# THE PHYSICAL PHENOMENA IN POWER BEAMS 

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#### Abstract

For research of the phenomena for a basis the concept of the author about general applicability of a principle of variability is accepted. Space-time ratios in electronic beams are examined. Expression for distances between charged particle in the accelerated beam is received. The criterion of existence of particle waves is received. Calculations of electric modes of the powerful device are carried out with use of computer modelling. The theory of the nonlinear physical phenomena in powerful tubes is developed. Problems of mathematical modelling arising at it are solved with use of object-oriented programming.


## INTRODUCTION

Intensive beams are used in forming x-ray, laser, microwave and ultra wideband radiation [1-4]. Characteristics and limiting parameters of accelerators of the charged particles always were defined by opportunities of radio engineering systems. Typical parameters of power electronics are huge: power up to $10^{14} \mathrm{~W}$, voltage $U$ up to $10^{7} \mathrm{~V}$, current up to $10^{6} \mathrm{~A}$, current density $J$ up to $10^{9} \mathrm{~A} / \mathrm{cm}^{2}$ [2].

It is obvious, that in such high current beams it is necessary to take into account the phenomena dependent on space-time ratios. For research of these phenomena for a basis the concept of the author about general applicability of a principle of variability [5] is accepted.

In free electron laser acceleration of the electron make with the help of radiofrequency tetrode amplifiers [4]. The basic purpose of the present report to present new results received by the author during development of the theory the powerful beams [5,6].

## METHODS

## Computer-Aided Method

Author offered new methods of direct calculation of charged particle beams allowing on the basis of a design to expect the basic electrical parameters and the characteristic. It allows at rather small resources of the computer to receive the large accuracy of calculation. The idea of a method consists in exact approach of model to real geometry of the device.
The theory is advanced which allows taking in account change of distribution of potential along electrodes of the device and influence of a magnetic field on movement of the charged particles. The new mathematical models are created. The processes in the active element and exterior circuits are analyzed simultaneously.
The clearances of a device are submitted by series connections of the nonlinear elements and ideal keys [5]. On the form of a current through a device, the essential
influence is rendered by initial velocities electrons and distribution of current between electrodes [5].
Maxwell-Lorenz equations, expressed in Euler variables, represented as departure. I produced the few sets of equations that described processes in high-current beam and in power device.
Then equations are transformed to Lagrange variables. I obtained three mathematical models: $a$ ) systems of the nonlinear integral Volterra equations of the second kind, b) systems of the nonlinear integrodifferential equations, c) systems of the nonlinear functional-differential equations.
Initial ratios for designing mathematical models are the equations of classical electrodynamics and the theory of circuits. According to [5] we present the equations modelling processes in a device. In this case diagram of electron movement in a tetrode looks like, as on fig. 3.2 [5, p. 74]. There 11 time intervals are allocated. To electron movement in a space the cathode-grid there correspond intervals I1-I5, in a space between managing and screen grids - intervals I6-I8, in a space a screen grid- anode - intervals I9-I11.

We accept, that initial electron velocity $\mathrm{V}_{0}$ at a start from the cathode. Let's enter designations:

$$
\psi=\frac{2 \cdot \omega \cdot d_{1}}{\sqrt{\frac{2 e}{m} U_{c 1}}}
$$

— angle of electron flight,

$$
\begin{gathered}
u_{1}(t)=\int_{0}^{t} E^{2}(s) d s \\
u_{2}(t)=\int_{0}^{t}(t-s) E(s) d s \\
u_{3}(t)=\int_{0}^{t} E(s) d s
\end{gathered}
$$

Also we shall write down system of the differential equations for interval I1 as:

$$
\begin{gathered}
\dot{E}(t)=\dot{f}_{y}(t)+u_{1}(t) / \psi^{2}+\gamma \cdot E(t) / \psi, \\
u_{1}(t)=E^{2}(t), \\
\dot{u}_{2}(t)=u_{3}(t), \dot{u}_{3}(t)=E(t), \\
t \in\left[0, t_{1}\right] .
\end{gathered}
$$

Entry conditions:

$$
E(0)=u_{1}(0)=u_{2}(0)=u_{3}(0)=0
$$

Condition for definition of value $t_{1}$ :

$$
2 u_{2}\left(t_{1}\right) / \psi^{2}+\gamma \cdot t_{1} / \psi-1=0
$$

Using designations

$$
\begin{gathered}
u_{4}(t)=\int_{t-\tau 1}^{t} E^{2}(s) d s \\
u_{5}(t)=\int_{t-\tau 1}^{t} E(s) d s
\end{gathered}
$$

write system of the differential equations:

$$
\begin{aligned}
& \dot{E}^{\prime}(t)=\dot{f}_{y}(t)+\psi^{-2}\left\{u_{4}(t)+E\left(t-\tau_{1}\right)\left[\tau_{1} \cdot E\left(t-\tau_{1}\right)-\right.\right. \\
& \left.\left.-2 u_{5}(t)\right]\right\}+\gamma \cdot \psi^{-1}\left[E(t)-E\left(t-\tau_{1}\right)\right], \\
& \dot{u}_{4}(t)=E^{2}(t)-E^{2}\left(t-\tau_{1}\right) F_{i 1}\left(t, \tau_{1}\right), \\
& \dot{u}_{5}(t)=E(t)-E\left(t-\tau_{1}\right), \\
& \dot{\tau}_{1}(t)=1-F_{i 1}\left(t, \tau_{1}\right), \\
& F_{i 1}\left(t, \tau_{1}\right)=2 \frac{u_{5}(t)-\tau_{1} \cdot E\left(t-\tau_{1}\right)+0,5 \gamma \psi}{\tau_{1}^{2} \cdot \dot{E}\left(t-\tau_{1}\right)+\gamma \psi} .
\end{aligned}
$$

Initial conditions:

$$
\begin{gathered}
\tau_{1}\left(t_{1}\right)=t_{1}, u_{4}\left(t_{1}\right)=u_{1}\left(t_{1}\right) \\
u_{5}\left(t_{1}\right)=u_{3}\left(t_{1}\right)
\end{gathered}
$$

The mathematical statements are possible as direct ( $D$ ) problem as reverse ( $R$ ) problem, and if $D$-problem is based on the equations with lagging argument. Corresponding R-problem is based on the equations with advancing argument. Analyses of the systems with retard fulfils by the solving of the D-problem. The mathematical models are possible to reverse in the sense of the time.
For a raise of effectiveness of designing the new method of inverses of tasks for systems with variable delay is offered. The model of an inverse task represents a nonlinear system of the differential equations with advance argument.

Let $f_{\mathrm{R}}(\mathrm{t}), \mathrm{t} \in\left[\mathrm{t}_{1}, \mathrm{t}_{3}\right]$ - vector function, defined solution $u_{D}(t)$ of a set of equations with lagging argument
$A\left(u_{\mathrm{D}}\right)=F_{\mathrm{D}}\left(\mathrm{t}, \mathrm{t}-\tau_{\mathrm{D}}\left(\mathrm{t}, \mathrm{u}_{\mathrm{D}}\right), \lambda, \mathrm{u}_{\mathrm{D}}(\mathrm{t}), \mathrm{u}_{\mathrm{D}}\left(\mathrm{t}-\tau_{\mathrm{D}}\left(\mathrm{t}, \mathrm{u}_{\mathrm{D}}\right)\right), V\left(\mathrm{t}, \mathrm{u}_{\mathrm{D}}\right)\right)$,
A and V-operators; $\lambda$ - parameter; $\mathrm{t}_{\mathrm{D}}>0$ - deviation of argument; the vector function $F_{D}$ is defined in $R^{n}$. The inferior index $D$ specifies that $u_{D}(t)$ there is a solution of a direct task. It is obtained for the given initial function $f_{\mathrm{D}}(\mathrm{t}), \mathrm{t} \in\left[\mathrm{t}_{0}, \mathrm{t}_{2}\right]$.

Statement of an inverse task: to deduce a set of equations with anticipating argument
$A\left(\mathrm{u}_{\mathrm{R}}\right)=F_{\mathrm{R}}\left(\mathrm{t}, \mathrm{t}+\tau_{\mathrm{R}}\left(\mathrm{t}, \mathrm{u}_{\mathrm{R}}\right), \lambda, \mathrm{u}_{\mathrm{R}}(\mathrm{t}), \mathrm{u}_{\mathrm{R}}\left(\mathrm{t}+\tau_{\mathrm{R}}\left(\mathrm{t}, \mathrm{u}_{\mathrm{R}}\right)\right), \mathrm{V}\left(\mathrm{t}, \mathrm{u}_{\mathrm{R}}\right)\right)$, such, that its solution $u_{R}(t)$ for the given vector function $f_{\mathrm{R}}(\mathrm{t}), t \in\left[\mathrm{t}_{1}, \mathrm{t}_{3}\right]$ has defined a vector function $f_{\mathrm{D}}(\mathrm{t}), t \in\left[t_{0}\right.$, $t_{2}$ ]. The solution of an inverse task corresponds to initial functions of a direct task, and solution direct - initial functions inverse. A criterion of a regularity of a solution of direct and inverse tasks for systems with variable delay from here follows: concurrence of initial functions of a direct task and functions $f_{\mathrm{D}}(t)$ solution, obtained in an outcome, of an inverse task.

Statement of a direct task assumes the representation of function of a velocity electrons $f_{V D}(t)$ and convection current $f_{\mathrm{KD}}(t)$ in a plane of the first electrode. In an outcome of a solution of a direct task to the given functions are put in the correspondence of function of a velocity $f_{\mathrm{VR}}(t)$ and convection current $f_{\mathrm{KR}}(t)$ in a plane of the second electrode.

The inverse task consists in calculation of the function $f_{\mathrm{VD}}(t)$ and $f_{\mathrm{KD}}(t)$, which set in a direct task, on known dependences $f_{\mathrm{VR}}(t)$ and $f_{\mathrm{KR}}(t)$ connected to a solution of a direct task. The conclusion of the equation of an inverse task makes a subject of analyze.
The method requires insignificant computing expenditures.

## De Broglie Wavelength And Distance Between Charged Particle In Beams

Properties of a wave are inherent in each elemental particle and, on the contrary, any wave has property, characteristic for particles. Wave properties considerably influence on behaviour of particles. The particle moving with a pulse $p$, has the length of a wave $\lambda$ connected to this pulse

$$
\lambda=h / p
$$

where $h$ - Planck constant.
This wave refers to as a De Broglie wave or the particle wave. Electron alongside with corpuscular properties have as well wave. Electron speed, past a potential $U$ with zero initial speed:

$$
\mathrm{v}(\mathrm{~m} / \mathrm{s})=\sqrt{\frac{2 e}{m} U}=5,95 \cdot 10^{5} \sqrt{U(V)}
$$

De Broglie wavelength for electron:

$$
\lambda=h / m v=h / \sqrt{2 e m U} .
$$

Here $e$ and $m$ - electron's charge and mass of the electron.

Now we shall estimate distance between electrons $R$ in a bunch, and then we shall compare it with $\lambda$. Electron properties depend on a ratio between $R$ and $\lambda$.

Electron-transit time of the distance 1 cm with constant velocity v is equal:

$$
\tau(\mathrm{s})=1 / \mathrm{v}(\mathrm{~cm} / \mathrm{s}) .
$$

In time $\tau$ through cross section of a beam with the area of $1 \mathrm{~cm}^{2}$ passes a charge

$$
Q=J \tau
$$

Having divided it on electron charge, we receive quantity of the electrons in cubic centimeter:

$$
n=J / \mathrm{ev}
$$

Electron occupies volume

$$
V=\mathrm{ev} / J
$$

We believe that the distance between next electrons is equal to the side of a cube:

$$
R=\sqrt[3]{V}=\sqrt[3]{e v / J}
$$

We receive equality criterion of length of a particle wave and electron distances in a beam. At equality $\lambda$ and $R$ the current density and voltage can be various. However relation $J / U^{2}$ should be always equal to
characteristic size

$$
C=5,04 \cdot 10^{9}\left(\mathrm{~A} / \mathrm{cm}^{2} \mathrm{~V}^{2}\right)
$$

Calculation under the deduced formulas was executed for two values of current density (in the table1).

Table 1. Results of calculation for various values $U$ and $J$ in electron beams

| $\boldsymbol{U}, \mathbf{V}$ | $\boldsymbol{\lambda}, \mathbf{m}$ | $\boldsymbol{J}, \mathbf{A} / \mathbf{c m}^{2}$ | $\lambda / \boldsymbol{R}$ | $\mathbf{J} / \mathbf{U}^{\mathbf{2}}$, <br> $\mathbf{A} / \mathbf{c m}^{2} \mathbf{V}^{\mathbf{2}}$ |
| :--- | :--- | :--- | :--- | :--- |
| $10^{-4}$ | $1,23 \cdot 10^{-7}$ | $10^{3}$ | 2,7 | $10^{11}$ |
| $10^{-4}$ | $1,23 \cdot 10^{-7}$ | $10^{9}$ | 270 | $10^{17}$ |
| $10^{-2}$ | $1,23 \cdot 10^{-8}$ | $10^{3}$ | 0,125 | $10^{7}$ |
| $10^{-2}$ | $1,23 \cdot 10^{-8}$ | $10^{9}$ | 12,5 | $10^{13}$ |
| 50 | $1,7 \cdot 10^{-10}$ | $10^{3}$ | $4,2 \cdot 10^{-4}$ | 0,4 |
| 50 | $1,7 \cdot 10^{-10}$ | $10^{9}$ | $4,2 \cdot 10^{-2}$ | $4 \cdot 10^{5}$ |
| $10^{4}$ | $10^{-11}$ | $10^{3}$ | $10^{-5}$ | $10^{-5}$ |
| $10^{4}$ | $10^{-11}$ | $10^{9}$ | $10^{-3}$ | 10 |

A trajectory of electron at presence of a magnetic field is bent under action of Lorenz force.
It is constructed mathematical model of a electron trajectory, leaving a cylindrical electrode with initial speed $v_{0}$. In a considered case we believe, that the vector of intensity of an electric field is directed to an electrode. We deduced the equation of electron movement. We shall choose cylindrical system of coordinates.
Electron takes off at the moment of time $t=0$ of a point with coordinate $r=r_{0}, z=0$, where $r_{0}$ - radius of an electrode. Thus its speed has a component only along an axis $z: ~ \dot{r}=\mathrm{v}_{0}, \dot{z}=0$.

In the crossed constants electric and magnetic fields the vector equation of electron movement:

$$
m \ddot{r}=-e(\boldsymbol{E}+[\mathbf{v B}])
$$

Here $e$ - absolute value of a charge particle (electron). Movement of electron occurs in a plane $r O z$. Projections of the vector equation to axes Or and Oz it is written down as:

$$
\begin{gathered}
m \ddot{r}=e E+e \dot{z} B \\
m \ddot{Z}=-e \dot{r} B .
\end{gathered}
$$

We shall copy system of two scalar equations, having expressed the second derivatives $\ddot{r}$ and $\ddot{Z}$ :

$$
\begin{gathered}
\ddot{r}=e E / m+e \dot{z} B / m \\
\ddot{Z}=-e \dot{r} B / m .
\end{gathered}
$$

These equations are solved analytically. Due to magnetic field the density of a current grows, and the distance between electrons decreases and with other things being equal becomes larger, than in case of absence of a magnetic field. Thus there is an opportunity of that the De Broglie wavelength appears comparable with electron trajectory length.

## CONCLUTION

The above decision of a problem evidently confirms validity of the concept according to which properties of object change at change of space-time ratios in this object. And always there is an opportunity to specify intervals of change of space-time parameters within the limits of which properties of object remain constant. In a considered case the border between such intervals is established by value of a constant $C$.

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