NUMERICAL OPTIMIZATION OF A PLASMA WAKEFIELD ACCELERATION EXPERIMENT*

K. V. Lotov and V. S. Tikhanovich, Budker INP, 630090, Novosibirsk, Russia

Abstract

One possible way to demonstrate both the efficiency and beam quality in a plasma wakefield accelerator is to prepare matched drive and accelerated beams by removing a central slice from a single high-quality electron bunch (parent beam). For parameters of the parent beam given, the question arises how to maximize the number and energy of accelerated particles and minimize their energy spread and emittance. This question is addressed by numerical simulations. The optimum shape of the beams, required plasma length, achievable energy gain and energy spread are found as functions of the plasma density and parent beam characteristics. The required control accuracy of adjustable beam and plasma parameters is determined.

INTRODUCTION

Since mid-1980th, the electron beam-driven plasma wakefield acceleration (PWFA) is actively studied as a possible way to future high-energy linear colliders [1, 2]. Already demonstrated are good agreement between theoretical predictions and experimental observations, high acceleration rate and high energy gain [3]. High efficiency of beam-to-beam energy transfer and high quality of the accelerated beam (witness) are not experimentally proven yet; this should be the next major step toward a competitive plasma-based accelerator.

One possible way to achieve both the efficiency and beam quality is to prepare matched drive and accelerated beams by removing a central slice from a single high-quality electron bunch (parent beam, Fig. 1). Given the parameters of the parent beam, the question arises how to maximize the number and energy of accelerated particles and minimize their energy spread and emittance. Other questions are what characteristics of the witness can be achieved with available parent beams and to what accuracy the adjustable parameters of the system must be held. These questions are addressed in the paper.

It is important to emphasize the degree of generality of the obtained results. As a reference point for optimization, we take the design parameters of PWFA experiment on the VEPP-5 injection complex [4, 5]. The numerical values obtained are therefore applicable to this experimental project only. The dimensionless relations are more general and valid for any experiment where the parent beam has a Gaussian-like shape. The scalings and order-of-magnitude estimates for accuracies are fully general and applicable wherever the driver and accelerated beam are cut from a single beam.

* Work supported by RFBR grants 03-02-16160, 05-02-16775, 06-02-16757, and SB RAS Lavrent'ev grant for young researchers.

OPTIMIZATION PROBLEM

There are two groups of input parameters in our problem. Parameters from the first group are stringently determined by the beamline, cannot be easily varied, and are not a subject of optimization. We take them close to the design beam parameters of Novosibirsk PWFA experiment [4, 5]: initial energy $W_0 = \gamma_0 mc^2 = 510$ MeV, number of particles in the parent beam $N_p = 2 \cdot 10^{10}$, normalized rms emittance $\epsilon_0 = 20$ mm mrad, initial energy spread δW_0 is negligibly low, minimum possible length $\sigma_{z,min} = 0.1$ mm, minimum possible radius $\sigma_{r,min} = 20\,\mu$m, cut-out sharpness $\delta z = 0.1$ mm.

Parameters from the second group can be adjusted relatively easily. They are the compressed beam length σ_z, projections of chopper edges onto the compressed beam z_1 and z_2, beam radius σ_r, plasma density n_0, and plasma length L_p. The optimum values are to be found with the constraints $\sigma_z \geq \sigma_{z,min}$, $\sigma_r \geq \sigma_{r,min}$, and $z_1 > z_2$.

Assume the parent beam at the entrance to the plasma is axisymmetric with the density distribution

$$n_{pb} = \frac{N_p\, e^{-r^2/2\sigma_r^2}}{2\sigma_r^2 \sigma_z\, (2\pi)^{3/2}} \left[1 + \cos\left(\sqrt{\frac{\pi}{2}}\frac{z}{\sigma_z}\right)\right] \qquad (1)$$

at $|z| < \sigma_z\sqrt{2\pi}$ if no slices are cut out. With the cutting on, the initial beam density is

$$n_b(r,z) = n_{pb}\big(g(z,z_2) + 1 - g(z,z_1)\big), \qquad (2)$$

where the function $g(z, z_i)$ describes a smooth cutout edge:

$$g(z,z_i) = \begin{cases} 1, & z < z_i - \delta z; \\ 0.5\left(1 - \sin\dfrac{\pi(z - z_i)}{2\,\delta z}\right), & |z - z_i| < \delta z; \\ 0, & z > z_i + \delta z. \end{cases}$$

The geometry of the cut beam is illustrated by Figure 1.

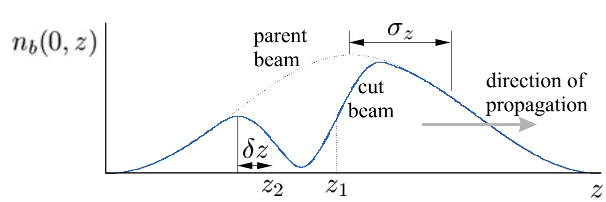

Figure 1: Beam shape at the entrance to the plasma.

Dynamics of thus defined beam in the uniform unmagnetized plasma of the length L_p and density n_0 is followed numerically with LCODE code [6, 7]. The output of the code is a 6-space distribution of beam macroparticles at the exit from the plasma. From these data, we find four goal parameters: the number of accelerated particles N_w, their average energy gain $\triangle W$, rms normalized emittance ϵ_w, and energy spread δW.

For realization of the optimization algorithm, it is necessary to combine the goal parameters into a single-valued criterion function F. The choice of the criterion function determines the convergence speed and, in principle, can strongly affect the result of optimization procedure. Fortunately, in the problem considered, there exists an acceleration regime for which all goal parameters are simultaneously good and there is no trade-off between, for example, energy gain and number of particles. Thus, the result weakly depends on the particular criterion function if the latter is reasonably chosen. Among several criterion functions tested, the function

$$F = \ln(N_w/N_p) + \ln(\triangle W/W_0) + \frac{1}{1+\left(\frac{\epsilon_w}{2\epsilon_0}\right)^2} + \frac{1}{1+\left(\frac{\delta W}{0.1\,\triangle W}\right)^2} \qquad (3)$$

provides the best convergence.

The above procedure defines the function $F(\sigma_r, \sigma_z, z_1, z_2, n_0, L_p)$ that has to be maximized in the 6-space of adjustable parameters. The function is noisy; a small variation of any argument can result in a large change of the value. This is because beam particles make about 10^2 betatron oscillations in a typical plasma length, and a very small variation of run parameters is sufficient to change the oscillation phase at the exit. Because of the noise, the search of maxima is best done with a simple step-by-step algorithm. The search starts from some random point and moves alternately in each coordinate until a local maximum is found with a given precision. Typically, one search takes about 200 code runs and 50 hours at Pentium-IV.

Because of the noise, the search started from different points results in different local maxima. The local maxima found are not uniformly distributed in the parameter space; they concentrate in regions of high F thus showing not only the location of optimum regimes in the parameter space but also the width of this regimes. Local maxima with low values of F can be easily culled.

OPTIMUM REGIME

The main result of the above optimization is that the optimum parameters form a curve in the 6-space of adjustable parameters. The curve has one free parameter; let it be the plasma density n_0. Other adjustable parameters are thus determined by n_0:

$$\sigma_z \approx 2.15\, c/\omega_p, \qquad \sigma_r = \sigma_{r,min}, \qquad (4)$$

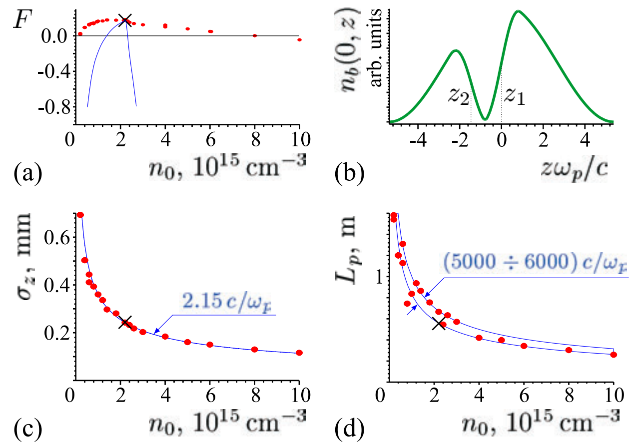

Figure 2: (a) Values of the criterion function on the optimum curve (dots) and on the line $(\sigma_r, \sigma_z, z_1, z_2, L_p) = \text{const}$ (line); (b) optimum beam shape; (c) optimum beam length; and (d) optimum plasma length. Dots show the local maxima obtained numerically.

$$z_1 \approx 0, \qquad z_2 \approx -\frac{\pi}{2}\, c/\omega_p, \qquad L_p \approx A\gamma_0 c/\omega_p, \qquad (5)$$

where the factor A depends on the parent beam population. In our case $A \approx 5$. Figure 2 illustrates these statements. From Figure 2a we can see that though $F \neq$ const on the optimum curve, this inconstancy is small compared to decrease of F as its arguments deviate from the curve.

It is instructive to look at evolution of main witness parameters in the plasma (Fig. 3). In this section, we use the input parameter set marked by crosses in Figure 2 and apply no halo filtering criteria. We see that the beam evolution occurs in three stages. After escape of the beam tail, the rest of the witness stably propagates in the plasma until the driver get considerably depleted. Then the driver widens, the wakefield structure changes, and the witness get defocused and lost. The end of the second stage corresponds to the optimum plasma length. The increase of emittance during the second stage is a numerical effect that disappears at shorter time steps, so it is a price for fast computing.

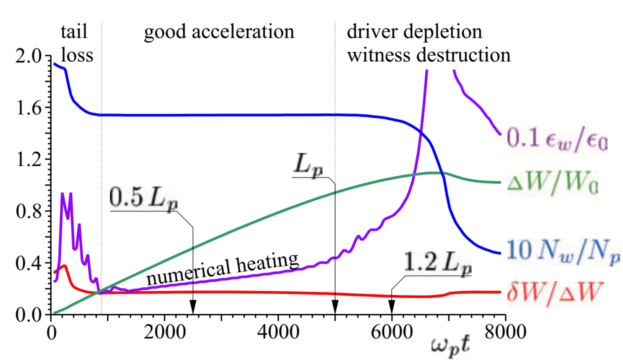

Figure 3: Evolution of witness emittance ϵ_w, energy gain $\triangle W$, population N_w, and energy spread δW as it propagates in the plasma.

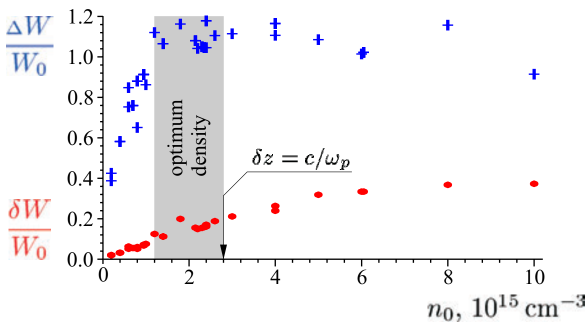

Figure 4: Dependence of the witness energy gain ΔW (crosses) and energy spread δW (ovals) on the plasma density.

The resulting energy spectrum depends on the plasma density (Fig. 4). The witness energy gain is limited to about the initial driver energy. At high plasma densities, the energy gain is high, but the energy spread is large because of the limited beam shape control. Namely, as the density increases, the width of the cut-out interval gets smaller than the cut-out sharpness δz, and the witness energy spread increases. In units of $eE_0 \equiv mc\omega_p$, the average accelerating rate is constant in a wide interval of plasma densities and equals A^{-1}. Together with the expression (5) for L_p, this suggests that the acceleration distance here is determined by the driver depletion. At low plasma densities, the acceleration length is limited by emittance-driven driver erosion, much of the energy remains with the driver (Fig. 4), and the witness energy gain is small. Thus, the optimum plasma density is the minimum one at which the acceleration length is determined by driver depletion ($n_0 \approx 2.2 \cdot 10^{15}$ cm^{-3} in our case).

The utilization efficiency of the driver energy is shown in Figure 5. Since the deceleration field is not uniform within the beam, about 50% of the energy is left with the driver. Extra 20-30% are left in the plasma, so the overall driver-to-witness efficiency is at 30% level in wide interval of plasma densities.

Note that the beam-plasma interaction in the optimum regime has most of the features of the efficient acceleration mode described in Ref.[8]. Namely, both drive and witness beams propagate in the blowout regime [9] in the electron-free cavern, the beams are shaped to flatten decelerating or accelerating field inside of them, and the length of the

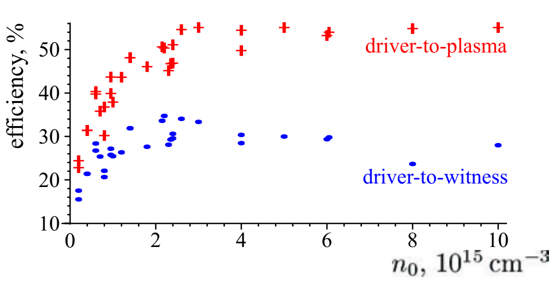

Figure 5: Efficiency of driver-to-plasma (crosses) and driver-to-witness (ovals) energy transfer.

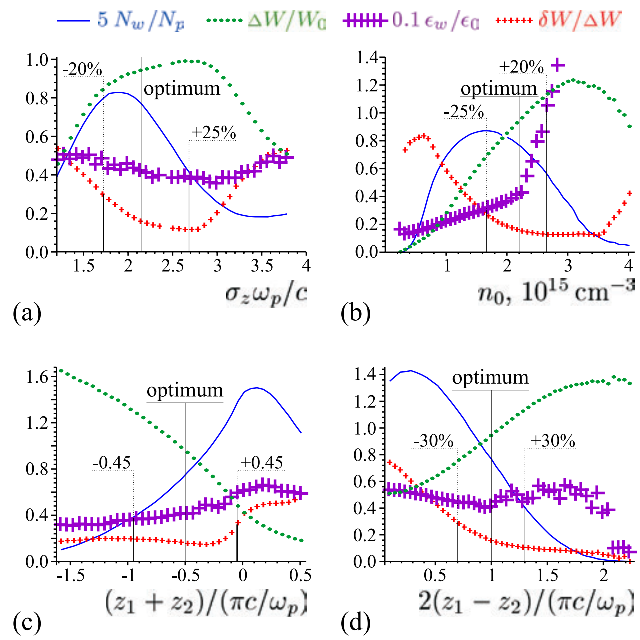

Figure 6: Variation of the witness population N_w, energy gain ΔW, emittance ϵ_w, and energy spread δW as (a) the parent beam length, (b) plasma density, (c) cut-out location, and (d) cut-out width deviate from the optimum values.

driver is about one wakefield half-wavelength. The only missed feature is the beam current, relatively low value of which limits the efficiency at 30% level.

Once the optimum set of adjustable parameters is found, the question arises of how precisely this values must be controlled. To answer this question, we follow the variation of main witness parameters as one of adjustable parameters of the system deviate from the optimum value (marked by crosses in Fig. 2). The results are illustrated by Figures 6 and 3. The tolerances listed correspond to twofold increase/decrease of any minimized/maximized witness parameter (marked by continuous lines in Fig. 6). We see that most of adjustable parameters of the system need to be controlled with 20-30% accuracy, and the most sensitive parameter is the plasma density. Note also that over-length plasma in much worse than an under-length one.

REFERENCES

[1] E. Esarey, et al., IEEE Trans. Plasma Sci. **24** (1996) 252.

[2] C. Joshi, et al., Phys. Plasmas **9** (2002) 1845.

[3] M. J. Hogan, et al., Phys. Rev. Lett. **95** (2005) 054802.

[4] A.V.Burdakov, et al., Plasma Physics Reports **31** (2005) 292.

[5] A. V. Petrenko, et al., “Experimental plasma wake-field acceleration project at the VEPP-5 injection complex,” EPAC’04, July 2004, Lucerne, p. 740, http://www.JACoW.org.

[6] K. V. Lotov, Phys. Rev. ST Accel. Beams **6** (2003) 061301.

[7] K. V. Lotov, Phys. Plasmas **5** (1998) 785.

[8] K. V. Lotov, Phys. Plasmas **12** (2005) 053105.

[9] J. B. Rosenzweig, et al., Phys. Rev. A **44** (1991) 6189.