# INFLUENCE OF BEAM SPACE CHARGE ON DYNAMICAL APERTURE OF TWAC STORAGE RING

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*Abstract*

High intensity ion beams are planned to store in Terra Watt Accumulator (TWAC) at ITEP (Moscow). A size of the phase-space domain where one can safely operate with the beam is related to the dynamic aperture (DA). The DA measured in units of Courant-Snyder invariant is studied numerically with account of the space charge forces (the “frozen core” model) and perturbations of the guiding magnetic field for particles with non-zero momentum deviations. An economical DA-algorithm used for SIS-100 studies with the “Micromap-code” developed at GSI (Darmstadt) has been implemented for MADX-code via its macro-scripts. The results have shown a significant dependence of DA on intensity, especially for particles with a non-zero momentum deviation.


## INTRODUCTION

A particle motion in a circular accelerator is basically governed by the magnetic field of the bending dipoles and the focusing quadrupoles. A presence of high-order multipole components in the guiding field makes the beam-dynamics non-linear and induces the trajectory distortions losses of particles with large amplitudes of betatron oscillations. As result, the motion is stable only for particles with amplitudes, which lie inside of restricted phase space area called as a stability domain. A size of this domain, i.e. the size of the phase-space region where one can safely operate with the beam, is related to the so-called dynamic aperture (DA). An accurate estimate of the DA is crucial for the specification of lattice parameters and accurate estimation of the particle losses during a storage and acceleration [1, 2]. The DA is defined usually as the radius (in units of Courant-Snyder invariant) of the largest circle inscribed inside the domain of stable initial conditions.

It is planned to store high intensity ion beams in TWAC facility at ITEP [3, 4]. For such intensity the space charge effects are crucially increased. In linear approximation and constant density beams the space charge (s.c.) field results in linear shift of the betatron tunes. The beams with more realistic density distribution (for example, Gaussian one) create non-linear components of the s.c. field, which (especially in a presence of non-linear perturbations of the guiding magnetic field) affect on the beam trajectories and DA.

In paper [5] DA of TWAC was studied without account of the s. c. effects. Some estimates of s.c. influence on the TWAC DA were made in paper [6]; however, in calculations we have assumed that the particle transverse invariants are equal and momentum deviation $\Delta p/p=0$. These assumptions are too rough as DA studies of FAIR (GSI, Germany) have shown [7]. A goal of the paper is more detailed study the DA dependence on the beam parameters (beam current and momentum deviation) with account of the non-linear s. c. effects and the guiding magnetic field non-linearity.

## NON-LINEARITY OF MAGNETIC FIELD

Data about non-linearity of the guiding magnetic field in ITEP accumulator are given in paper [5]. The magnetic lattice includes two kinds of the magnetic blocks: 1) the combined functions C-blocks manufactured at the factory; 2) E-blocks made at ITEP’s workshop with the vacuum chamber and the neutral pole placed inside the magnet.

The magnetic structure consists of 8 periods; each period includes 11 C-blocks and one E-block. The magnetic field in the blocks is expressed by Taylor series for deviations of the vertical magnetic field $\Delta B_y$ according to the formula $\Delta B_y=\rho B_0\Sigma(c_n/n!)x^n$, where $x$ is the horizontal coordinate and $\rho$ is the orbit curvature. The values of $c_n$ ($n$=1…6) for all blocks are given in Table 1 using the units [$m^{-2}$], [$m^{-3}$], [$m^{-4}$], [$m^{-5}\times10^4$], [$m^{-6}\times10^6$], [$m^{-7}\times10^8$], respectively.

C-blocks have comparatively small fluctuations of non-linearity. Therefore, we can neglected these fluctuations assuming the same magnetic field nonlinearities in all these blocks. The magnetic field in C-blocks is calculated by numerical solution of two-dimensional Laplace equation for magnetostatic potential. On the contrary E-blocks have significant fluctuations, which were measured before the block’s installation. The expansion coefficients $c_n$ for every $E_i$-block of the $i$-th period had been found by the least square method.

Table 1: The coefficients $c_n$ for TWAC.

| Block name | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ | $c_6$ |
|---|---|---|---|---|---|---|
| C-blocks | 0.32 | -0.07 | -9.4 | -0.08 | -0.07 | -0.07 |
| $E_1$ | 0.31 | 1.11 | 81.9 | -1.84 | -1.11 | 1.21 |
| $E_2$ | 0.31 | 1.16 | 88.2 | -1.67 | -1.14 | 1.03 |
| $E_3$ | 0.32 | 0.71 | -9.50 | -1.32 | 0.11 | 0.18 |
| $E_4$ | 0.32 | -0.29 | -29.5 | 0.12 | 0.17 | -1.16 |
| $E_5$ | 0.32 | 0.52 | -24.6 | -0.52 | 0.55 | -1.21 |
| $E_6$ | 0.31 | 0.70 | 136.1 | -1.80 | -2.34 | 2.417 |
| $E_7$ | 0.32 | 0.92 | 68.8 | -1.99 | -0.98 | 1.34 |
| $E_8$ | 0.31 | 0.89 | 105.7 | -1.88 | -1.41 | 1.47 |

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## THE DA CALCULATION ALGORITHM

A particle dynamics of the coasting beam in four-dimensional phase space $(x,x',y,y')$ is considered, where $x,y$ are the transverse coordinates, and $x',y'$ are derivatives on longitudinal variable $s$. To define the DA numerically, one must scan all possible combinations of four coordinates. Such direct evaluation of the DA is very CPU time consuming. Therefore, an economical algorithm for the DA calculations using a fast tracking is required [2]. In this report, the DA-algorithm used for SIS-100 studies [7] with the Micromap-code has been implemented for MADX-code [8] via its macro-scripts.

The space charge field is calculated in the "frozen s.c. field" approximation, which assumes that initial distribution of the space charge density is unperturbed. A numerical realization deals with the s.c. force created by thin elements (kicks), which are inserted around the ring according to some integration method. This approach has been already used with several codes, e. g. FRANKENSPOT [9] and MAD8 [10], while the beam with Gaussian distribution is usually assumed. Thus the beam density for element with number $n$ is calculated for the beam with Gaussian distribution $\rho_n(x,y)\sim\exp[-x^2/2(\sigma_{nx})^2-y^2/2(\sigma_{ny})^2]$.

The beam sizes $\sigma_{nx,y}\sim(\varepsilon_{x,y}\beta_{nx,y})^{1/2}$ have been calculated with TWISS-command of MADX-code using an iteration procedure for the definition of $\beta_{nx,y}$-functions at the given beam emittances $\varepsilon_x$ and $\varepsilon_y$.

In the MAD8 code and its successor MADX code the 4-dimensional s.c. kicks can be simulated using the BEAMBEAM element, which describes the "beam-beam" interaction. Unfortunately, TRACK module of MAD8 does not permit to have more than 200 BEAMBEAM elements.

Principally, a number of BEAMBEAM elements is not restricted in MADX code in contrary to the "frozen" and obsolete MAD8 code. Our calculations for TWAC ring have been performed with MADX using 336 BEAMBEAM elements simulating s.c. kicks.

The magnetic field non-linearities were also simulated as a set of thin non-linear lenses placed at the center of the magnetic blocks. The non-linearity amplitudes are chosen in correspondence with Table 1.

In a frame of the procedure developed in [7], MADX tracking for set of particles with initial coordinates $x_{in}$, $y_{in}$ and zero initial derivatives ($x'_{in}=0, y'_{in}=0$) has been performed. The initial coordinates are chosen along 72 radial rays with fixed ratio $y_{in}/x_{in}$. Figure 1,a shows the resulting boundary of the stability domain on the $(x_{in},y_{in})$-plane (number of tracked turns $n_{turns}$=1000).

Using the relation $\varepsilon_u=\gamma_u u^2+2\alpha_u uu'+\beta_u(u')^2$, points on $(x,y)$-plane ca be transferred on the $(\varepsilon_x,\varepsilon_y)$-plane of Courant-Snyder invariants. Figure 1,b shows the points of the boundary of the stability domain on the $(\varepsilon_x,\varepsilon_y)_{in}$-plane. Note, every four rays of the same ratio $|y_{in}/x_{in}|$ on the $(x_{in},y_{in})$-plane are transferred onto one ray on the $(\varepsilon_x,\varepsilon_y)_{in}$-plane having a fixed ratio $(\varepsilon_y/\varepsilon_x)_{in}=(\gamma_y/\gamma_x)\cdot(y_{in}/x_{in})^2$. The resulting four points on every $(\varepsilon_y/\varepsilon_x)_{in}$-ray correspond to oscillations with different initial phases.

The DA algorithm follows to a conservative approach. The sufficient ("conservative") stability condition requires that for each turn particle invariants should be less than the minimal value of $\varepsilon_x,\varepsilon_y$ and their sum $\varepsilon_\Sigma=\varepsilon_x+\varepsilon_y$. Therefore, the minimum values of invariants $\varepsilon_{xmin},\varepsilon_{ymin}$ and their sums $\varepsilon_{\Sigma min}$ for every "boundary" trajectory over all turns are calculated and saved for the further DA-calculations.

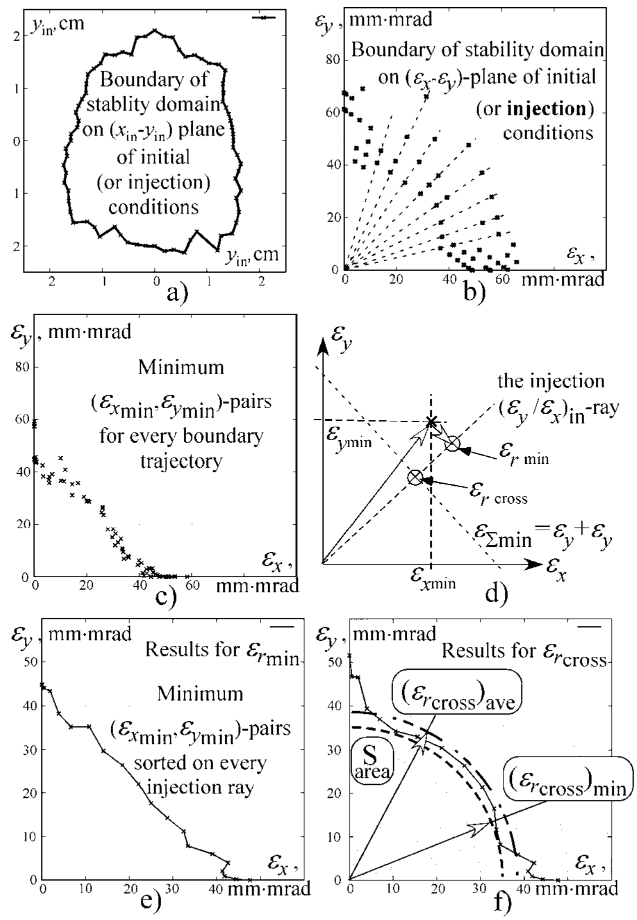


Fig.1 The example of short-term (1000) DA-calculations for SIS-300 lattice using the code MADX [11].

Every $(\varepsilon_{xmin},\varepsilon_{ymin})$-pair defines a point on the $(\varepsilon_x,\varepsilon_y)$-plane, and the equation $\varepsilon_{\Sigma min}=\varepsilon_x+\varepsilon_y$ defines a straight line. As an example, a family of $(\varepsilon_{xmin},\varepsilon_{ymin})$-pairs for the "boundary" trajectories is presented at Fig.1,c. It is seen the resulting boundary has a smear shape. The similar uncertain situation exists in the case of $\varepsilon_{\Sigma min}$-lines too.

In order to draw unambiguous boundaries in both cases we apply a "heuristic" sorting procedures using the initial rays with a constant ratio $(\varepsilon_y/\varepsilon_x)_{in}$ (see Fig.1,d). In the first case, every $(\varepsilon_{xmin},\varepsilon_{ymin})$-point is projected onto its "parental" ray $(\varepsilon_y/\varepsilon_x)_{in}$ with a conservation of the parameter $\varepsilon_{rmin}=(\varepsilon^2_{xmin}+\varepsilon^2_{ymin})^{1/2}$, i.e. by a rotation. In the second case, the cross-point of the line $\varepsilon_{\Sigma min}=\varepsilon_x+\varepsilon_y$ with its "parental" ray $(\varepsilon_y/\varepsilon_x)_{in}$ results in the "cross radius" $\varepsilon_{rcross}$. As result, up to 4 radii $\varepsilon_{rmin}$ and up to 4 radii $\varepsilon_{rcross}$ corresponding to different initial oscillation phases are located on every ray $(\varepsilon_y/\varepsilon_x)_{in}$. Using their minimum values, the final limiting curves can be drawn. Figures 1,e and 1,f present typical limiting curves plotted respectively by use of the first and the second sorting procedures. It is seen that the procedures provide different stability areas.

The minimum and the average radii on the ($\varepsilon_x$,$\varepsilon_y$)-plane are introduced in order to quantify these limiting curves. Totally, for two procedures two pairs of these radii $(\varepsilon_{r\min})_{\min}$, $(\varepsilon_{r\min})_{\rm ave}$ and $(\varepsilon_{r\rm cross})_{\min}$, $(\varepsilon_{r\rm cross})_{\rm ave}$ are calculated.

## RESULTS AND DISCUSSION

All calculations were made for protons with energy 700 MeV. Firstly, we have checked a normalization by comparison of analytical and calculational values of betatron tune shifts $\Delta Q_{x,y}$; the results plotted at Fig. 2 have shown a good coincidence. The maximum number of protons $N_{\rm p}=6\cdot10^{13}$ corresponds to $\Delta Q_{x,y}$~0.3.

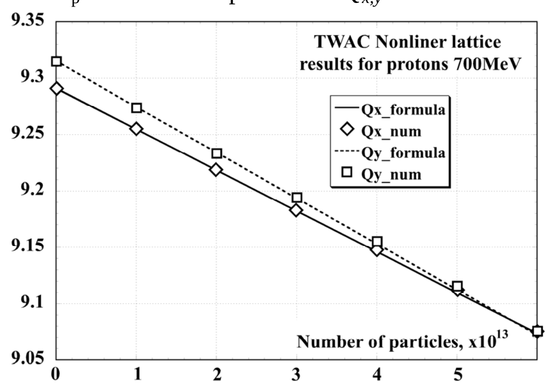


Fig.2 Betatron tunes vs. beam intensity.

Then the dependencies of DA on the intensity and momentum deviation have been studied. We have seen that "in average" for zero momentum the DA spread is comparatively insensitive relative to the intensity. An interesting feature is a significant DA diminishment (especially for parameter $(\varepsilon_{r\min})_{\min}$ ) for some chosen $N_{\rm p}$. It seems to be caused by appearance of strong non-linear resonances along some rays. The effects are drastically increased for $\Delta p/p\neq0$, e.g., Figure 3 shows the dependence of DA on $N_{\rm p}$ for $\Delta p/p$=-0.5%.

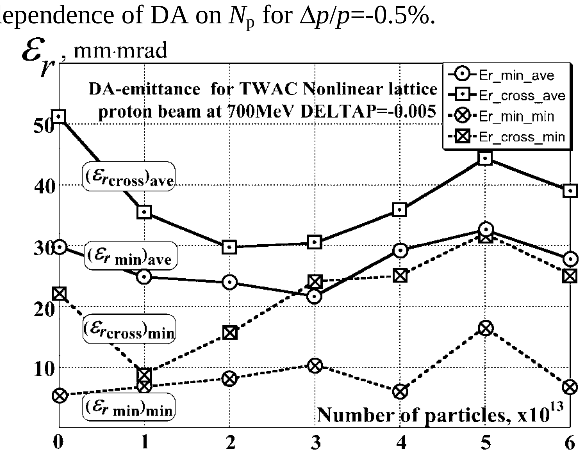


Fig.3 DA vs. $N_{\rm p}$ at $\Delta p/p$=-0.5%.

Figure 4 shows the dependence of DA on $\Delta p/p$ at $N_{\rm p}=2\cdot10^{13}$. A strong asymmetry relative to momentum deviation is seen. Thus, DA, which "feels" individual particle, strongly oscillates during the accumulation due to the intensity change and momentum diffusion before of an intra-beam scattering (IBS). It means that in TWAC the DA shortage can result in essential increase of particle losses. Now we study a possibility to include this DA shortage in MOCAC code [12] for simulation of particle losses in TWAC.

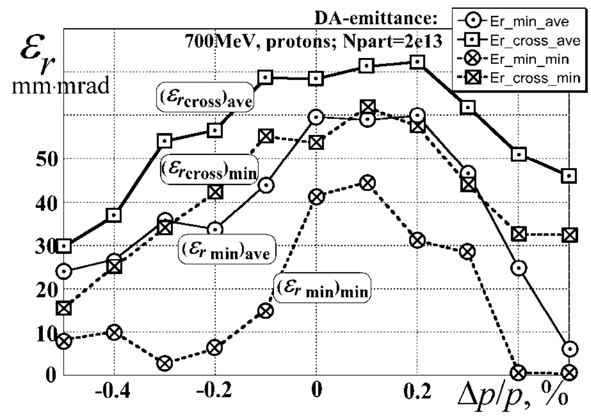


Fig.4 DA vs. $\Delta p/p$ at $N_{\rm p}=2\cdot10^{13}$.

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