

# A Discrete Hysteresis Model for Piezoelectric Actuators and its Parameter Identification

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## Abstract

Hysteresis is an important nonlinear effect exhibited by piezoelectric actuators (PEA) and its modelling has been drawing considerable attention. This paper presents the development of a novel discrete model based on the concept of auto-regressive moving average (ARMA) for the piezoelectric-actuator hysteresis, and its parameter identification method as well. Experiments were carried out to verify the effectiveness of the developed model. The result obtained shows that the developed model can well represent the hysteresis of the PEA.

## Introduction

### What is Hysteresis?

- A memory effect of PEA
- The hysteresis exhibited at an given time instant depends on not only the input at the present time but also the operational history of the system considered.

### Hysteresis Model

- Preisach Model
- Ferromagnetic Material Model
- Nonlinear Auto-regressive Moving Average Model with Exogenous Input (NARMAX model)

### Research Issue

- Not all the continuous controllers can work on the sampled digital system as desired since the discrete sampling can sometimes make the continuous system unstable. Therefore, it is advantage to develop a discrete hysteresis model of PEA for its digital controller design.

### Content of the paper

- The ferromagnetic material hysteresis model is discretized and, by combining it with the concept of auto-regressive moving average (ARMA), a novel model is developed to represent the hysteresis of PEA.

## Discrete ARMA-based Hysteresis Model

### Discrete the Ferromagnetic Material Model

$$\dot{y} = |\dot{x}|(ax + cy) + b\dot{x}$$

- Integral

$$y(k+1) - y(k) = \frac{1}{2}a[x^2(k+1) - x^2(k)] + c \int_{x(k)}^{x(k+1)} y dx + b[x(k+1) - x(k)]$$

- Using trapezoid equation to estimate the integral term

$$y(k+1) = a \frac{\alpha(k+1)}{2 - c\beta(k+1)} + b \frac{2\beta(k+1)}{2 - c\beta(k+1)} + \frac{2 + c\beta(k+1)}{2 - c\beta(k+1)} y(k)$$
$$\alpha(k+1) = x^2(k+1) - x^2(k)$$
$$\beta(k+1) = x(k+1) - x(k)$$

$$y(k+1) = ay_1(k+1) + by_2(k+1)$$

- $\dot{x} > 0$

$$y_1(k+1) = \frac{\alpha(k+1)}{2 - c\beta(k+1)} + \frac{2 + c\beta(k+1)}{2 - c\beta(k+1)} y_1(k)$$

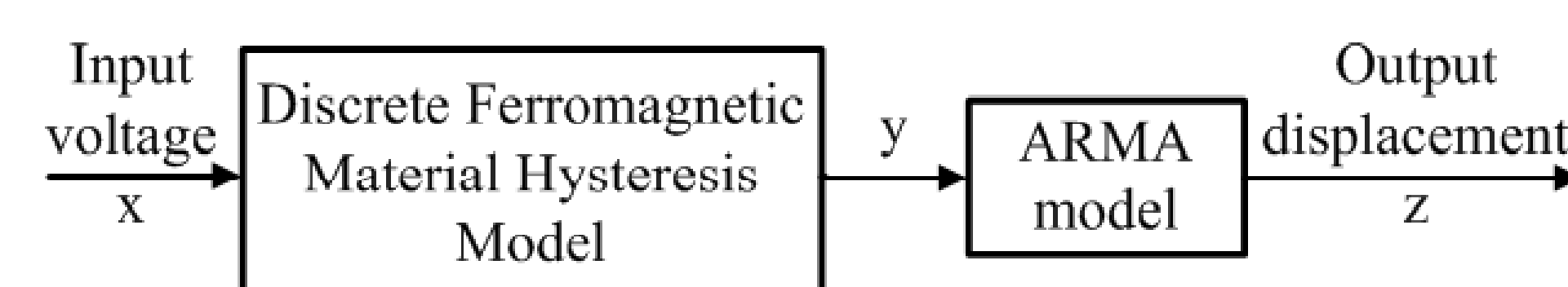
$$y_2(k+1) = \frac{2\beta(k+1)}{2 - c\beta(k+1)} + \frac{2 + c\beta(k+1)}{2 - c\beta(k+1)} y_2(k)$$

- $\dot{x} < 0$

$$y_1(k+1) = \frac{-\alpha(k+1)}{2 + c\beta(k+1)} + \frac{2 - c\beta(k+1)}{2 + c\beta(k+1)} y_1(k)$$

$$y_2(k+1) = \frac{2\beta(k+1)}{2 + c\beta(k+1)} + \frac{2 - c\beta(k+1)}{2 + c\beta(k+1)} y_2(k)$$

### Combined with ARMA model



$$z(t) = a_1 z(t-1) + a_2 z(t-2) + b_0' y_1(t) + b_0'' y_2(t) + b_1' y_1(t-1) + b_1'' y_2(t-1) + b_2' y_1(t-2) + b_2'' y_2(t-2)$$

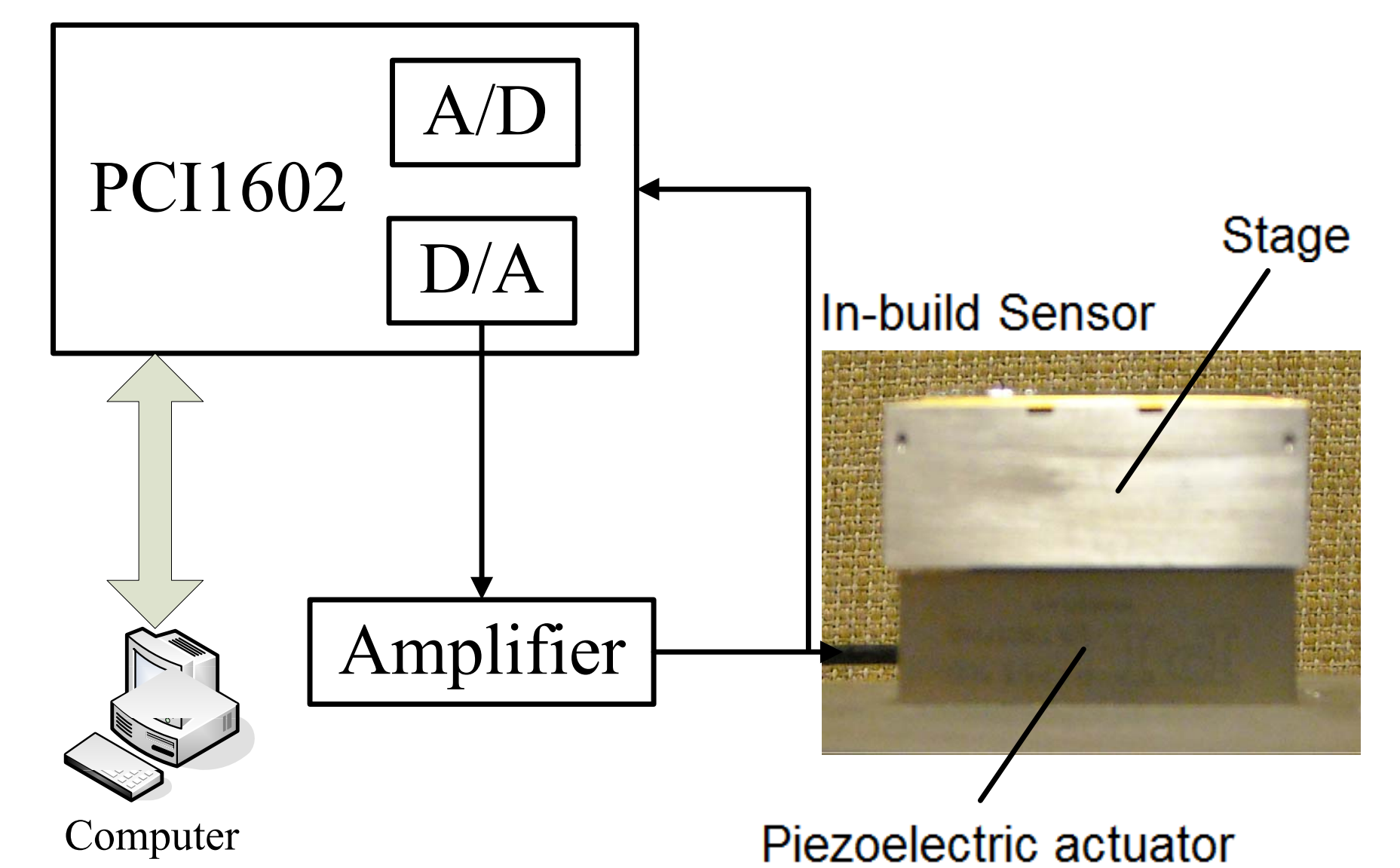
## Parameter Identification

- Online Estimation for the linear parameters
- Box-Kanemasu method for nonlinear parameter c

## Experiment

- P-753, Physik Instrumente
- Resolution: 0.5nm
- Sampling Interval: 0.05ms

Figure1. Piezoelectric Driven Stage



### Parameter Identification

Parameters	$a_1$	$a_2$	$b_0'$	$b_0''$
Value	1.6531	-0.676	-0.00276	-0.0732
Parameters	$b_1'$	$b_1''$	$b_2'$	$b_2''$
Value	0.0064	0.174	-0.00353	-0.0976

### Model Verification

Frequency (Hz)	10	50	200	400
Discrete				
ARMA-based hysteresis model	0.0943	0.0989	0.1112	0.1603
General				
discrete form of hysteresis	0.0946	0.0996	0.1128	0.1627

## Conclusions

- The discrete ARMA-based hysteresis model can better predict the hysteresis of PEA.
- The model shows a larger estimation error in high frequency application than in low frequency application due to the estimation of the integral term in the discrete hysteresis equation.

## Reference

- [1] Santosh Devasia and S. O. Reza Moheimani, A Survey of Control Issues in Nano-positioning, IEEE Transactions on Control System Technology, Vol.15, No.5, Sep.2007
- [2] H. J. Zhang, a. S. J. Chen, and M. Zhou, Fast tool servo control for diamond-cutting microstructured optical components, J. Vac. Sci. Technol. B 27,3..., May/Jun 2009
- [3] H. Zhang, D. Chapman, Z. Zhong, C. Parham, M. Gupta, Crystal tilt error and its correction in diffraction enhanced imaging system, Nuclear Instrument and Methods in Physics Research A 572 (2007) 961-970
- [4] I. D. Mayergoyz, Mathematical Models of Hysteresis, Physical Review Letters, Vol.56, No. 15, April 14th, 1986
- [5] Han J.M.T.A. Adriaens, Willem L. de Koning and Reinder Banning, Modeling piezoelectric actuators, IEEE/ASME transactions on mechatronics, VOL.5, No.4, Dec. 2000
- [6] Liang Deng, Yonghong Tan, Modeling hysteresis in piezoelectric actuators using NARMAX models, Sensors and actuators, A149, 2009, P106-112
- [7] Jozef Vörös, Modeling and Identification of Hysteresis using Special Forms of the Coleman-Hodgdon Model, Journal of Electric Engineering, Vol.60, No.2, 2009, 100-105
- [8] X. B. Chen, Q. S. Zhang, D. Kang, and W. J. Zhang, On the dynamics of piezoelectric positioning systems, Review of scientific instrument 79. 116101, 2008