# RACE-TRACK MICROTRON ON 50 MEV WITH SMALL NUMBER OF ORBITS 

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#### Abstract

The description of the mathematical model of the racetrack microtron on 50 MeV is represented. One of the features of the accelerator is that the accelerating section of the microtron has large transversal size (e.g. a cryostat of superconducting section). For detour of this section on the first revolution frequency rate of the acceleration is enlarged up to four. Other feature is application of the injection directly into an accelerating section at low energy of the beam. Some characteristics of the represented microtron are: increase of the energy of the beam per revolution -10 MeV , number of revolutions -5 , energy of the injection - $(0.8-1) \mathrm{MeV}$. The carried out modeling has shown, that phase-energy area, from which the particles are captured in a mode of acceleration, allows to receive on the exit the intensive accelerated beam. Transversal focusing of the beam is carried out by the poles creating the field of an opposite direction along the gap of the bending magnets. Also before an input into the accelerating section and after it the single quadruples are put. The similar system of the focusing provides transversal stability of the beam for all orbits.


## INTRODUCTION

One of directions of development the accelerator techniques is creation of the economic high-current accelerators of continuous action with the energy in a
range (30-200) MeV , the average current of the beam $(10-100) \mu \mathrm{A}$, with an energy dispersion about $5 \cdot 10^{-3}$ and with small transversal emittance of the beam (about $\pi \cdot 10^{-4} \mathrm{~cm} \cdot \mathrm{rad}$ ). These requirements are answered most full with electron accelerators of type a race-track microtron with the superconducting accelerating section.
Limitation for application of the superconducting accelerating section in the race-track microtron is rather big transversal sizes of the cryostat containing accelerating structure, and put obstacles in the way of the first orbits. In the present work the variant of a race-track microtron on 50 MeV with the superconducting section giving increase of the energy up to 10 MeV with frequency of 1500 MHz is considered. For the purpose of detour of the accelerating section on the first revolution the multiplicity of acceleration is enlarged up to four. For obtaining the necessary output energy is enough five orbits. The similar accelerator can be used as independent accelerator, and also as the injector in the cascade scheme.
Unobstructed passing of the beam on orbits is provided with application of focusing with the help of the reverse field, created poles, placed along edges of the magnets [1]. Also after an exit and before an input into an accelerating section the magnetic single quadruples are put. The scheme of the accelerator is shown in a Fig. 1.
Mathematical modeling was carried out by programs MathCAD 7 and POISSON. The technique of calculations is detailed in the article [2].


Figure 1: The scheme of the race-track microtron. 1 - The pole of the bending magnet. 2 - The pole of the reverse field magnet. 3 - Single quadruples. 4 - The cryostat with accelerating section.

## Parameters of a Microtron:

Frequency of accelerating voltage $\ldots \mathrm{f}=1500 \cdot 10^{6} \mathrm{~Hz}$.
Wavelength of accelerating voltage $\ldots \lambda \mathrm{g}=19.993 \mathrm{~cm}$. Increase of circulation time of the particles after passing magnets.. $\Delta \mathrm{T}_{\text {pass }}=4 \mathrm{~T}_{\text {gen }}$. Equilibrium increase of energy $\ldots \ldots . . \Delta \mathrm{Es}=10.0 \mathrm{MeV}$. Induction of magnetic field ............ B = 2695 Gs (*). Total length of the drift sites ........... Ldr = 226.5 cm (*) $^{*}$. Energy of injection (complete) axial particles $\qquad$ $\mathrm{Eo}=0.875 \mathrm{MeV}(*)$.
$\left(^{*}\right)$ - values at which the optimal mode of acceleration is provided are marked.

## MODELING OF PHASE MOVEMENT

At modeling acceleration in the race-track microtron, as accelerating section the wave-guide with evenly distributed cells is considered (Fig. 2).


Figure 2: The scheme of superconducting accelerating section.

The equations describing movement of electrons in accelerating section look like:

$$
\begin{align*}
& \frac{\mathrm{dE}}{\mathrm{ds}}=\mathrm{U}_{0} \cdot \sin \left(\frac{2 \pi}{\lambda \mathrm{~g}} \cdot \mathrm{~s}\right) \cdot \cos (\varphi) \\
& \frac{\mathrm{d} \varphi}{\mathrm{ds}}=\frac{2 \pi}{\lambda \mathrm{~g}} \cdot \frac{\mathrm{E}}{\sqrt{\mathrm{E}^{2}-\mathrm{E}_{0}^{2}}} \tag{1}
\end{align*}
$$

The system (1) was integrated by the numerical method. Entry conditions for the first orbit were chosen from a range of energies: $E 0_{\text {in }}=(0.81-1.0) \mathrm{MeV}$ with step $\Delta \mathrm{E}_{\text {in }}=0.0065 \mathrm{MeV}$ and phases: $\varphi 0_{\text {in }}=-(2.21-1.95) \mathrm{rad}$. with step $\Delta \varphi 0_{\text {in }}=0.0027 \mathrm{rad}$.

Output values of energy and phase for each orbit after integration were written in the matrix and on the received data isolines of output energies were plot depending on entry conditions (Fig. 3).
If in the accelerator the bunched beam getting in area, allocated on the figure 3 is injected, with energy in a range ( $0.92-0.82$ ) MeV and phase extent of 0.1 rad , energy and phase then dispersions of output energy of the beam of the fifth orbit will be equal ( $50 \pm 0.25$ ). MeV and 0.25 rad .

## FOCUSSING OF THE BEAM BY RECIRCULATION SYSTEM

The path of recirculation for all orbits where particles move without change of energy, allocates between the points C and D (see. Fig. 1) also include sequentially: the entry quadruple - the entry bending magnet - the drift path - the output bending magnet - the output quadruple. The complete matrix of transition of path of recirculation is calculated by multiplication of the matrixes of separate parts of the paths. During recirculation the beam in all orbits passes sequentially the same recirculation system. Therefore, for organization of steady movement of the accelerated particles research by the differential equations Hill's with periodic coefficients is applicable [3].
At steady movement, after passing path of recirculation, the parameters of the output transversal emittances will be equal to entry them. The matrix of transition for one period of the recirculation path (Twiss matrix) will be:

$$
M c=\left(\begin{array}{cc}
{\left[\cos \mu+\alpha_{0} \sin \mu\right]} & \beta_{0} \sin \mu  \tag{2}\\
-\frac{1+\alpha_{0}^{2}}{\beta_{0}} \sin \mu & {\left[\cos \mu-\alpha_{0} \sin \mu\right]}
\end{array}\right)
$$

The condition of stability calculated as a half-sum of the units on the main diagonal of the matrix (2) is defined as:

$$
\begin{equation*}
|\cos \mu|<1 \tag{3}
\end{equation*}
$$



Figure 3: The isolines of output energies for last 5-th orbit.

Units of the matrix (2) are expressed through parameters of phase ellipse: $\alpha$ - characterizes an inclination of the ellipse, $\beta$ determines envelope of the beam.
For finding parameters of entry and output emittances the real matrix of transition is calculated and by matching its units with units of the matrix (2) parameters entry (output) emittances are calculated.

In the median plane the transition matrix of recirculation path will consist of the sequential matrix product: the entry focusing quadruple, the entry bending magnet, the drift path, the output bending magnet and the output focusing quadrupole.


Figure 4: The matched horizontal emittances of the beam for each orbit.

The matrix of the magnet is calculated with help of two independent quotient solutions by numerical integration of the equations of movement for the median plane. For transition in normal coordinate system the axial trajectory is calculated also. In Fig. 4 the emittances of all orbits for horizontal movement in the median plane are represented.
Ellipses of the emittances have a canonical form ( $\alpha_{0}=0$ ) with semiaxes: $\mathrm{A}_{0}^{\prime}=\left[\varepsilon\left(1+\alpha^{2}\right) / \beta\right]^{1 / 2}$ and $\mathrm{B}_{0}=(\varepsilon \beta)^{-1 / 2}$; $\varepsilon=\pi \mathrm{A}^{\prime}{ }_{0} \mathrm{~B}_{0}-$ emittance of the beam.

The matrix of transition for vertical movement will consist of the same units, as horizontal. But in a vertical direction the quadruple and the bending magnet operate on a beam differently. The quadrupole will defocus thebeam, the bending magnet can both to focusing and to defocusing, depending on the value of the reverse field. Results of calculation of the vertical matched emittances are shown in Fig. 5. The entry and output emittances also represent canonical ellipses with semiaxes $\mathrm{A}_{0}^{\prime}$ and $\mathrm{B}_{0}$.

## CONCLUSION

With the help of the created mathematical model research of the race-track microtron with five orbits and with multiplicity to four is carried out. It is shown, that


Figure 5: The matched vertical emittances of the beam for each orbit.
particles can be injected in accelerating section at low energy . ( $0.8-1.0$ ) MeV.
At development of technique of creation of transversal stability of the beam the method of research of the equations of movement with periodic coefficients is used.

Application for focusing a reverse field and single quadrupoles has allowed, using the indicated method, to find regimens at which the beam steadily moves on all orbits.

The procedure of the research developed in work [2] and applied for accelerator with small number of orbits can be used for calculations of various variants of the accelerator of type a race-track microtron and recirculation accelerators.
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