OPTIMAL BEAMLINES FOR BEAMS WITH SPACE CHARGE EFFECT

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Abstract

Space charge effect is ever of fundamental importance for low-energy parts of accelerators. Simple and robust estimations of the emittance degradation in various space charge affected beamlines have been obtained analytically and numerically. Nonuniform longitudinal and transverse distributions of current, accelerating and bunching were taken into account. The parameters of optimal beamlines for space charge affected beams have been estimated.

INTRODUCTION

Any machine starts from an injector where space charge effect is significant and causes emittance degradation. Then an important goal is to minimize this effect and to find the optimal parameters of an injector. A technique known as "emittance compensation" was put forward in [1] and substantially formalized in [2]. We shall use a similar approach.

First of all, one should isolate a part of a machine where the space charge effect is significant. We shall ever use Kapchinsky-Vladimirsky equation [3] as a basis (for rms-values here):

$$\begin{cases} x'' = \dfrac{\varepsilon_x^2}{x^3} + \dfrac{I}{I_0(\beta\gamma)^3}\dfrac{1}{x+y} - \dfrac{e}{p}G_x x, \\ y'' = \dfrac{\varepsilon_y^2}{y^3} + \dfrac{I}{I_0(\beta\gamma)^3}\dfrac{1}{x+y} - \dfrac{e}{p}G_y y, \end{cases} \tag{1}$$

where x and y mean horizontal and vertical coordinates respectively; $''$ means second derivative by independent coordinate z; ε, I, G, e and p are emittance, current, focusing gradient particle charge and its longitudinal momentum respectively. Two criteria of validity of the chargeless model can be derived then [4]

$$\begin{gathered} \frac{I}{I_0(\beta\gamma)^3}\frac{1}{x+y} << \frac{\varepsilon_{x\text{ or }y}^2}{(x\text{ or }y)^3}, \\ \int_L \sqrt{\frac{I}{I_0(\beta\gamma)^3}\frac{1}{(x+y)(x\text{ or }y)}}\,dz << 1, \end{gathered} \tag{2}$$

where L is the total length of the beamline.

Effects of transverse, longitudinal and combined charge inhomogeneity in uniform and nonuniform beamlines in the view of emittance compensation were considered in [5]. It was found that emittance degradation in an optimal beamline can be ever estimated as

$$\varepsilon \cong \xi n r \sqrt{\frac{I}{I_0(\beta\gamma)^3}} \tag{3}$$

where I is the peak current; r is the rms beam size; $n = \varphi/2\pi$, the number of periods of charge oscillations; and ξ is the coefficient depending on the beamline and charge distribution. The effect of longitudinal inhomogeneity is stronger than of transverse one. Half-integer n give worse result than integer ones. It was found also that the effects in uniform and nonuniform beamlines are almost equal.

BASICS

Consider a bunch as a set of round, uniformly charged, emittanceless, and independently moving slices, as in [2]. Beamlines are ever consider circular symmetric. If the bunch moves through a uniform beamline matched to its slice carrying maximum current, that is

$$\frac{I_{\max}}{I_0(\beta\gamma)^3}\frac{1}{2x^2} = \frac{e}{p}G, \tag{4}$$

the phase portraits of the slices spread as in Fig. 1.

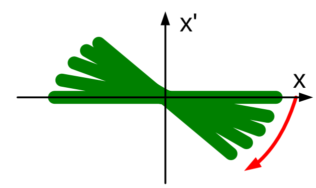

Figure 1: Phase portraits of slices.

If the current density depends on the radius, the transverse force and the motion through this beamline looks as in Fig. 2.

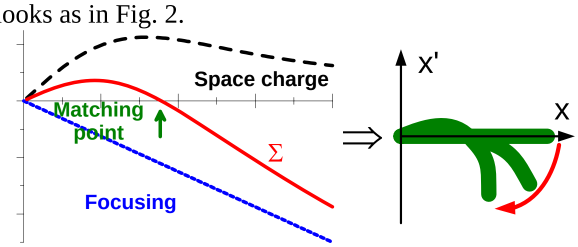

Figure 2: Phase portrait of a slice.

A slice is regarded then as a set of charged rings. Two important assumptions are to be added to consider successfully the effect of transverse inhomogeneity and the combined effect:

- the motion is laminar, that is the ring trajectories don't cross;
- the effects are independent.

Now consider an arbitrary slice with a given start state, carrying current I_p and moving through a given beamline. Name its phase trajectory "principal". Then the principal trajectories of other slices are proportional to it with the coefficients $\sqrt{I/I_p}$. Consider first the motion of a slice as a small perturbation of the principle trajectory. Analogous arguments can be produced for rings. Thus the linearized equation can be derived:

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$$\delta'' + \left(2\frac{x'}{x} + \frac{(\beta\gamma)'}{\beta\gamma} \right)\delta' = \begin{cases} -\dfrac{j}{x^2}\delta : \text{longitud.}, \\ -\dfrac{4\tilde{j}}{x^2}\delta : \text{transverse.} \end{cases} \quad (5)$$

where x is the principal trajectory, $\delta = \delta x/x$ is its dimensionless perturbation, and $j = I/I_0(\beta\gamma)^3$. $\tilde{j}$ means the current within the given ring. In the case of transverse inhomogeneity, x ever means the ring radius. Phase portraits of slices or rings are aligned if $\delta' = 0$.

BUNCHING

Slice current is being changed through a beamline while bunching. Using results of [5] we need to estimate only one, most convenient, type of beamlines to obtain a result valid for others. Our beamline is matched to one of the slices, so that $g \equiv eG/p = j/2x^2$ and principal trajectories are uniform; similar for the rings. As the longitudinal momentum is also conserved, the basic equation is then

$$\delta'' = -\frac{j \text{ or } 4\tilde{j}}{x^2}\delta. \quad (6)$$

Its Hamiltonian is

$$H = \frac{p^2}{2} + \frac{j(z) \text{ or } 4\tilde{j}(z)}{2x^2}\delta^2, p \equiv \delta' \quad (7)$$

Adiabatic damping of the relative amplitude $a \propto j(z)^{-1/4}$ occurs while bunching if the condition $g'/g^{3/2} << 1$ is met. If the linear charge phase advance through a beamline is 2π, its nonlinear correction is

$$\Delta\varphi \approx \frac{\pi}{4}a_0^2\upsilon^{-1/4}, \quad (8)$$

where a_0 is the initial relative amplitude of charge oscillations and $\upsilon = j_{end}/j_{start}$, the bunching coefficient. Combining the above formulae one obtains

$$\varepsilon \propto r' \propto \Delta\varphi a\sqrt{j} \propto \upsilon^{-1/4}\upsilon^{-1/4}\upsilon^{1/2} = const(\upsilon), \quad (9)$$

As the adiabatic condition is violated quite often, the conclusion should be checked numerically. Gaussian distribution of current is ever considered. All the slices start with x = 1 and $x' = 0$ and move as consistent with a nonlinear equation (10) derived from (1):

$$x'' = j/2x - gx. \quad (10)$$

j and g increase synchronously as $\exp(\ln\upsilon \cdot z/L)$. In the case of longitudinal inhomogeneity, the results are placed in Fig. 3. Charge phase advance in an optimal beamline

$$\varphi = \int_0^L \frac{\sqrt{j}}{x}dz = \int_0^L \sqrt{2g}\,dz \quad (11)$$

ever occurs $\approx 2\pi$. Optimum was searched within a rectangle $g < 5$ and $1.5 < \varphi < 12$. Optimal focusing (the starting value) fluctuates within 0.08 and 0.11.

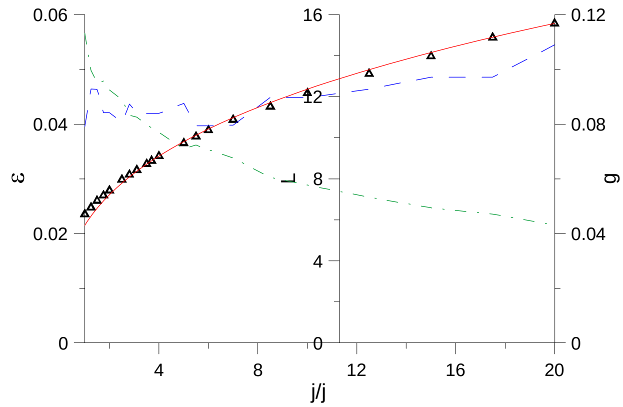

Figure 3: Effect of longitudinal inhomogeneity: minimum emittance (triangles; red solid line – $0.0215 \cdot \upsilon^{1/3}$), optimal focusing (blue dashed line), and optimal length (green dash-dot line), $\approx 15.7 \cdot \upsilon^{-1/3}$.

Longitudinal inhomogeneity ever exists while bunching, so there is no sense to estimate the effect of transverse inhomogeneity separately. The parameters of optimal beamlines in the case of the combined effect are placed in Fig. 4. Each slice was divided into a set of charged rings. Each ring started with $x' = 0$ and moved through a beamline as consistent with a nonlinear equation

$$x'' = 2\tilde{j}/x - gx. \quad (12)$$

$\tilde{j}$ and g increased synchronously as $\exp(\ln\upsilon \cdot z/L)$. Totally Gaussian bunches were considered. Optimum was searched within the same limits as above. Charge phase advance in an optimal beamline ever occurs $\approx 2\pi$. Optimal focusing (the starting value) fluctuates within 0.10 and 0.14.

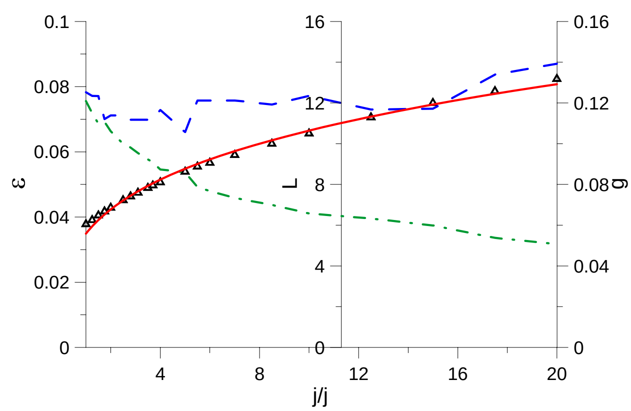

Figure 4: Effect of combined inhomogeneity: minimum emittance (triangles; red solid line – $0.0349 \cdot \upsilon^{0.28}$), optimal focusing (blue dashed line), and optimal length (green dash-dot line), $\approx 12.7 \cdot \upsilon^{-0.28}$.

ACCELERATING

In this case $\beta\gamma$ is not preserved, so the basic equation is

$$\delta'' + \frac{(\beta\gamma)'}{\beta\gamma}\delta' = -\frac{j \text{ or } 4\tilde{j}(z)}{x^2}\delta. \quad (13)$$

Its Hamiltonian is

$$H = \frac{p^2}{2\beta\gamma} + \frac{\beta\gamma j}{2x^2}\delta^2,\ p = \beta\gamma\delta'. \qquad (14)$$

Adiabatic damping (rather rise) $a \propto (\beta\gamma)^{1/4}$ occurs while accelerating if the condition $x(\beta\gamma)'/\beta\gamma\sqrt{j} << 1$ is met. If the linear charge phase advance through a beamline is constant, its nonlinear correction is

$$\Delta\varphi \propto \frac{\pi + \sqrt{\ln\alpha}}{4}, \qquad (15)$$

where $\alpha = (\beta\gamma)_1/(\beta\gamma)_0$, the accelerating coefficient. Combining the above formulae one obtains

$$\varepsilon_n \propto r'\beta\gamma \propto \Delta\varphi a\sqrt{j}\beta\gamma \propto \alpha^{1/4}\alpha^{-3/2}\alpha = \alpha^{-1/4}. \qquad (16)$$

Normalized emittance is estimated here.

Of course, the adiabatic condition is violated quite often, so the estimate should be checked numerically. In the case of pure longitudinal inhomogeneity, equation (10) with initial conditions x = 1 and x' = 0 was solved for all the slices through a beamline. Longitudinal momentum was increased linearly $\beta\gamma \propto 1 + (\alpha - 1)z/L$, and focusing $g \propto (\beta\gamma)^{-3}$. The results are placed in Fig. 5.

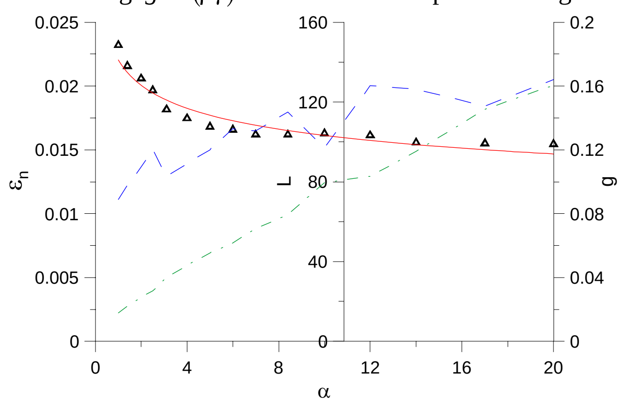

Figure 5: Effect of longitudinal inhomogeneity: minimum normalized emittance (triangles; red solid line – $0.0220{\cdot}\alpha^{-0.136}$), optimal focusing (blue dashed line), and optimal length (green dash-dot line), ≈ 11.96 + 6.05α.

Longitudinal charge distribution was Gaussian. Optimum was searched within a rectangle $g < 5$ and $1.5 < \varphi < 12$. Charge phase advance in an optimal beamline ever occurs $\approx 2\pi$. Optimal focusing varies within 0.1 and 0.16.

Combined effect in accelerating beamlines was also estimated. The scheme was similar to the one for bunching beamlines. The difference is that βγ was varied instead of *j*, as above. The results are placed in Fig. 6.

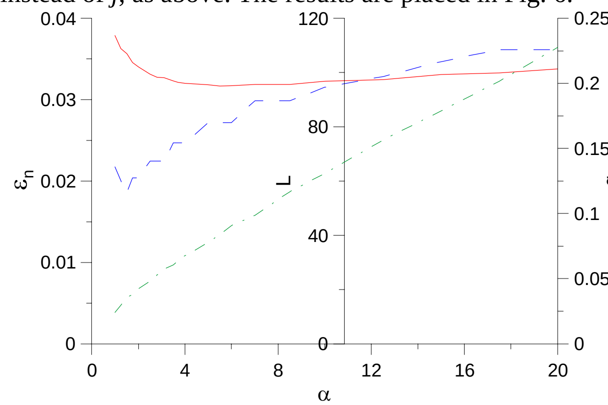

Figure 6: Effect of combined inhomogeneity: minimum normalized emittance (red solid line), optimal focusing (blue dashed line) $\approx 0.115{\cdot}\alpha^{0.227}$, and optimal length (green dash-dot line), ≈ 10.89 + 5.03α.

SUMMARY

Thus, one can estimate the emittance degradation and the parameters of the optimal beamline as:

$$\varepsilon_n \cong \xi r\sqrt{\frac{I}{I_0\beta\gamma}}, \qquad (17)$$

$$G \cong \varsigma\frac{p}{ex^2}\frac{I}{I_0(\beta\gamma)^3}, \qquad (18)$$

$$L \cong \lambda x\sqrt{\frac{I_0(\beta\gamma)^3}{I}}, \qquad (19)$$

where all the parameters belong to the beginning of a beamline. ξ, ς, and λ for the situations considered are placed in Table 1.

Table 1: Parameters of optimal beamline

Beamline	Beam	ξ	ς	λ
Bunching, $\upsilon = j_{end} / j_{start}$	Transverse : uniform; longitudinal : Gaussian	$0.0215{\cdot}\upsilon^{1/3}$	0.08…0.11	$15.7{\cdot}\upsilon^{-1/3}$
	Totally Gaussian	$0.0349{\cdot}\upsilon^{0.28}$	0.1…0.14	$12.7{\cdot}\upsilon^{-0.28}$
Accelerating, $\alpha = (\beta\gamma)_1/(\beta\gamma)_0$	Transverse : uniform; longitudinal : Gaussian	$0.0220{\cdot}\alpha^{-0.136}$	0.1…0.16	11.96 + 6.05α
	Totally Gaussian	0.033	$0.115{\cdot}\alpha^{0.227}$	10.89 + 5.03α

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