IMPEDANCE ESTIMATION FOR THE ALBA STORAGE RING

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Abstract

In the framework of the Spanish Light Source CELLS project [1], an analysis of coupling impedance of the ALBA storage ring has been carried out. Broad-band impedance has been evaluated using the MAFIA code and analytical formulae, for various components of the ALBA vacuum chamber, such as cross-section transitions, bellows, button-electrodes, strip-lines, high-order modes of RF cavities, etc.

IMPEDANCE AND INSTABILITIES

An intensive particle beam moving in a vacuum chamber induces quite strong electromagnetic fields affecting the beam itself. The most significant results of collective effects are various instabilities of beam motion, which can lead to beam losses or beam quality deterioration.

To describe the beam-environment interaction, each part of a vacuum chamber can be represented as frequency-depended coupling impedance. The real (resistive) part of the impedance results in energy loss of beam particles, the imaginary (reactive) one produces a phase shift of particles oscillation .

For almost all cases, impedance of a vacuum chamber section can be approximated by a resonant RLC-circuit for each oscillation mode:

$$Z(\omega) = \frac{R_s}{1 + iQ\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)}\,, \tag{1}$$

where R_s is the shunt resistance of the longitudinal (Ω) or transverse (Ω/m) mode, ω_r is the resonance frequency and Q is the quality factor.

Because damping (rising) time of an oscillation mode is $\tau = \frac{2Q}{\omega_r}$, we can separate the narrow-band impedance (long-time wake field) and the broad-band impedance (short-time wake field). The narrow-band impedance leads to bunch-by-bunch interaction and can result in multi-bunch instabilities, whereas the broad-band one leads to intra-bunch particle interaction and can cause single-bunch instabilities.

ALBA IMPEDANCE BUDGET

For a whole ring, sum of broad-band impedances of all the vacuum chamber components, generally called "broad-band coupling ring impedance", is used as a global stability criterion for intra-bunch particle interaction. Since longitudinal broad-band impedance is inductive for almost all practical cases, the normalized impedance $Z_\parallel/n$ is used, where $n = \omega/\omega_0$ is the revolution frequency number.

Let's evaluate the broad-band impedance contribution introduced by various parts of the ALBA vacuum chamber.

Resistive Wall

For a round vacuum chamber of radius b and length L, contribution of wall resistivity to the total broad-band impedance can be evaluated using the formula [2]:

$$Z_\parallel = (1+i)\frac{L}{2\pi}\frac{\omega Z_0 \delta_s}{2cb}, \quad Z_\perp = \frac{\pi^2}{12}(1+i)\frac{L}{2\pi}\frac{Z_0\delta_s}{b^3}, \tag{2}$$

where $Z_0 = 120\pi\,\Omega$ is the free-space impedance and $\delta_s = \sqrt{2\rho/\mu\omega}$ is the skin depth.

For the ALBA stainless steel vacuum chamber ($\rho = 0.73\ \mu\Omega$m, $\mu = \mu_0$, $L = 268.8$ m, $b = 14$ mm), resistive wall impedance at the cut-off frequency $\omega_c = c/b = 2\pi \cdot 3.41$ GHz is $Z_\parallel/n = 0.1(1+i)\ \Omega$, $Z_\perp = 36(1+i)\ k\Omega/$m.

Cross-section Transitions

For a cylindrical vacuum chamber, there are formulae of low-frequency impedance of untapered cross-section step from radius b up to d [2]:

$$\frac{Z_\parallel}{n} = i\frac{Z_0\,(d-b)}{2\pi R}\ln\frac{d}{b}, \quad Z_\perp = i\frac{Z_0\,(d-b)}{\pi b^2}\frac{d^2-b^2}{d^2+b^2}. \tag{3}$$

Tapering with a slope of $\tan\theta = \frac{l}{d-b}$, where l is the taper length, reduces the impedance roughly by factor of $\sin\theta$. For a small-angle circular tapered transition, there are formulae [3] valid at frequencies $\omega \ll cl/b^2$. If the taper is linear, $\tan\theta = const$, the impedance is:

$$\frac{Z_\parallel}{n} = i\frac{Z_0\omega_0\tan\theta}{4\pi c}\,(d-b)\,, \quad Z_\perp = i\frac{Z_0\tan\theta}{2\pi}\left(\frac{1}{b} - \frac{1}{d}\right). \tag{4}$$

Transverse impedance of a rectangular taper with the horizontal half-size h and the vertical half-size b ($h \gg b$) is one half of the impedance of a flat tapered collimator [4]. For a linear taper, it is:

$$Z_\perp = i\frac{Z_0 h\tan\theta}{8}\left(\frac{1}{b^2} - \frac{1}{d^2}\right). \tag{5}$$

Longitudinal impedance of a rectangular taper is roughly h/b larger than the impedance of a circular taper with the same size.

Formulae (3-5) are used for rough impedance estimation of a taper with complex geometry, for more accurate impedance evaluation, the MAFIA code is used.

Regular part of the ALBA vacuum chamber has an octagonal cross-section with the aperture of $72\times28\ \text{mm}^2$.

In the bending magnets, the vertical size decreases from 28 mm down to 22 mm, in the narrow gap vessels for insertion devices (ID) it decreases down to 8 mm, and in the RF cavity sections the vacuum chamber cross-section changes from the 72×28 mm^2 octagon to a circle of 74 mm diameter. All these transitions are tapered with a slope of 1/10. Longitudinal impedance of the RF cavity taper and the ID taper calculated using MAFIA is presented in Figure 1 in comparison with the formulae for a circular taper and for a rectangular one correspondingly. Sum broad-band

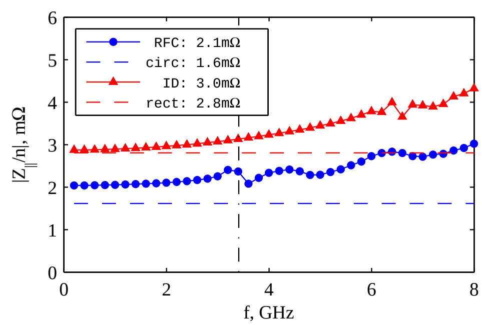

Figure 1: Impedance of the ALBA tapers.

impedance of 32 bending magnet tapers is $Z_\parallel/n = 18i$ mΩ, $Z_\perp = 9.6i$ kΩ/m, the impedance of 6 RF cavity tapers is $Z_\parallel/n = 12i$ mΩ, $Z_\perp = 4.1i$ kΩ/m, and the impedance of 16 ID tapers is $Z_\parallel/n = 48i$ mΩ, $Z_\perp = 44.1i$ kΩ/m.

Bellows

The ALBA bellows have the length $l = 48$ mm, the inner radius $r_i = 51$ mm and outer one $r_o = 66$ mm. The corrugation depth is $\delta r = r_o - r_i = 15$ mm and the corrugation period is $w = 2.824$ mm. At frequencies $\omega \ll c/w$, the impedance of unshielded bellow is inductive [5]:

$$\frac{Z_\parallel}{n} = i\frac{Z_0 l}{2\pi R}\ln\left(1+\frac{\delta r}{b}\right), \quad Z_\perp = \frac{2R}{b^2}\frac{Z_\parallel}{n}. \quad (6)$$

Total impedance of 48 ALBA bellows is $2.37i$ Ω, this value is too big, so all the bellows should be shielded. According to experience of other facilities, shielding reduces drastically the broad-band impedance of bellows. RF shield of the ALBA bellow has 30 narrow slots of 20×3 mm^2. At frequencies lower than $c/2\pi l = 2.4$ GHz, the impedance of a slot is inductive [2]:

$$\frac{Z_{||}}{n} = -i\frac{Z_0\omega_0}{c}\frac{\alpha_e+\alpha_m}{4\pi^2 b^2}, \quad Z_\perp = -iZ_0\frac{\alpha_e+\alpha_m}{\pi^2 b^4}, \quad (7)$$

where α_e is the electric polarizability and α_m is the magnetic susceptibility. For a rectangular slot, $\alpha_e+\alpha_m = w^3\left(0.1814 - 0.0344\frac{w}{l}\right)$. Total low-frequency broad-band impedance of 48 shielded bellows calculated with (7) is small enough: $Z_\parallel/n = -7.8i$ mΩ, $Z_\perp = -6.8i$ kΩ/m.

Button-electrode BPMs

To estimate broad-band impedance of the ALBA button-electrode BPM, let's take the following parameters: $r_b = 6$ mm is the button radius, $C_b = 0.5$ pF is the button-electrode capacitance, $\theta = r_b/b = 0.429$ rad is the effective semi-angular aperture. At frequencies lower than $\omega_c = c/r_b = 2\pi\cdot 8$ GHz, the impedance of a button-electrode is inductive [6]:

$$\frac{Z_\parallel}{n} = i\frac{2Z_c r_b}{R}\left(\frac{\theta}{\pi}\right), \qquad Z_\perp = \frac{2R}{b^2}\frac{Z_\parallel}{n}, \quad (8)$$

where $Z_c = \frac{r_b}{\pi c C_b}$ is the characteristic impedance.

At frequencies higher than ω_c, the impedance has resonance peaks at the frequencies $\omega_k = \frac{c}{r_b}(2k-1)$:

$$\frac{Z_\parallel}{n}(\omega_k) = \frac{\omega_0}{\omega_k}\frac{Z_c^2}{Z_T}\left(\frac{\theta}{\pi}\right)^2, \quad (9)$$

where $Z_T = 50$ Ω is the termination impedance.

Figure 2 shows longitudinal impedance of one 4-button ALBA BPM calculated using (8,9) in comparison with the MAFIA simulation. There are 120 button-

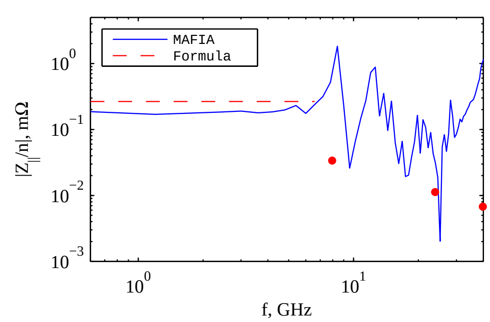

Figure 2: Impedance of the button-electrode BPM.

electrode BPMs in ALBA, so their broad-band impedance is $Z_\parallel/n = 32i$ mΩ, $Z_\perp = 13.9i$ kΩ/m.

Strip-line BPM

Low-frequency impedance of a pair of strip-lines is calculated with the formulae [2]:

$$Z_\parallel = 2Z_T\left(\frac{\phi}{2\pi}\right)^2\left(2\sin^2\frac{\omega l}{c} - i\sin\frac{2\omega l}{c}\right)$$
$$Z_\perp = \frac{Z_\parallel}{\omega}\frac{c}{b^2}\left(\frac{4}{\phi}\right)^2\sin^2\frac{\phi}{2}, \quad (10)$$

where Z_T is the termination impedance, l is the strip-line length, and ϕ is the angle subtending to pipe axis. Figure 3 shows the impedance of the ALBA strip-line BPM, calculated using the following parameters: $Z_T = 50$ Ω, $l = 178$ mm, $\phi = \pi/4$. Broad-band low-frequency impedance is $Z_\parallel/n = 26i$ mΩ, $Z_\perp = 21.5i$ kΩ/m.

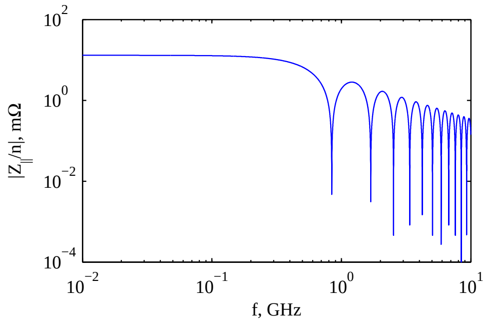

Figure 3: Impedance of the strip-line BPM.

High-order Modes of RF cavities

High-order modes (HOM) of RF cavities usually introduce a considerable contribution to the broad-band ring impedance. Although the impedance of each HOM is narrow-band, a broad-band component appears due to integral effect of all the HOMs. Broad-band impedance of a RF cavity is a sum of impedances of all the high-order modes [7]:

$$Z_{BBR}^{RF} = \sum_{m=1}^{M} Z_m^{HOM}. \tag{11}$$

The impedance of m-th HOM can be written as (1), at low frequencies this impedance is inductive:

$$Z_m^{HOM}(\omega) = i\frac{\omega R_m}{Q_m \omega_m}. \tag{12}$$

In Figure 4, there are longitudinal and transverse impedances of the ALBA RF cavity high-order modes, calculated using the MAFIA code. The fundamental mode is subtracted from the longitudinal impedance. Numerical integration of the MAFIA data in the frequency

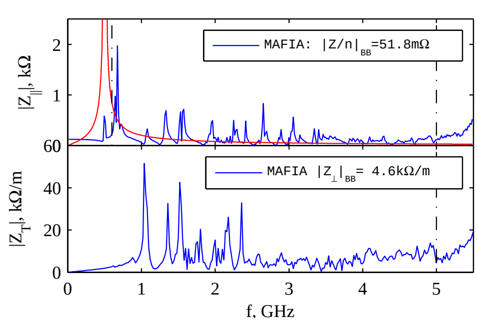

Figure 4: RF cavity HOM impedance.

range of $0.6 - 5.0$ GHz gives the low-frequency broad-band impedance of one cavity: $|Z_{\parallel}/n|_{BB}^{HOM} = 51\text{m}\,\Omega$, $|Z_{\perp}|_{BB}^{HOM} = 4.6\ \text{k}\Omega/\text{m}$.

Broad-band Impedance of the Ring

A summary of the ALBA broad-band impedance is presented in Table 1. Figure 5 shows the relative impedance contributions.

Table 1: ALBA broad-band impedance

	$\lvert Z_{\parallel}/n\rvert$ mΩ	$\lvert Z_{\perp}\rvert$ kΩ/m
Resistive wall @ cut-off frequency	140	50.3
ID tapers (16)	47	44.1
Bending magnet tapers (32)	18	9.6
RF cavity tapers (6)	17	4.1
Shielded bellows (48)	8	6.8
Button-electrode BPMs (120)	32	13.9
Strip-line BPM	26	21.5
RF HOMs (6 cavities)	300	27.6
Other (flanges, antechamber, etc)	50	10.0
Total	530	103.3
note: $\lvert\sum Z\rvert \neq \sum \lvert Z\rvert$		

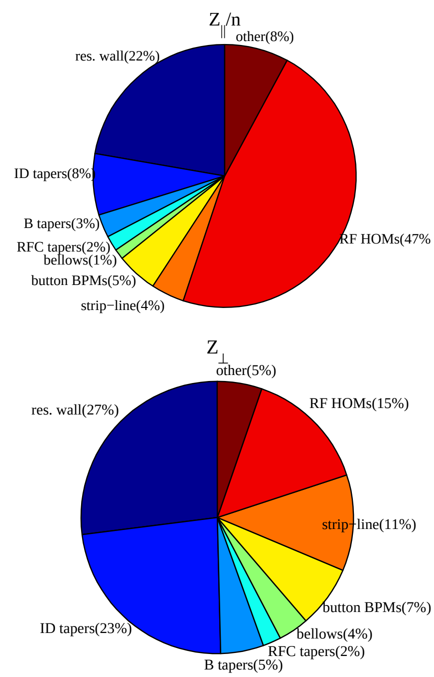

Figure 5: Broad-band impedance contributions.

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