ION BEAM ACCELERATION IN INDEPENDENTLY PHASED CAVITIES*

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Abstract

Ion superconducting linac is based on an array of short identical niobium cavities. By specific phasing of the RF cavities one can provide a stable particle motion in the whole accelerator. The longitudinal and transverse ion beam dynamics are studied in this linac. The equation of motion in the Hamiltonian form is devised by the smooth approximation. The focusing methods by the solenoid field and RF field are studied. The results of this investigation are compared with the matrix calculation of ion beam dynamics in superconducting linac.

INTRODUCTION

Ion superconducting linac is usually based on the superconducting (SC) interdigital cavities. This linac consists of the niobium cavities which can provide typically 1 MV of accelerating potential per cavity. Such structures can be used for ion acceleration with different mass-charge ratio in the low energy region [1]. The geometrical velocity β_G of the RF wave is constant for cavities. The identical cavities operate at the some initial drive phase φ. By controlling the driven phase of the accelerating structure and the distance between the cavities, the beam can be both longitudinally stable and accelerated in the whole system.

Beam focusing can be provided with help of SC solenoid lenses, following each cavity and with help of special RF fields. A schematic plot of one period of the accelerator structure is shown in Fig. 1. The low-charge-state beams and the low velocity require stronger transverse focusing than one is used in existing SC ion linac.

In this paper methods of the beam dynamics investigation are compared for low ion velocities and for the charge-to-mass ratio $Z/A = 1/66$. This comparison can be demonstrated with an example a post-accelerator of radioactive ion beams (RIB) linac, where beam velocity increases from $\beta = 0.01$ to $\beta = 0.06$ [1].

PARTICLE MOTION IN SC LINAC

The general axisymmetric equations of motion for ion moving inside an accelerator can be written as

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\left(\gamma\frac{\mathrm{d}z}{\mathrm{d}t}\right) &= \frac{eZ}{Am}E_z(\vec{r},t) - \frac{e^2Z^2}{2A^2m^2\gamma}\frac{\partial}{\partial z}A_\varphi^2 \\
\frac{\mathrm{d}}{\mathrm{d}t}\left(\gamma\frac{\mathrm{d}r}{\mathrm{d}t}\right) &= \frac{eZ}{Am}E_r(\vec{r},t)(1-\beta\beta_G) - \frac{e^2Z^2}{2A^2m^2\gamma}\frac{\partial}{\partial r}A_\varphi^2
\end{aligned} \qquad (1)$$

In every cavity the acceleration RF field of periodic H-cavity is represented as an expansion in spatial harmonics

$$\left.\begin{aligned}
E_z &= E_0\sum \mathrm{I}_0(h_n r)\cos(h_n(z-z_i))\cos(\omega t) \\
E_r &= E_0\sum \mathrm{I}_1(h_n r)\sin(h_n(z-z_i))\cos(\omega t)
\end{aligned}\right\} \qquad (2)$$

where E_0 is amplitude of RF field at the axis ($E_0 \neq 0$ if $-L_r/2 < z-z_i < L_r/2$), $h_n = \pi/D + 2\pi n/D$, $n = 0, 1, 2, \ldots$, $D_i = \beta_G\lambda/2$ is the period length of the cavity, L_r is the cavity length, z_i is the coordinate of the i-th cavity center. I_0, I_1 are modified Bessel function. In our case the reference particle velocity β_c and the geometrical velocity β_G are closely in each class of the identical cavities. Retaining in (2) only zeroth harmonic we can use the traveling wave system. In this system ωt can be replaced by $h_0(z-z_i) + \varphi_{0i}$, where φ_{0i} is the RF phase when the reference particle traverses the cavity center. In equation (1) the value A_φ is the azimuthal vector-potential of the magnetic field in every solenoid ($\boldsymbol{B} = \mathrm{rot}\boldsymbol{A}$).

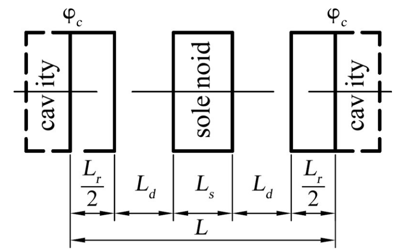

Figure 1: Layout of structure period.

Superconducting cavities provide high accelerating gradient in linear accelerating. Together with the higher accelerating rate in SC linac the defocusing factor is much higher in comparison to the normal conducting linear accelerator. The beam focusing can be provided by SC solenoids which follow each the cavity [1]. The conditions of longitudinal and transverse beam stabilities for the structure consisting from the periodic sequence of the cavities and solenoids were studied early using transfer matrix calculation [2]. In SC linac design, it is very important to know the bucket size since it relates to the longitudinal RF focusing. But the linac longitudinal acceptance cannot be obtained by matrix method because of the assumption that the particles have small longitudinal oscillation amplitude. In order to investigate the nonlinear ion beam dynamics in such accelerated structure and to calculate the longitudinal and transverse acceptances it can be used smooth approximation [3,4]. In this paper, three dimensional equation of motion for ion beam in the Hamiltonian form is derived in the smooth approximation for superconducting linac.

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In periodical structure, which was shown in Fig.1., RF field can be expanded into a Fourier series as

$$E_z = \frac{U}{L} \mathrm{I}_0(k_0 r)\left\{ f_{z.0} + \sum_1^\infty f_{z,n}^c \cos k_n z + f_{z,n}^s \sin k_n z \right\}$$
$$E_r = \frac{U}{L} \mathrm{I}_1(k_0 r)\left\{ f_{r.0} + \sum_1^\infty f_{r,n}^c \cos k_n z + f_{r,n}^s \sin k_n z \right\} \quad (3)$$

Here $f_{z,0} = S_0 \cos(\varphi_c + \psi)$, $f_{r,0} = -S_0 \sin(\varphi_c + \psi)$, $f_{z,n}^c = (-1)^n T_n^+ \cos(\varphi_c + \psi)$, $f_{r,n}^c = (-1)^{n+1} T_n^+ \sin(\varphi_c + \psi)$, $f_{z,n}^s = (-1)^n T_n^- \sin(\varphi_c + \psi)$, $f_{r,n}^s = (-1)^{n+1} T_n^- \cos(\varphi_c + \psi)$
$T_n^\pm = S_n^+ \pm S_n^-$, $S_n^\pm = \sin(Y_n^\pm)/Y_n^\pm$, $Y_n^\pm = (k_c \pm k_n)L_r/2$.

In this expressions: $E = 2U/L_r$, U is the cavity voltage amplitude; $k_n = 2\pi n/L$, $n = 0, 1, 2, \ldots$; k_c is slipping factor, $k_c = (2\pi/\lambda)(1/\beta_c - 1/\beta_G)$. In the coefficients $f_n^{c,s}$ the phase relative to the reference particle ψ defined by $\psi = \omega(t - t_c)$, t_c is the flight time of the reference particle.

In the simple case the vector-potential of the magnetic field $A_\varphi = Br/2$ can be approximated by the step function for every solenoid. If L_s is effective solenoid length and L is a lattice period, the external solenoid magnetic field can be represented as an expansion into spatial harmonics too.

BEAM DYNAMICS IN SMOOTH APPROXIMATION

Let us consider particle acceleration in the polyharmonic fields of the cavities (3) and solenoids. The ion dynamics in such periodic structure is complicated. The particles trajectories can be presented as a sum of the slowly term and a fast oscillation term with a period L. The normalized particle velocity deviation with respect to the reference particle velocity, $\Delta\beta$, can be represented as a sum of a slow motion term and a fast oscillation term too.

Following Ref. [5] one can apply averaging over the fast oscillations and obtain the phase (ψ) and radial ($\rho = h_0 r$) motion equations in smooth approximation.

$$\frac{d^2\psi}{d\xi^2} + 3\left[\frac{d}{d\xi}(\ln\beta\gamma)\right]\frac{d\psi}{d\xi} = -\frac{\partial \overline{U}_{eff}}{\partial\psi}$$
$$\frac{d^2\rho}{d\xi^2} + \left[\frac{d}{d\xi}(\ln\beta\gamma)\right]\frac{d\rho}{d\xi} = -\frac{\partial \overline{U}_{eff}}{\partial\rho} \quad (4)$$

where $U_{eff} = U_0 + U_1 + U_2$ is effective potential function. We use the following designations:

$$U_0 = 4\alpha[\mathrm{I}_0(\rho)\sin(\varphi_c + \psi) - \psi\cos\varphi_c - \sin\varphi_c]S_0 + \frac{1}{2} b \frac{L_c}{L}\rho^2$$
$$U_1 = \alpha^2 \sum_1^\infty \left[\frac{\mathrm{I}_0^2(\rho)}{(2\pi n)^2}\left(g_{z,n}^{c\ 2} + g_{z,n}^{s\ 2}\right) + \frac{\mathrm{I}_1^2(\rho)}{(2\pi n)^2}\left(g_{r,n}^{c\ 2} + g_{r,n}^{s\ 2}\right)\right], \quad (5)$$
$$U_2 = -4\alpha b\rho \mathrm{I}_1(\rho)\frac{L_s}{L}\sum \frac{g_{r,n}^c}{(2\pi n)^2}\frac{\sin X_n}{X_n} + b^2 \sum \frac{1}{(2\pi n)^2}\left(\frac{\sin X_n}{X_n}\right)^2 \rho^2$$

Here $\alpha = \dfrac{\pi eZUL}{2A\lambda mc^2 \beta_g^3 \gamma_g^3}$ is interaction parameter, $b = (eZBL/2Amc\beta_c\gamma_c)^2$ is focusing coefficient, $X_n = \pi n L_s/L$,

$$g_{z,0} = S_0(\cos\varphi_c - \cos(\varphi_c + \psi)),$$
$$g_{r,0} = -S_0(\sin\varphi_c - \sin(\varphi_c + \psi)),$$
$$g_{z,n}^c = (-1)^n T_n^+ (\cos\varphi_c - \cos(\varphi_c + \psi)),$$
$$g_{z,n}^s = (-1)^n T_n^- (\sin\varphi_c - \sin(\varphi_c + \psi))$$
$$g_{r,n}^c = (-1)^{n+1} T_n^+ (\sin\varphi_c - \sin(\varphi_c + \psi)),$$
$$g_{r,n}^s = (-1)^{n+1} T_n^- (\cos\varphi_c - \cos(\varphi_c + \psi))$$

In this expression for U_{eff} we take into account the coherent oscillations of bunches and the effective potential function describe slowly oscillations in the reference particle frame. Earlier, in [5] the effective potential function was found in the frame where averaged velocity of reference particle, $\overline{\beta}_c = 0$. Now, it is interesting to compare these two cases and matrix method, which was used in [2].

The effective potential U_{eff} provides the full description of the ion dynamics in the smooth one-particle approximation. In our case the analysis of the effective potential (5) makes it possible to study the condition at which the radial and phase stability of the beam is achieved. We begin our analysis with expanding U_{eff} in the vicinity of its minimum ($\psi = 0$, $\rho = 0$):

$$U_{eff} = U_{eff}(0,0) + \frac{1}{2}\Omega_z^2\psi^2 + \frac{1}{2}\Omega_r^2\rho^2 + \ldots \quad (6)$$

The expansion coefficients here depend on the parameter of interaction α, the values of L_r/L, L_s/L and the slipping factor k_c. The radial and phase stability of the beam will be provided when $\Omega_z^2 > 0$, $\Omega_r^2 > 0$, where Ω_z, Ω_r are dimensionless frequencies of small longitudinal and transverse oscillations.

In the simplest case when the phase velocity β_G changes from cavity to cavity and $k_c = 0$

$$\Omega_\psi^2 = -4\alpha\sin\varphi_1 + 4\chi\alpha^2\sin^2\varphi_1$$
$$\Omega_\rho^2 = 2\alpha\sin\varphi_1 + \chi\alpha^2\sin^2\varphi_1 + \tilde{B}\frac{L_{sol}}{L} \quad (7)$$

Here the value of χ depends on the ratio of L_r/L. For some of L_r/L the value of χ is listed in Table 1.

Table 1: The value of χ for some ratio of L_r/L

L_r/L	0	1/4	1/2	1
χ	1/3	3/16	1/12	0

In single wave approximation when $L_r/L = 1$ and fast oscillation terms are absent, the value of $\chi = 0$. In this case, the dimensionless frequencies Ω_z, Ω_r are equal to the longitudinal and transverse phase advances per a cavity μ_z and μ_r which were founded by transfer matrix calculation. But the conditions of focusing are changed if the parameter α is large (β_c is small) and the influenced of fast oscillations is greatly. The functions $\Omega_z(\beta)$ and $\Omega_r(\beta)$ for different values of β are shown in Fig 2. In case, when the effective potential function was found in the reference particle frame, the dimensionless frequencies Ω_z, Ω_r (green lines) are close to the phase advances per a period μ_z and μ_r (red lines). In this case the smooth approximation gives very good agreement with the transfer matrix calculation. In the other case when effective potential was found in the frame, where $\overline{\beta}_c = 0$, it appears the sharp distraction for Ω_z (blue line) if $\beta < 0.02$. Therefore first variant of smoothing will be used for analysis of 3D dynamics.

This approximation has been applied to find the longitudinal acceptance. Fig. 3 shows the phase acceptance (thin line) and maximum energy width inside the RF bucket (thick line) for $L_r/L = 1/4$, $\varphi_c = -20°$ and different β. In case, when the effective potential function was found in the reference particle frame the phase acceptance and maximum energy width shown by red lines, and when higher harmonics is absent (single wave approximation) by blue lines. The influence of the fast oscillations is negligibly for maximum energy width.

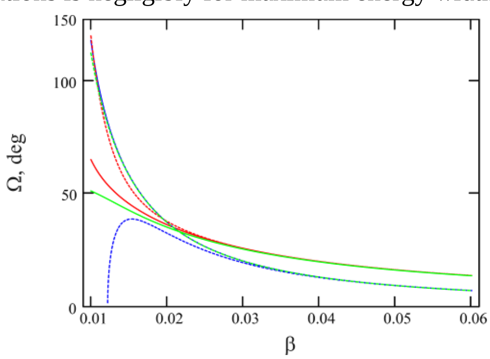

Figure 2: The frequencies of longitudinal Ω_z (dot lines) and transverse Ω_r (solid lines) oscillations for B = 20 T.

For the charge-to-mass ratio $Z/A = 1/66$, $\varphi_c > -20°$, $\beta = 0.01$ and the transverse emittance $V_r = 0.1\pi\cdot$mm$\cdot$mrad the beam focusing can be realized for the solenoid field above $B \sim 20$ T. Smooth approximation gives very good agreement with the transfer-matrix calculation.

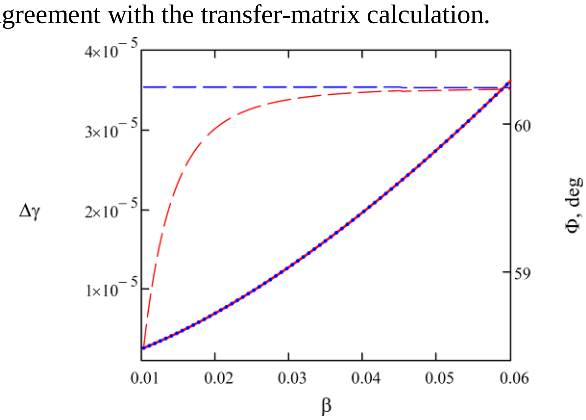

Figure 3: The phase acceptance (Φ) and maximum energy width ($\Delta\gamma$) within bunch for different β.

APF AND SOLENOID FOCUSING

The smooth approximation has been applied to the study of alternating phase focusing (APF) in RIB linac. By adjusting the drive phase of the two cavities, we can achieve the acceleration and the focusing by less magnitude of magnetic field B [2]. New effective potential U_{eff} must be find for this accelerating structure. The analysis of the effective potential makes it possible to study the condition at which the phase and radial stability of the beam is achieved.

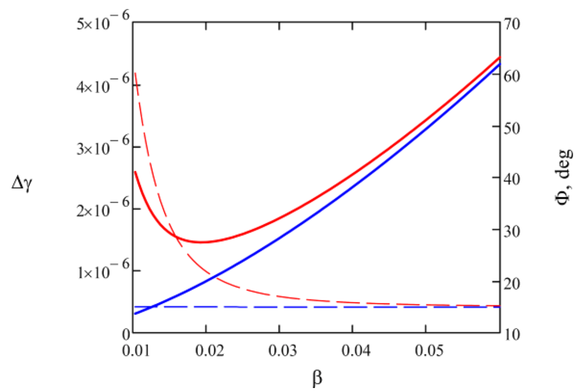

Figure 4: The phase acceptance (Φ) and maximum energy width ($\Delta\gamma$) within bunch for different β.

In the simplest case when a slipping factor $k_c = 0$, $\varphi_1 = -30°$, $\varphi_2 = 20°$ and $\rho = 0$ the longitudinal acceptance is shown in Fig. 4. Now the influence of the fast oscillations is considerable. The maximum energy width inside the RF bucket has minimum in $\beta = 0.017$ and the phase acceptance decreases from 60° to 15°. The value of magnetic field B_{min} can be reduced to 9 T in this case.

CONCLUSION

The methods of the focusing analysis are compared for low ion velocities. It is shown that the smooth approximation gives very good agreement with the transfer matrix calculation if the effective potential function is found in the reference particle frame. By the smooth approximation it is studied nonlinear ion beam dynamics in linac with combined focusing. The borders of the beam stability area can be found by smooth approximation.

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