

# TRANSMISSION LINE ANALYSIS OF BEAM DEFLECTION IN A BPM STRIPLINE KICKER

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## Abstract

In the usual treatment of impedances of beamline structures the electromagnetic response is computed under the assumption that the source charge trajectory is parallel to the propagation axis and is unaffected by the wake of the structure. For high energy beams of relatively low current this is generally a valid assumption. Under certain conditions the assumption of a parallel source charge trajectory is no longer valid and the effects of the changing trajectory must be included in the analysis. Here the usual transmission line analysis [1] that has been applied to BPM type transverse kickers is extended to include the self-consistent motion of the beam in the structure.

## 1 INTRODUCTION

The desire to use one induction accelerator to provide multiple lines of sight for advanced radiography [2] has stimulated work in the use of cylindrical stripline kickers to deflect kiloampere electron beams. We consider a cylindrical stripline kicker as shown in Fig. 1 consisting of four electrodes. Two of the electrodes are grounded while the remaining two are driven from the downstream end by opposite polarity cable signals. For simplicity, we will assume that the kicker impedance is matched to that of the drive cables but that the upstream termination cables have an arbitrary impedance  $Z_t$ .

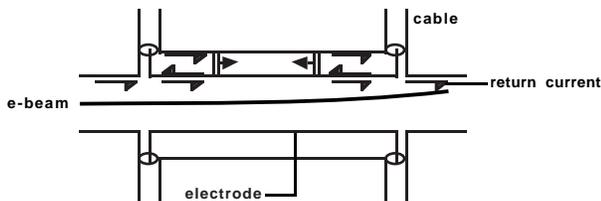


Fig. 1 Schematic of return currents in the stripline kicker

## 2 TRANSMISSION LINE EQUATIONS

The transmission line model of the kicker structure is shown in Fig. 2. The quantity  $I_r$  represents the beam return current which is introduced into the transmission lines formed by the electrodes and the outer vacuum housing at the gaps at either end of the striplines. These are the usual sources used in the analysis of reference [1]. Also shown is the voltage source representing the pulser.

The schematic is shown only for one of the driven plates and  $I_r$  is interpreted as the dipole return current flowing on that strip (since the monopole return current will not generate a net deflecting force). To these sources we must add distributed shunt current sources to account for the fact that the beam is changing its transverse position within the structure. If we follow a given "slice" of the beam as it enters the kicker imagine that it enters on axis so that there is no dipole return current at  $z=0$ . We now allow the beam slice to deflect due to the action of, say, an external bias coil. That slice will then generate a dipole return current on the strip. Since the current on the strip must be conserved, an equal and opposite current must be induced on the other side of the strip, i.e., in the transmission line. This can be represented by the distributed shunt current source  $g(z, \tau)$  given by

$$g(z, \tau) = -\frac{\partial}{\partial z} [I_r(z, \tau) - I_r(0, \tau)]. \quad (1)$$

Here the variable  $\tau$  is the slice label given by  $\tau \equiv t - L/c - z/c$ . We will assume that the axial velocity is  $c$ , vacuum light speed and  $L$  is the length of the kicker. We will solve the transmission line equations in the variables  $z$  and  $t$  so that we will need to convert  $g$  to the proper form. The transmission line equations become

$$\frac{\partial V}{\partial t} = -L \frac{\partial i}{\partial t} \quad (2)$$

and

$$\frac{\partial i}{\partial t} = -C \frac{\partial V}{\partial t} + g(z, t) \quad (3)$$

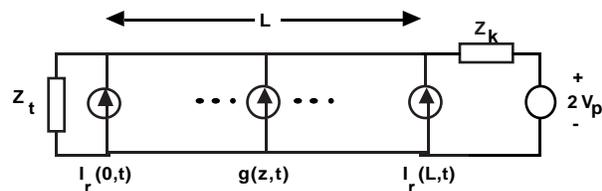


Fig. 2 Transmission line circuit showing distributed sources

where  $C$  is the capacitance per unit length of the line and  $L$  in the inductance per unit length of the line. We take  $Z_k = \sqrt{L/C}$  and  $c = 1/\sqrt{LC}$  vacuum light speed which

is also the propagation speed on the line. We have two boundary conditions for the problem. At  $z=0$  we have that

$$V(0,t) = -Z_t(i(0,t) - I_r(0,t)) \quad (4)$$

and

$$V(L,t) = 2V_p + Z_k(i(L,t) + I_r(L,t)). \quad (5)$$

We will also need to compute the total Lorentz force on an electron passing through the structure. It can be shown that  $\vec{E} + \vec{v} \times \vec{B}$  is proportional to the quantity  $V^*$  defined as

$$V^* = V - Z_k i \quad (6)$$

We solve these equations by Laplace transforming in  $t$  to  $s$ . By using the method of variation of parameters we find that

$$\begin{aligned} \tilde{V}^*(z,s) = & \left[ 2\tilde{V}_p(s) + \tilde{I}_r(L,s)Z_k e^{-\frac{2sL}{c}} \right] e^{-\frac{s}{c}(L-z)} \\ & + Z_k \int_z^L dz' \tilde{g}(z',s) e^{-\frac{s}{c}(L+z') + \frac{s}{c}(z-z')} \end{aligned} \quad (7)$$

Note that the force does not depend on the upstream termination impedance. This is due to the fact that waves moving in the positive  $z$  direction have the magnetic force canceling the electric force. Only waves moving upstream will exert a force on the beam. Since the downstream termination is matched to the line any waves reflecting off the upstream termination exert no force and leave the problem when they arrive at the downstream termination.

### 3 BEAM DYNAMICS

In order to compute the behavior of a slice of the beam we need  $V^*(z,\tau)$ . We can invert equation (7) and use the definition of  $\tau$  to obtain

$$\begin{aligned} V^*(z,\tau) = & 2V_p \left( \tau + \frac{2z}{c} \right) + I_r \left( L, \tau - \frac{2L}{c} + \frac{2z}{c} \right) Z_k \\ & + Z_k \int_z^L dz' g \left( z', \tau + \frac{2z}{c} - \frac{2z'}{c} \right) \end{aligned} \quad (8)$$

Let us examine the consequences of equation (8) for the usual case. We set  $V_p=0$  and take a parallel beam trajectory for the source charge so that  $g$  vanishes and

$$I_r = \frac{Q\eta x_0}{b} \delta(\tau) \quad (9)$$

where  $x_0$  is the particle offset,  $Q$  is its charge,  $b$  is the kicker electrode radius and  $\eta$  is a geometric factor. We thus find that  $V^*$  is given by

$$V^*(z,\tau) = \frac{Q\eta x_0 Z_k}{b} \delta \left( \tau - \frac{2L}{c} + \frac{2z}{c} \right) \quad (10)$$

We can integrate the transverse force over the length of the kicker to obtain the wake function as

$$W(\tau) = \frac{\alpha Q \eta x_0 c}{2b} \left[ \theta(\tau) - \theta \left( \tau - \frac{2L}{c} \right) \right] \quad (11)$$

where  $\alpha$  is another geometric factor. We can now find the transverse impedance by taking the Fourier transform of the wake function.

$$Z_{\perp}(\omega) = \frac{i}{Qx_0} \int_{-\infty}^{\infty} W(\tau) e^{-i\omega\tau} d\tau \quad (12)$$

to obtain

$$Z_{\perp}(\omega) = \frac{\alpha \eta Z_k L}{2b} \left[ \frac{1 - e^{-\frac{2i\omega L}{c}}}{\frac{\omega L}{c}} \right] \quad (13)$$

the normalized real and imaginary parts of which are plotted in Fig. 3 and 4 respectively as a function of  $x \equiv \omega L / c$ . These forms match those found previously [1].

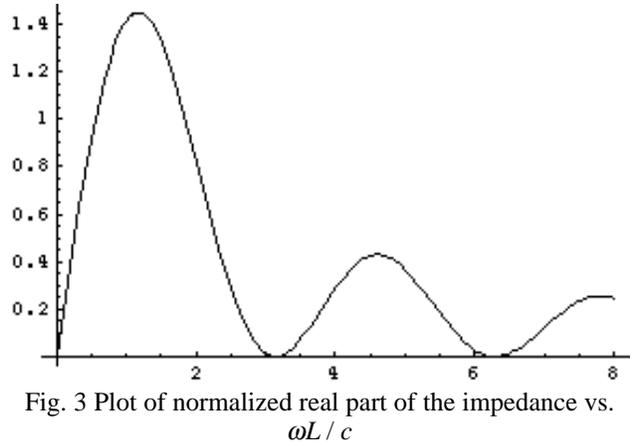


Fig. 3 Plot of normalized real part of the impedance vs.  $\omega L / c$

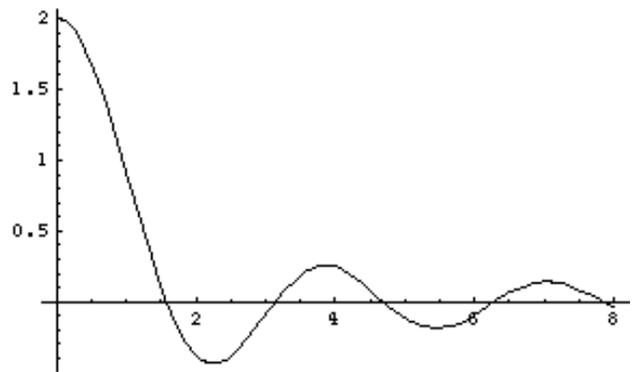


Fig. 4 Normalized plot of the imaginary part of the impedance vs.  $\omega L / c$

#### 4 ASYMPTOTIC DEFLECTION

We may use the expression for  $V^*$  to find the self-consistent displacement of the beam inside the kicker due to the action of the wakefields. Let us consider the case of a continuous beam with no applied voltage. The dipole return current is given by

$$I_r = -\frac{2I_b(\tau)x(z, \tau)}{\pi b} \sin\left(\frac{\phi_0}{2}\right) \quad (14)$$

where  $I_b$  is the beam current ( $I_b$  is  $<0$  for electrons) and where  $\phi_0$  is the angle subtended by the driven stripline. Let us consider the case of a long electron beam and put  $I_b = -I_B$  where  $I_B$  is a positive constant. Inserting the appropriate geometric factors we may write the differential equation of motion for a slice of the beam as

$$\frac{\partial^2 x(z, \tau)}{\partial z^2} = \frac{2I_B}{I_c L^2} \left[ x\left(L, \tau - \frac{2L}{c} + \frac{2z}{c}\right) - \int_z^L \frac{\partial}{\partial z'} x\left(z', \tau + \frac{2z}{c} - \frac{2z'}{c}\right) dz' \right] \quad (15)$$

Let us now Laplace transform this equation in  $\tau$  to  $s$ .

$$\frac{\partial^2 \tilde{x}}{\partial z^2} = \frac{2I_B}{I_c L^2} \left[ \tilde{x}(L, s) e^{-\frac{2sL}{c} + \frac{2sz}{c}} - \int_z^L \frac{\partial}{\partial z'} \left[ \tilde{x}(z', s) e^{\frac{2sz}{c} - \frac{2sz'}{c}} \right] dz' \right] \quad (16)$$

The quantity  $I_c$  is the "critical current" and is given by

$$I_c \equiv \frac{\pi Z_0 b^2}{16 Z_k L^2} \frac{\gamma\beta^2 I_0}{\sin^2\left(\frac{\phi_0}{2}\right)} \quad (17)$$

where  $Z_0$  is the impedance of free space (377 ohms) and  $I_0$  is approximately 17kA.

Upon integrating (16) by parts we obtain

$$\frac{\partial^2 \tilde{x}(z, s)}{\partial z^2} = \frac{2I_B}{I_c L^2} \left[ \tilde{x}(z, s) - \frac{2}{c} \int_z^L \tilde{x}(z', s) e^{\frac{2sz}{c} - \frac{2sz'}{c}} dz' \right] \quad (18)$$

which can be solved in the asymptotic limit for large  $\tau$ . This limit corresponds to the limit  $s \rightarrow 0$ . Therefore, in the limit  $\tau \rightarrow \infty$  we have

$$\frac{\partial^2 x(z, \tau)}{\partial z^2} = \frac{2I_B}{I_c L^2} x(z, \tau) \quad (19)$$

which can be solved to yield

$$x(z, \tau) = x(0, \tau) \cosh\left(\sqrt{\frac{2I_B}{I_c}} \frac{z}{L}\right) + \frac{x'(0, \tau)L}{\sqrt{\frac{2I_B}{I_c}}} \sinh\left(\sqrt{\frac{2I_B}{I_c}} \frac{z}{L}\right) \quad (20)$$

Thus the input position and angle are both amplified by the factor  $\cosh\left(\sqrt{2I_B/I_c}\right)$  at the exit of the kicker.

#### 5 CONCLUSIONS

We have studied the deflection due to beam induced voltages in a stripline transverse kicker. The asymptotic displacement of the beam position at the kicker output is predicted as a function of the beam current, kicker impedance and dimensions of the structure. In the limit of infinitely stiff beams the usual result is recovered for the transverse impedance.

#### 6 ACKNOWLEDGMENTS

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