mA and  $R=2$ .

## ELECTRON COOLING AND VERTICAL DIFFUSION HEATING

The optimal horizontal emittance  $\epsilon_{m0}$  after multiturn injection is a few times larger usually than the vertical ring acceptance  $\epsilon_{yac}$ ,  $\epsilon_{m0} \gg \epsilon_{yac}$ . The equilibrium stack emittance  $\epsilon_{st}$  is one order of magnitude smaller than the vertical emittance  $\epsilon_{y0}$  of newly injected ions  $\epsilon_{y0} \gg \epsilon_{st}$ .

The ratio of the stack emittance to its intensity is a constant [1,2] defined by the Laslet tune shift  $\Delta Q_{sc}$ :

$$\epsilon_{st}/N_{st} = (r_p/2\pi) \cdot (Z^2/A) / (\beta^2 \gamma^3 \Delta Q_{sc}), \quad (1)$$

where  $r_p$  is the proton radius,  $Z$  and  $A$  are the ion charge and atomic number,  $\beta$  and  $\gamma$  are relativistic parameters. The stack density  $N_{st}/\epsilon_{st}$  remains constant; when the stack emittance linearly increases with increasing stack intensity. This stack density behavior takes place at cooling together with heating produced by stack noise. Below we consider IBS and diffusion heating, caused by a high level of the stack noise, which is stronger than IBS [7]. The electron cooling together with ion diffusion heating leads to an equilibrium stack emittance  $\epsilon_{st}=D/\lambda_{cool}$ , where  $D$  is the diffusion coefficient and  $\lambda_{cool}$  is the stack cooling rate. According to (1) the ratio of the diffusion coefficient  $D$  to the cooling rate is proportional to the stack intensity  $N_{st}$ :

$$D(N_{st})/\lambda_{cool} = N_{st} \cdot (\epsilon_{st-inj}/N_{inj}), \quad (2)$$

where  $N_{inj}/\epsilon_{st-inj} = N_{st}/\epsilon_{st}$  is the constant stack density conserved at different stack intensities,  $\epsilon_{st-inj}$  is the equilibrium stack emittance at injection intensity  $N_{inj}$ .

The cooling rate of newly injected ions is reduced to  $\lambda = \lambda_{cool} \cdot \epsilon_1^{3/2} / (\epsilon_1 + \epsilon_x)^{3/2}$  at the large horizontal emittance  $\epsilon_x$ , where  $\epsilon_1 \equiv \beta_x \theta_e^2$  is a constant defined by  $\theta_e$ , an effective electron angle spread and by  $\beta_x$ , the cooler horizontal beta function. The constant  $\epsilon_1$  is a few times larger than the stack emittance and considerably smaller than the horizontal emittance  $\epsilon_x$ . The cooling rate of newly injected ions is  $\lambda \equiv \lambda_{cool} \cdot \epsilon_1^{3/2} / \epsilon_x^{3/2}$  at  $\epsilon_{st} \ll \epsilon_1 \ll \epsilon_x$ . The decrease in the horizontal emittance of newly injected ions is defined by electron cooling:

$$d\epsilon_x/dt = -(\lambda_{cool} \cdot \epsilon_1^{3/2} / \epsilon_x^{3/2}) \cdot \epsilon_x. \quad (3)$$

The input of horizontal diffusion heating at  $D_x=D$  is neglected for newly injected ions at typical parameters when  $\epsilon_{st} < \epsilon_1 \cdot (\epsilon_{y-ac}/\epsilon_x) \cdot (\epsilon_1 \epsilon_x)^{1/2}$ . The vertical emittance of newly injected ions is defined by the electron cooling and the vertical diffusion heating at  $D_y=D$ :

$$\frac{d\epsilon_y}{dt} = -\lambda_{cool} \frac{\epsilon_1^{3/2}}{\epsilon_x^{3/2}} \cdot \epsilon_y + D. \quad (4)$$

The vertical emittance  $\epsilon_y$  is

$$\epsilon_y(t) = \epsilon_{y0} \cdot \frac{\epsilon_x(t)}{\epsilon_{x0}} + 2\epsilon_x \cdot \frac{D \cdot (\epsilon_{x0}^{1/2} - \epsilon_x(t)^{1/2})}{\lambda_{cool} \cdot \epsilon_1^{3/2}}$$

at the initial vertical emittance  $\epsilon_{y0}$  and horizontal one  $\epsilon_{x0}$ . The vertical emittance reaches the maximum

$$\epsilon_{y-max}(N_{st}) = \frac{8}{27} \cdot \frac{D(N_{st})}{\lambda_{cool}} \cdot \frac{\epsilon_{x0}^{3/2}}{\epsilon_1^{3/2}},$$

at  $\epsilon_{y0} \ll \epsilon_{x0}$ , when the horizontal emittance is cooled down to  $\epsilon_{x-cool} = 4 \cdot \epsilon_{x0}/9$ . With further cooling the vertical emittance decreases to a small value due to a reduction of the horizontal betatron amplitude.

## INCOHERENT VERTICAL ION LOSSES

The ions injected at large horizontal betatron amplitudes are lost in the vertical direction, when the vertical emittance  $\epsilon_{y-max}$  reaches the vertical acceptance  $\epsilon_{yac}$ :  $\epsilon_{y-max} = (8D/27\lambda_{cool}) \cdot (\epsilon_{x0-los}^{3/2}/\epsilon_1^{3/2}) = \epsilon_{yac}$ . The injected ions are lost at horizontal betatron amplitude corresponded to the horizontal emittance larger than

$$\epsilon_{x0-los}(N_{st}) = \frac{9}{4} \cdot \epsilon_1 \cdot \left( \frac{\epsilon_{y-ac}}{D(N_{st})/\lambda_{cool}} \right)^{2/3}. \quad (5)$$

The vertical losses of newly injected ions per injection cycle are equal to  $N_{inj-los} = N_{inj} \cdot (\epsilon_{m0} \cdot \epsilon_{x0-los}) / (\epsilon_{m0} \cdot \epsilon_{st-ac})$ . The lifetime of the injected ions is estimated as

$$\tau_{inj} = T_{inj} \cdot \frac{\epsilon_{m0} - \epsilon_{st-ac}}{\epsilon_{m0} - \epsilon_{x0-los}}.$$

The simulated lifetime of newly injected protons is in good agreement with S-LSR experimental dates (Fig.4).

The number of newly injected ions captured in the stack at each injection cycle corresponds to

$$N_{inj-capt} = \frac{\alpha \cdot (N_{inj}/N_{st})^{2/3} - \epsilon_{st-ac}/\epsilon_{m0}}{1 - \epsilon_{st-ac}/\epsilon_{m0}},$$

where  $\alpha = 9/4 \cdot (\epsilon_1/\epsilon_{m0}) \cdot (\epsilon_{y-ac}/\epsilon_{st-inj})^{2/3}$ . This number is becomes zero when the stack intensity reaches the maximum possible level

$$N_{st-max} \leq N_{inj} \cdot \left[ \frac{9}{4} \cdot \frac{\epsilon_1}{\epsilon_{st-ac}} \cdot \frac{\epsilon_{y-ac}^{2/3}}{\epsilon_{st-in}^{2/3}} \right]^{3/2} \quad (6)$$

At this stack intensity all newly injected ions are lost in the vertical direction during the injection cycle because of strong ion diffusion heating. This condition is realized when the stack lifetime is a few orders of magnitude larger than the lifetime of newly injected ions, which occurs at S-LSR cooling stacking injection.

The stack lifetime is only twice as large as the lifetime of newly injected ions at HIMAC cooling stacking injection (Fig.1). The maximum stack intensity is defined by the stack losses. The stack losses per injection cycle are  $N_{st-loss} = N_{st} \cdot T_{inj} / \tau_{st}$  at  $T_{inj} \ll \tau_{st}$ . The equilibrium stack intensity is defined by the equilibrium between the number of newly injected ions captured in the stack and the stack losses:  $N_{inj-capt} = N_{st-loss}$ . At  $\epsilon_{x0-loss} \gg \epsilon_{st-ac}$ ,

$$N_{st-eq} = N_{inj} \cdot \left[ \frac{9}{4} \cdot \frac{\tau_{st}}{T_{inj}} \cdot \frac{\epsilon_1}{\epsilon_{m0}} \cdot \frac{\epsilon_{y-ac}^{2/3}}{\epsilon_{st-in}^{2/3}} \right]^{3/5} \quad (7)$$

The vertical diffusion heating leads to the nonlinear dependence of the equilibrium stack intensity on the stack ion lifetime  $N_{st-eq} \propto N_{inj} \cdot (\tau_{st} / T_{inj})^{3/5}$ . This dependence agrees with the S-LSR experimental data, where the stack lifetime is 2 orders of magnitude large than the injection repetition time; however, the maximum of the stack intensity is only one order of magnitude higher than the injection intensity [5,6] (Fig.4).

The dependence of the equilibrium stack intensity on the vertical scraper aperture  $N_{st-eq} \propto a_{y-ac}^{4/5}$  is in good agreement with the HIMAC experimental data at range of 5-15 mm [4] (Fig.3).

## BETACOOOL SIMULATIONS

The calculations of the cooling stacking injection by BETACOOOL code [8] are performed with allowance the following effects: cooling of newly injected ions, intra beam scattering, ion interaction with residual gas atoms and diffusion heating. The ion losses are related to the ion interaction with residual gas atoms, the bump orbit displacement at each injection cycle and the synchrotron ring acceptance. Newly injected ions are lost because of the bump orbit displacement if they do not come to a space available for the stack during the injection cycle. Multi scattering on residual gas atoms and ion diffusion heating are sources of incoherent ion losses in the case of small vertical synchrotron acceptance and weak cooling at large horizontal betatron amplitudes [7]. The simulated stack intensity at the HIMAC cooling stacking injection is shown in Fig.5 at the stack lifetime 8 s and lifetime of newly injected ions 4 s (Fig.1).

The peculiarity of the S-LSR cooling stacking experiments is a very large difference between lifetime of stack protons and lifetime of newly injected particles [5,6]. The simulated lifetime of stack protons is about 1500 s and the lifetime of newly injected protons is 10 s at high stack intensity (Fig.4 and Fig.6). The decrease in

the simulated lifetime of newly injected protons (Fig.4) is related to growth of the vertical emittance heating rate  $\dot{\epsilon}_v = 5 \cdot 10^{-3} \cdot (N_{st} / N_{inj}) \pi \cdot \text{mm} \cdot \text{mrad} / \text{s}$  introduced in a new version of BETACOOOL code. However, this heating does not effect the stack lifetime.

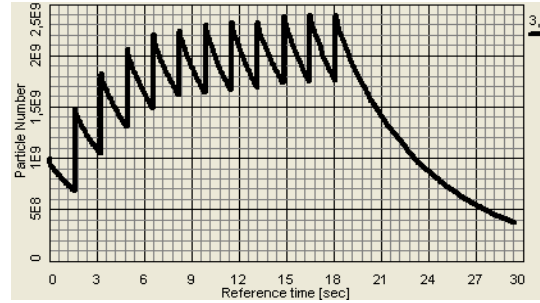


Fig.5 Dependence of the simulated stack intensity on time at HIMAC for I=100 mA and R=2.

The BETACOOOL simulations are performed with the following parameter values: ring acceptances  $\epsilon_{ac} = 2500/30 \pi \cdot \text{mm} \cdot \text{mrad}$ , stack acceptances  $\epsilon_{st-ac} = 20/30 \pi \cdot \text{mm} \cdot \text{mrad}$ , emittances of newly injected ions  $\epsilon_{i-inj} = 80/10 \pi \cdot \text{mm} \cdot \text{mrad}$ . The cooling friction force is simulated on the basis of the Parkhomchuk model at the effective temperature of 10 meV. The results of the BETACOOOL simulations (Fig. 6) are in rather good agreement with the S-LSR experimental data obtained with the feedback system [6]. The saturation of the S-LSR stack intensity is related to the vertical losses of newly injected ions caused by a vertical stack noise which realized at a fixed critical stack density.

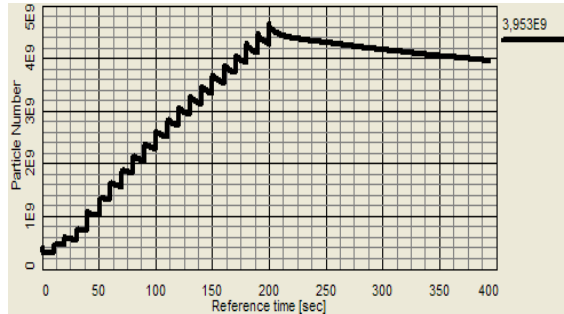


Fig.6 Dependence of the simulated stack intensity on time at the S-LSR for I=100 mA and R=2.

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