BEAM-SIZE EFFECT AND PARTICLE LOSSES AT COLLIDERS*

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Abstract

In the modern colliders, the macroscopically large impact parameters give a substantial contribution to the standard cross section of the $e^+e^- \to e^+e^-\gamma$ process. These impact parameters may be much larger than the transverse sizes of the colliding bunches. It means that the standard cross section of this process has to be substantially modified. Such a beam-size effect has been discovered in BINP (Novosibirsk) about twenty five years ago. In this report we give simple qualitative description of this effect and present two novel topics. First, we discuss how to take into account quantitatively the particle correlations in the beams. Second, we present our calculations related to bremsstrahlung at B-factories KEKB and PEP-II. We find out that beam-size effect reduces beam losses by about 20%.

INTRODUCTION

The so called beam-size or MD-effect is a phenomenon discovered in experiments at the MD-1 detector on the VEPP-4 collider, Novosibirsk (1981). It was observed [1], that the number of measured photons in the process $e^+e^- \to e^+e^-\gamma$ was considerable smaller than expected. A qualitative explanation of the effect was given by Yu.A. Tikhonov [2], who pointed out that those impact parameters ϱ, which give an essential contribution to the standard cross section, reach values of $\varrho_m \sim 5$ cm whereas the transverse size of the bunch is $\sigma_\perp \sim 10^{-3}$ cm. The limitation of the impact parameters to values $\varrho \lesssim \sigma_\perp$ is just the reason for the decreasing number of observed photons. The first calculations of this effect have been performed in Refs. [3] and [4]. The detailed description of the MD-effect can be found in review [5]. Later on, the effect of limited impact parameters was taken into account using the bremsstrahlung reaction for measuring the luminosity at the VEPP–4 collider [6] and at the LEP-I collider [7].

A general scheme to calculate the finite beam-size effect (which starts from the quantum description of collisions as an interaction of wave packets that form bunches) had been developed in paper [8]. Since the effect under discussion is dominated by small momentum transfer, the general formulas can be considerable simplified. The corresponding approximate formulas were derived in [8]. In the second step, the transverse motion of the particles in the beams can be neglected. The less exact (but simpler) formulas, which are then found, correspond to the results of Refs. [3, 4]. It has also been shown that similar effects have to be expected for several other reactions such as bremsstrahlung for colliding ep–beams [9], [10], e^+e^-– pair production in $e^\pm e$ and γe collisions [8]. The corresponding corrections to the standard formulas are now included in programs for simulation of events at linear colliders. The influence of MD-effect on polarization was considered in Ref. [11]. In 1995 the MD-effect was experimentally observed at the electron-proton collider HERA [12] at the level predicted in [10].

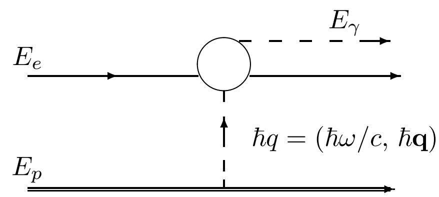

Figure 1: Block diagram of radiation by the electron

The possibility to create high-energy colliding $\mu^+\mu^-$ beams is now wildly discussed. For several processes at such colliders a new type of beam-size effect will take place — the so called linear beam-size effect [13]. The calculation of this effect was performed by method developed for the MD-effect in [8].

In the present paper we discuss two new features related to the MD-effect: 1) an account the influence of the particle correlations in the beams on the MD-effect; 2) an influence of the MD-effect on the beam losses at the existing B-factories.

QUALITATIVE DESCRIPTION OF THE MD-EFFECT

Qualitatively we describe the MD–effect using the $ep \to ep\gamma$ process as an example and use the following notations: N_e and N_p are the numbers of electrons and protons (positrons) in the bunches, σ_H and σ_V are the horizontal and vertical transverse sizes of the proton (positron) bunch, $\gamma_e = E_e/(m_e c^2)$, $\gamma_p = E_p/(m_p c^2)$ and $r_e = e^2/(m_e c^2)$ is the classical electron radius. This reaction is described by the diagram of Fig. 1 which corresponds to the radiation of the photon by the electron (the contribution of the photon radiation by the proton can be neglected). The large impact parameters $\varrho \gtrsim \sigma_\perp$, where $\sigma_\perp$ is the transverse beam size, correspond to small momentum transfer $\hbar q_\perp \sim (\hbar/\varrho) \lesssim (\hbar/\sigma_\perp)$. In this region, the given reaction can be represented as a Compton scattering of the equivalent photon, radiated by the proton, on the electron. The equivalent photons with frequency ω form a “disk” of radius $\varrho_m \sim \gamma_p c/\omega$ where $\gamma_p = E_p/(m_p c^2)$ is the Lorentz-factor of the proton.

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In the reference frame connected with the collider, the equivalent photon with energy $\hbar\omega$ and the electron with energy $E_e \gg \hbar\omega$ move toward each other and perform the Compton scattering. The characteristics of this process are well known. The main contribution to the Compton scattering is given by the region where the scattered photons fly in a direction opposite to that of the initial photons. For such a backward scattering, the energy of the equivalent photon $\hbar\omega$ for the typical emission angles $\theta_\gamma \lesssim 1/\gamma_e$ and the energy of the final photon E_γ is related by

$$\hbar\omega \sim \frac{E_\gamma}{4\gamma_e^2(1-E_\gamma/E_e)} \,. \tag{1}$$

As a result, we find the radius of the "disk" of equivalent photons with the frequency ω (corresponding to a final photon with energy E_γ) as follows:

$$\varrho_m = \frac{\gamma_p c}{\omega} \sim 4\,\lambda_e\,\gamma_e\gamma_p\,\frac{E_e - E_\gamma}{E_\gamma}\,, \quad \lambda_e = \frac{\hbar}{m_e c}\,. \tag{2}$$

Equation (2) is also valid for the $e^-e^+ \to e^-e^+\gamma$ process with replacement the protons by the positrons. For the KEKB and PEP-II colliders it leads to

$$\varrho_m \gtrsim 1 \text{ cm} \quad \text{for} \quad E_\gamma \lesssim 0.1 \text{ GeV} \,. \tag{3}$$

The standard calculation corresponds to the interaction of the photons (that form the "disk") with the unbounded flux of electrons. However, the particle beams at the HERA collider have finite transverse beam sizes of the order of $\sigma_\perp \sim 10^{-2}$ cm. Therefore, the equivalent photons from the region $\sigma_\perp \lesssim \varrho \lesssim \varrho_m$ cannot interact with the electrons from the other beam. This leads to the reduction of the number of the observed photons.

MD-EFFECT AND CORRELATIONS OF PARTICLES IN A BEAM

Correlations of particle coordinates in the beams are ignored in earlier papers [3, 4, 8], since usually these correlations are small. However, more accurate measurements may be sensitive to them. In the paper [14] we derive formulas which necessary to take into account quantitatively the effect of particle correlations in the spectrum of bremsstrahlung as well as in pair production. The corresponding additional term is determined by the correlation function for the density of particles in the beam. It should be mentioned that the same problem was considered in paper [15] using unrealistic assumptions. As a result, an application of formulas obtained in this paper to the HERA experiment is ungrounded.

MD-EFFECT FOR PEP-II AND KEKB

It was realized in last years that the MD-effect in bremsstrahlung plays an important role in the beam lifetime problem. At storage rings TRISTAN and LEP-I, the process of single bremsstrahlung was the dominant mechanism for the particle losses in beams. If an electron loses more than $1\ \%$ of its energy, it leaves the beam. Since the MD-effect considerably reduced the effective cross section of this process, the calculated beam lifetime in these storage rings was larger by about $25\ \%$ for TRISTAN [16] and by about $40\ \%$ for LEP-I [17] (in accordance with the experimental data) then without taken into account the MD-effect.

Usually in experiments the cross section is found as the ratio of the number of observed events per second $d\dot{N}$ to the luminosity L. Also, in our case it is convenient to introduce the "observed cross section", defined as the ratio

$$d\sigma_{\rm obs} = \frac{d\dot{N}}{L}\,. \tag{4}$$

Contrary to the standard cross section $d\sigma$, the observed cross section $d\sigma_{\rm obs}$ depends on the parameters of the colliding beams. To indicate explicitly this dependence we introduce the "correction cross section" $d\sigma_{\rm cor}$ as the difference between $d\sigma$ and $d\sigma_{\rm obs}$:

$$d\sigma_{\rm obs} = d\sigma - d\sigma_{\rm cor}\,. \tag{5}$$

The relative magnitude of the MD-effect is given, therefore, by the ratio

$$\delta = \frac{d\sigma_{\rm cor}}{d\sigma}\,. \tag{6}$$

Let us consider the number of photons emitted by electrons in the process $e^-e^+ \to e^-e^+\gamma$. The standard cross section for this process is well known. The correction cross section depends on the r.m.s. transverse horizontal and transverse vertical bunch sizes σ_{jH} and σ_{jV} for the electron, $j = e$, and positron, $j = p$, beams (for detail see [18]). In calculations we used data from Review of Particle Physics–2002 and 2006. Besides, for the KEKB collider we have to take into account that its e^+e^- beams of the length $l_e = l_p = 0.65$ cm collide to a crossing angle $2\psi = 22$ mrad. Formulas of the correction cross section for this case have been obtained in [9]. In the above notations the correction cross section is as follows:

$$d\sigma_{\rm cor}^{(e)} = \frac{16}{3}\alpha r_e^2 \frac{dy}{y}\left[\left(1 - y + \frac{3}{4}y^2\right)L_{\rm cor} - \frac{1-y}{12}\right] \tag{7}$$

where $y = E_\gamma/E_e$ and

$$\begin{aligned} L_{\rm cor} &= \ln\frac{2\sqrt{2}\gamma_e\gamma_p(1-y)(a_H+a_V)\lambda_e}{a_H a_V y} - \frac{3+C}{2}\,, \\ a_H &= \sqrt{\sigma_{eH}^2 + \sigma_{pH}^2 + (l_e^2 + l_p^2)\,\psi^2}\,, \\ a_V &= \sqrt{\sigma_{eV}^2 + \sigma_{pV}^2}\,, \quad C = 0.577\ldots\,. \end{aligned} \tag{8}$$

The observed number of photons is smaller due to MD-effect than the number of photons calculated without this effect. The relative magnitude of the MD-effect is given by quantity δ from Eq. (6) (see Table 1). It can be seen

Table 1: Relative magnitude of the MD-effect for different photon energies

$y = E_\gamma/E_e$	0.005	0.01	0.05	0.1	0.5
δ, % PEP-II	26	24	19	16	6.0
δ, % KEKB	29	26	21	18	8.9

from Table 1 that the MD-effect considerably reduces the differential cross section.

To estimate the integrated contribution of the discussed process into particle losses, we should integrate the differential observed cross section from some minimal photon energy. We take into account (as it is usually assumed) that an electron leaves the beam when it emits the photon with the energy 10 times larger than the beam energy spread. In other words, the relative photon energy should be $y = E_\gamma/E_e \geq y_{\min}$ where $y_{\min} = 0.0061$ for PEP-II and $y_{\min} = 0.007$ for KEKB. After integration of the differential observed cross section from $y_{\min} \ll 1$ up to $y_{\max} = 1$, we obtain

$$\sigma_{\rm obs}^{(e)} = \frac{16}{3}\alpha r_e^2 \left\{ \left(\ln\frac{1}{y_{\min}} - \frac{5}{8} \right) \times \left[\ln\frac{\sqrt{2} a_H a_V}{(a_H + a_V)\lambda_e} + \frac{2 + C}{2} \right] + \frac{1}{12}\left(\ln\frac{1}{y_{\min}} - 1 \right) \right\}, \quad (9)$$

this leads to $\sigma_{\rm obs}^{(e)} \approx 2.5 \cdot 10^{-25}$ cm^2 for the discussed colliders. Let us note that the standard cross section $\sigma^{(e)}$ integrated over the same interval of y, is larger than the observed cross section by about 20 % (see Table 2).

To understand the importance of the bremsstrahlung channel for particle losses, we estimate the corresponding partial beam lifetime. The number of particles, which the single electron bunch losses during a second, is equal to

$$\Delta\dot{N}_e = L\sigma_{\rm obs}^{(e)}/n_b \,, \quad (10)$$

where L is the luminosity and n_b is the number of bunches. Therefore, the partial lifetime of the electron bunch, corresponding to the bremsstrahlung process at a given luminosity, can be estimated as

$$\tau_{\rm brem}^e = \frac{N_e}{\Delta\dot{N}_e} = \frac{N_e n_b}{L\sigma_{\rm obs}^{(e)}} \,. \quad (11)$$

The obtained numbers for the electron and positron beams are presented in Table 2 (date from Review of Particle Physics–2002/2006). They can be compared with the luminosity lifetime $\tau_L = 2.5$ h for the PEP-II and $\tau_L = 3.4$ h for the KEKB (from RPP-2002), which is some average characteristic of lifetimes for both beams. More detailed comparison with the experimental numbers for lifetimes of beams at KEKB shows that the bremsstrahlung process is important for the electron beam lifetime, but has rather small influence on the positron beam lifetime.

Table 2: Integrated contribution of the MD-effect

	$\sigma/\sigma_{\rm obs}$	$\tau_{\rm brem}^e$, hr	$\tau_{\rm brem}^p$, hr
PEP-II	1.20	4/8.7	12/44
KEKB	1.23	8.9/6	14/13

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