SPACE CHARGE LIMIT ON THE INTENSITY OF AN ION COASTING BEAM DURING ITS ELECTRON COOLING

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Abstract

The cooling of ion beams increases the strengths if their space charge forces and the nonlinearities of such forces. These variations change the stability conditions for transverse coherent oscillations of the ion beam and the shape of the stability diagram of these oscillations. The beam cooling holds until the coherent frequency shift of the beam approach the border of its stability diagram. Without special efforts this phenomenon defines a non-resonant limit on the attainable value of the Laslett tune shift of the ion beam during its e.g. electron cooling.

INTRODUCTION

One of the important limitations on the operation performance of ion beams occurs due to their space charge fields. For many reasons the strengths of these limitations are described using the Laslett tune shift of the beam, which is defined as the value of the tune shift of small betatron oscillations of ions due to space charge fields of the beam. For a coasting beam of a round cross section consisting of non-relativistic ions the Laslett tune shift reads

$$\Delta\nu_L = \frac{Ne^2}{4\pi M v^2 \epsilon}. \tag{1}$$

Here, Ne is the charge of the beam, M is the ion mass, v is its velocity, ϵ is the transverse emittance of the beam. Due to repulsion of particles the space charge fields decrease the tunes of the betatron oscillations. For small amplitudes of betatron oscillations relevant tune shift is equal to $(-\Delta\nu_L)$. Outside the beam the space charge fields decay. Hence, for the particles oscillating with the amplitudes exceeding transverse beam sizes the tune shifts due to space charge fields tend to zero. It means that the space charge fields increase the tune spreads of the betatron oscillations in the beams. If the footprint of the beam in the plane of the tunes of the vertical (ν_y) and horizontal (ν_x) oscillations covers some resonant regions, the transverse sizes of the beam blowup resulting in the degradation of its phase space density. The sizes of the beam footprint are proportional to the value of $\Delta\nu_L$. So that $\Delta\nu_L$ should not exceed some threshold value $\Delta\nu_{L0}$ which, generally, depends on the machine parameters. In particular, it depends on betatron tunes and on the azimuthal symmetry of the lattice. The calculations of such threshold values $\Delta\nu_{L0}$ is not easy even for simplified cases. Therefore, in particular designs of the ion storage rings one uses as $\Delta\nu_{L0}$ some "world average value" which for the storage rings usually does not exceed 0.05.

In particles accelerators and storage rings the beam intensity can be limited due to interactions of the coherent oscillations of the beam with the wakefields, which the beam induces in surrounding electrodes. In such cases and neglecting the frequency spread of the beam, coherent oscillations can become unstable due to dissipations of the wakefields. The most unstable are the dipole coherent modes. These conventional instabilities can limit the beam intensity in wide frequency spectra complementary to the limitations due to space charge fields. Generally, increases in the frequency spreads of the beam increase the Landau damping of the beam coherent modes and can eliminate the instability. The beam cooling reduces the frequency spreads due to the lattice chromaticity and due to the lattice nonlinearities, but increases the Laslett tune shift of the beam.

The space charge forces depend on the distance of a particle to the center of gravity of the beam. In a storage ring with an idealized lattice providing the linear and achromatic focusing the equations describing coherent dipole oscillations and incoherent oscillations of particles inside the beam are uncoupled . In such a lattice, the frequency spreads due to space charge fields of the beam do not contribute to Landau damping of dipole coherent oscillations. In a coasting ion beam Landau damping of dipole oscillations can appear due to the revolution frequency spread, due to the lattice chromaticity, or due to e.g. a cubic nonlinearity of the lattice focusing (see, e.g. in Refs. [1] – [6]). We shall define the frequency spread of the beam due to these sources using $\delta\omega_{ext}$. Specific features of Landau damping strongly depend on the frequency distribution in the beam. The width and the shape of this distribution is modified by the tune spreads due to space charge of the beam. Correspondingly, the stability conditions of the coherent oscillations of such a beam vary with the variations of the value of $\Delta\nu_L$.

In this report, we describe the variations of the stability conditions for dipole coherent oscillations in the coasting beam for the cases, where the values of the Laslett tune shift and of $\delta\omega_{ext}$ vary due to the beam cooling.

STABILITY DIAGRAMS

For simplicity we consider e.g. vertical coherent oscillations of a coasting beam and assume that the variations of the amplitudes of coherent oscillations due to interactions with wakes are substantially faster than that due to the beam cooling. Correspondingly, we describe coherent oscillations of the beam using the Vlasov equation. For the same reasons, we use the model where the space charge forces of the beam are described using the expressions: $F_y(y-d,x) \simeq F_y(y,x) - d(\phi,t)(\partial F_y/\partial y)$ and $F_x(y,x)$. Here, $d(\phi,t)$ is the vertical dipole offset of the beam, $\phi = \theta - \omega_0 t$, $\theta = s/R_0$ is the particle azimuth, ω_0 is the revolution frequency of the reference particle. Simple calculations with the linearized Vlasov equation result in the following dispersion equation (see e.g. in Ref.[6] and [7] for more detail):

$$1 = -\int_0^\infty dI_x dI_y I_y \frac{\partial f_0}{\partial I_y}\left[\Omega_{m,n} + m_y\Omega_y(I)\right] \quad (2)$$
$$\times \int_{-\infty}^{\infty} d\Delta p \frac{\rho(\Delta p)}{\Delta\omega_m - g\Delta p - m_y\Delta\omega_3 + m_y\Omega_y(I)}.$$

Here, $\mathrm{Im}\omega > 0$, $f_0(I_y, I_x)\rho(\Delta p)$ is the distribution function of the beam without coherent oscillations, $I_{x,y}$ are the action variables of the vertical and of the horizontal betatron oscillations, $\Delta p = p - p_0$ is the deviation of the particle momentum from the nominal value, $g\Delta p$ is the chromatic frequency shift of the betatron oscillations of a particle, $\Delta\omega_3$ is its betatron frequency shift due to e.g. lattice octupole fields, $\Delta\omega_m = \omega - m_y\omega_0\nu_y$ is unknown value of the frequency shift ($m_y = \pm 1$), $\Omega_{m,n}$ is the coherent frequency shift of the monochromatic beam:

$$\Omega_{m,n} = \frac{m_y N e^2 Z_\perp(m_y\omega_0\nu_y + n\omega_0)}{4\pi p_0 \nu_y}, \quad (3)$$

$Z_\perp(\omega)$ is the transverse broadband wake impedance. The value $\Omega_y(I) \propto \Delta\nu_L$ in Eq.(2) describes the frequency shift of the vertical betatron oscillations of a particle due to the beam space charge fields. The dispersion equation (2) can be used either for the calculations of the eigenfrequencies of coherent modes or for the calculations of the stability diagram of the transverse coherent oscillations of the beam. For the last purpose, the value $\Omega_{m,n}$ is calculated from Eq.(2) where the variable $\Delta\omega_m$ is varied slightly above the real axis ($\Delta\omega_m \to \Delta\omega_m + i0$). If we define $\Delta\omega_3 = aI_y - bI_x$, then the equation for the stability diagram reads

$$\Omega_{m,n} = \Delta\omega_m - \frac{gQ_{yp} + aQ_{yy} - bQ_{yx}}{Q_y(\Delta\omega_m)}, \quad (4)$$

where

$$Q_y = \int_0^\infty dI \int_{-\infty}^{\infty} d\Delta p \frac{I_y(\partial f_0/\partial I_y)\rho(\Delta p)}{\Delta\omega_m - \delta\omega(\Delta p, I) + i0}, \quad (5)$$

$$Q_{yp} = \int_0^\infty dI \int_{-\infty}^{\infty} d\Delta p \frac{I_y(\partial f_0/\partial I_y)\Delta p\rho(\Delta p)}{\Delta\omega_m - \delta\omega(\Delta p, I) + i0}, \quad (6)$$

$$Q_{yy} = \int_0^\infty dI \int_{-\infty}^{\infty} d\Delta p \frac{I_y^2(\partial f_0/\partial I_y)\rho(\Delta p)}{\Delta\omega_m - \delta\omega(\Delta p, I) + i0}, \quad (7)$$

$$Q_{yx} = \int_0^\infty dI \int_{-\infty}^{\infty} d\Delta p \frac{I_x I_y(\partial f_0/\partial I_y)\rho(\Delta p)}{\Delta\omega_m - \delta\omega(\Delta p, I) + i0}, \quad (8)$$

and $\delta\omega(\Delta p, I) = g\Delta p + m_y\Delta\omega_3(I) - m_y\Omega_y(I)$. Equation (4) shows that without external frequency spreads ($g = 0$, and $a, b = 0$) for any strength of the beam space charge the stability diagram coincides with the real axis of $\Omega_{m,n}$. This fact confirms the so-called Merle–Möhl rule [1]. The beam cooling results in gradual decrease in the external frequency spread $\delta\omega_{ext}$ as well as in the increase in the value of the Laslett tune shift. Correspondingly, the shape of the stability diagram deforms during the beam cooling. In particular calculations of these deformations we took a Gaussian momentum distribution in the beam with the rms width σ_p. We also assumed a flat beam $\epsilon_x \gg \epsilon_y$ and the exponential distribution in the horizontal plane with the width $I_{x0} = p\epsilon_x$. The initial strength of the space charge field was specified using the parameter $q = \Delta\nu_L\omega_0/(\delta\omega_{ext})_{in}$.

Several examples given in Figs.1–4 show that during the beam cooling the height of the stability diagram decreases. Increases in the Laslett tune shifts due to transverse cooling of the beam shift the stability diagrams to the lower values of the coherent frequency shifts $\Omega_{m,n}$. Even for the cases, where the momentum cooling dominates (Figs.1 and 2) that can break the symmetry of the stability diagram

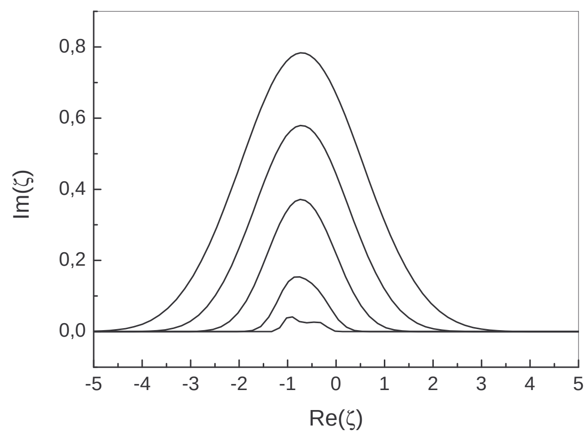

Figure 1: Stability diagrams for the vertical coherent oscillations ($\zeta = \Omega_{m,n}/(\delta\omega_{ext})_{in}$). The oscillations are stable below the border curve, $a = b = 0$, from top to bottom $(\sigma_p/\sigma_{in}) = 1$, 0.75, 0.5, 0.25 and 0.1, no transverse cooling; $m_x = 1$.

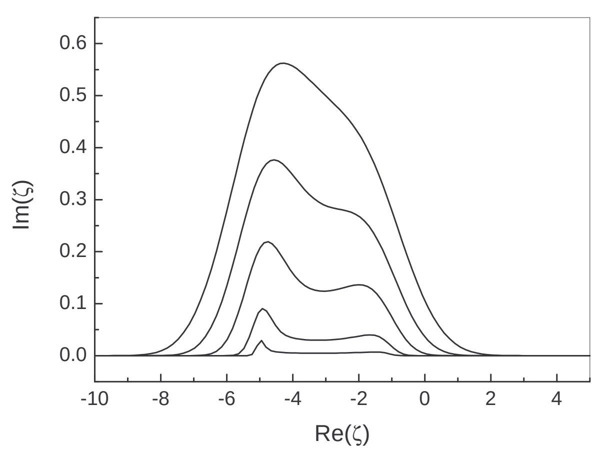

Figure 2: Same as in Fig.1, but $q = 5$.

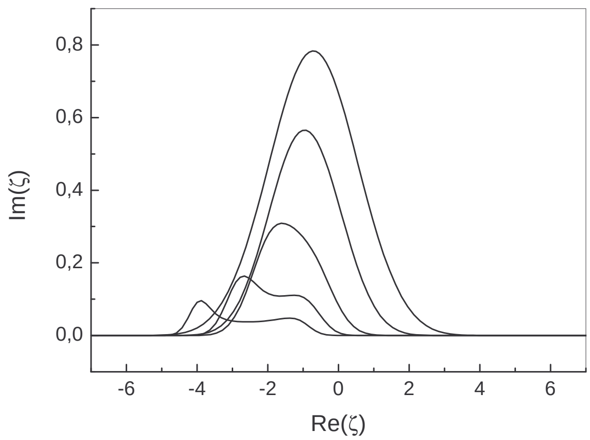

Figure 3: Same as in Fig.1, but equal rates of the transverse and longitudinal cooling.

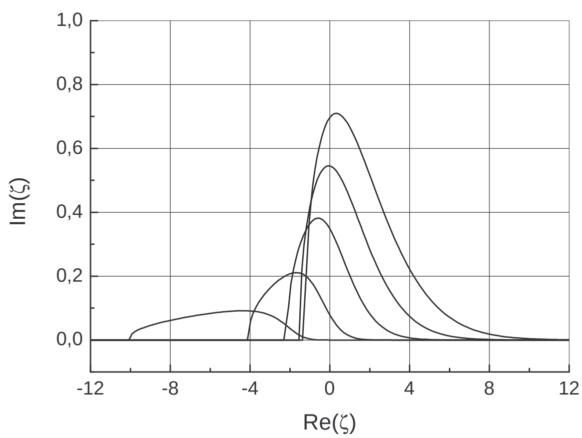

Figure 4: Stability diagrams for the vertical coherent oscillations. The oscillations are stable below the border curve, $g = 0$, $a = 0$, $b < 0$, flat beam ($\epsilon_y \ll \epsilon_x$) from top to bottom $(\epsilon_x/\epsilon_{in}) = 1$, 0.75, 0.5, 0.25 and 0.1, $q = 1$, $m_x = 1$.

provided that the Laslett tune shift is large enough (compare e.g. Fig.1 and Fig.2). For a given impedance budget of the ring, it means that during the cooling the stability diagram approaches the nearest thresholds of instabilities of its coherent oscillations. Apart from the possible loss of the beam intensity an approaching these thresholds increases the Schottky noise background of the beam (see e.g. in Ref.[7]). On its turn, that can result in additional transverse heating of the beam due to the diffusion of the ions on these coherent fluctuations (see e.g. in Ref.[6], or in Ref.[7]). The strengths of relevant diffusion coefficients increase inversely proportional to the distance to a threshold [7].

CONCLUSION

The beam cooling decreases the thresholds of instabilities of transverse coherent oscillations of the beam. The loss of the beam stability becomes more severe due to transverse cooling which increases the Laslett tune shifts of the beam. This phenomenon poses additional limitation on the attainable values of the Laslett tune shifts in the ion storage rings with the beam cooling. Provided that the instabilities occur due to a wideband beam environment and contrary to ordinary cases, relevant threshold value $\Delta\nu_{L0}$ will not depend on the betatron tunes, but will be determined by the impedance budget of the ring. This threshold can be increased using appropriate feedback systems which can damp unstable modes of the dipole coherent oscillations.

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