KV-BEAM IN A DISPERSIVE CHANNEL

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Abstract

The interplay between dispersion and space charge in circular accelerators or storage rings is investigated by looking for self-consistent solutions of the Vlasov-Poisson equation that generalize the KV-distribution to the case where dispersion is present. The results show a growth of the rms quantities describing the beam distribution with the momentum deviation and tune depression. The growth, however, is modest for realistic values of these parameters.

1 INTRODUCTION

The combined effect of space charge and dispersion is one of the issues to be investigated in the Electron Ring at the University of Maryland [1]. The goal is to produce, maintain and study a beam with a depressed-tune factor in the range $0.2 \div 0.4$. In this range of highly space charge dominated beams the answer to the question of whether the usual single-particle treatment of dispersion is still justified is not obvious and very little can be found in accelerator physics literature on this topic [2]. In order to get an insight into the scale of the problem, we studied a simplified model of the Electron Ring dynamics. In the model we assume the smooth approximation in which the external focusing functions and the radius of curvature are constant. Moreover, all the nonlinearities due to the external focusing are neglected. The study has been carried out by looking for self-consistent solutions of the Vlasov-Poisson equations.

2 DISPERSION

Dispersion is usually characterized in terms of the dispersion function $D(z)$, solution of the equation (see e.g. [3]):

$$D(z)'' + k(z)D(z) = \frac{1}{\rho(z)}.$$

where $\rho(z)$ is the local radius of curvature and $\delta = \frac{\Delta p}{p_0}$ describes the relative deviation from the designed momentum p_0.

In a multi-particle perspective we are interested in describing how the beam distribution is affected by the presence of dispersion. Consider a beam of non-interacting particles with a gaussian distribution in both the transverse variables and the longitudinal momentum. In other words, let the beam be described by the following distribution function:

$$f(x, p_x, y, p_y, \delta) = \frac{f_o}{\delta_o\sqrt{\pi}} e^{-\frac{\delta^2}{\delta_o^2}} \exp(-\frac{I}{T}),$$

where I is an invariant for the system and T a constant. An obvious choice for I is

$$I = \frac{1}{2}(p_x - \delta D(z)')^2 + \frac{1}{2}k(x - \delta D(z))^2 + \frac{1}{2}p_y^2 + \frac{1}{2}ky^2,$$

where, for semplicity, we have supposed that the focusing function k is constant. The second moment of the distribution can be easily seen to be:

$$< x^2 >_{\delta_o} = \frac{< x^2 >_o}{1 - \frac{\delta_o^2 D(z)^2}{2 < x^2 >_o}} \simeq < x^2 >_o + D(z)^2 < \delta^2 > . \tag{1}$$

Here we wrote $< x^2 >_o$ to indicate the second moment of the distribution when $\delta_o \to 0$ and the last equality holds for small δ_o. We see that if we neglect the mutual interaction among the particles, the dispersion function turns out to be a measure of the second moment of the distribution.

One of the questions we want to address in this paper is how the relation (1) changes when we allow space charge effects to enter the picture. A consequence of the presence of space charge is to modify the strength of the effective focusing forces acting on the particles and therefore to *depress* the tune ν_o. In the smooth approximation and absence of space charge $D = \rho_o/\nu_o^2$. We can question whether in presence of space charge the expression for the second moment of the distribution can be recovered from (1) by the change $\nu \leftrightarrow \nu_o$ in the expression for the dispersion (ν_o is the undepressed tune, ν is the depressed tune as calculated for an equivalent KV-beam in absence of dispersion). As we will see, the estimate we get in this way, while working for a moderate space charge, fails for higher tune depression giving a very high upper limit.

Finally, notice that (1) provides the natural generalization for the definition of dispersion in the presence of space charge.

3 THE VLASOV-POISSON EQUATIONS

Our model is described by a Hamiltonian [3] $H = H_\perp + H_\parallel$, where $H_\parallel = \frac{m^2c^4}{E_o^2}\delta^2$ is a purely longitudinal term and

$$H_\perp = \frac{1}{2}(p_x^2 + p_y^2) + \frac{k}{2}(x^2 + y^2) - \frac{x}{\rho_o}\delta + g_o\psi,$$

(with $g_o = q/mv_z^2\gamma^3$).

The Hamiltonian refers to a beam of particles of charge q and mass m in a smooth circular channel. The self-force is described by the potential ψ. The designed momentum, longitudinal velocity, and the corresponding relativistic factor are p_o, v_z, and γ.

Since the Hamiltonian is time independent the momentum deviation δ is a constant of the motion. Clearly $H_\perp$ is also an integral of the motion.

We want to search for self-consistent solutions of the Vlasov-Poisson equation associated with H:

$$\frac{\partial f}{\partial z} + \{f, H\} = 0, \qquad \nabla^2\psi = -\frac{q}{\epsilon_0} n(x, y),$$

where

$$n(x, y) = \int\int\int d\delta dp_x dp_y f(x, p_x, y, p_y, \delta). \quad (2)$$

In particular, we want to look for a stationary solution $\frac{\partial f}{\partial z} = 0$. We recall that any function of integrals of motion of an Hamiltonian system is a stationary solution of the corresponding Vlasov equation. Therefore, a particular stationary solution of the Vlasov equation associated with the Hamiltonian H is given by:

$$f(x, p_x, y, p_y, \delta) = f_\parallel(\delta) f_\perp(H_\perp).$$

4 GENERALIZED KV BEAM

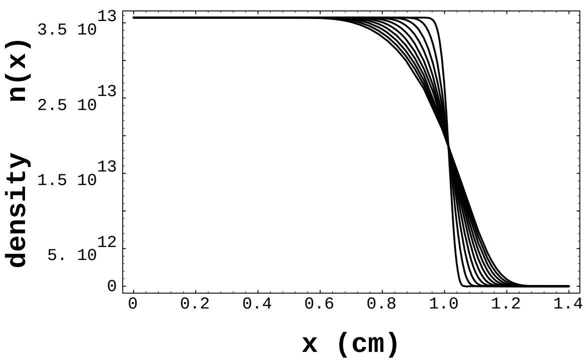

Figure 1: Section of $n(x, y)$ at $y = 0$ for different values of δ_o, ranging from .001 to .01, ($I = .105$ A, ν/ν_o =0.317).

We look for solutions of the Poisson-Vlasov equation describing a beam with a gaussian distribution of the longitudinal momentum and a KV-beam like distribution in the transverse plane:

$$\begin{aligned} f_\parallel(\delta) &= \frac{1}{\delta_o\sqrt{\pi}} e^{-\frac{\delta^2}{\delta_o^2}}, \\ f_\perp(H_\perp) &= f_0 \delta(H_\perp - H_0). \end{aligned}$$

In the limit $\delta_o \to 0$, we recover the usual KV distribution. The corresponding space density (see eq. 2) can be expressed in terms of the Gauss error function:

$$n(x, y) = \pi f_o [erf\left(\frac{\lambda(x, y)\rho_o}{\delta_o |x|}\right) + 1]. \quad (3)$$

with

$$\lambda(x, y) = H_o - \frac{k_x}{2} x^2 - \frac{k_y}{2} y^2 - g_o \psi(x, y).$$

Therefore the Poisson equation then reads:

$$\nabla^2\psi = -\frac{q}{\epsilon_o} \pi f_o [erf\left(\frac{\lambda(x, y)\rho_o}{\delta_o |x|}\right) + 1]. \quad (4)$$

The two parameters f_o and H_o are related respectively to the density of the beam and its size. They depend on each other through the normalization equation

$$\begin{aligned} N_L = & \int\int n(x, y) dx dy \\ = & \pi f_o \int\int [erf\left(\frac{\lambda(x,y)\rho_o}{\delta_o |x|}\right) + 1] dx dy, \end{aligned} \quad (5)$$

where N_L is the linear density of the beam, which is related to the current I by the relation $N_L = I/(qv_z)$. When we solve equation (4) for different values of the parameter δ_o we will be interested in comparing solutions corresponding to beams that carry the same current (i.e. same N_L). After setting f_o to a fixed value, we shall use equation (5) to determine H_o.

4.1 Emittance Calculation

The beam distribution can be charcaterized in terms of the emittance and related rms quantities. The rms emittance in the horizontal plane is:

$$\epsilon_x = (<x^2><p_x^2> - <xp_x>^2)^{1/2}.$$

For a KV round beam of radius a in absence of dispersion $\epsilon_{xo} = a\sqrt{H_o/8}$. When dispersion is present it is possible to reduce the expression for the rms quantities to the calculation of double integrals over x and y. Although these integrals cannot be carried out analytically, it is possible to find out the scaling with respect to δ_o using simple arguments. It turns out that

$$(\epsilon_x - \epsilon_{xo}) \propto \delta_o^2. \quad (6)$$

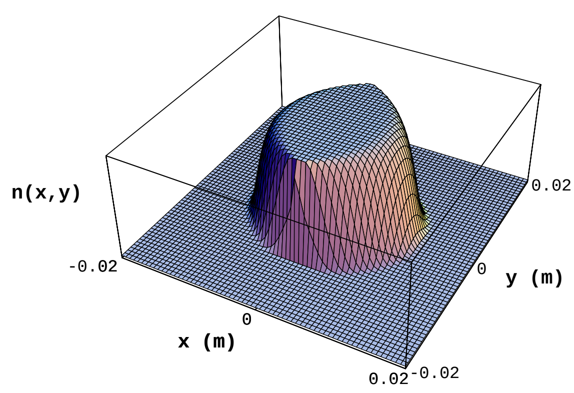

Figure 2: Density distribution $n(x, y)$ for $I = .105$ A, ν/ν_o =0.317, $\delta_o = .01$.

4.2 The Numerical Solution

Beam Energy E_o	10 keV
Tune ν_o	7.6
Focusing func. k	17.437 m^{-2}
Radius of curv. ρ_o	1.82 m

In solving numerically equation (4) we have used the Succesive Overrelaxation Method (SOR) [4].

We show two sets of solutions. The first set of solutions describes beams carrying the same current (I=.105 A, corresponding to a tune depression factor $\nu/\nu_o = .317$. [1]) for various values of the rms momentum spread. In particular, δ_o ranges between 10^{-2} and 10^{-3}. The results, in terms of the horizontal profile ($y = 0$) of the beam density $n(x, y)$, see (3), are shown in Fig. 1.

Fig. 2 shows the full density function in the (x, y) plane for $\delta_o = .01$. The scaling of the emittance with respect to δ_o (6) has also been checked and is shown in Fig. 3. The curve in the picture is a parabola obtained by numerical fitting of the first four points.

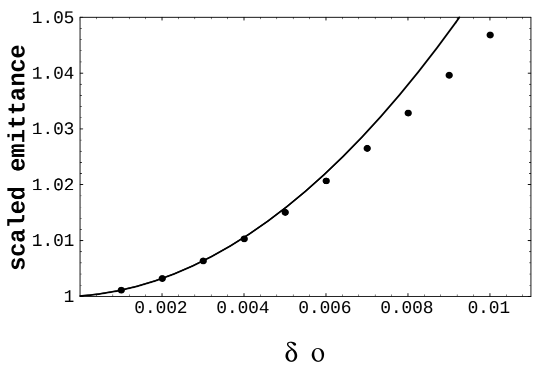

Figure 3: Scaled horizontal emittance (ϵ_x/ϵ_{xo}) as a function of δ_o, (I= .105 A, ν/ν_o =0.317).

In the second set of solutions the parameter δ_o is kept fixed ($\delta_o = .01$), while the value of the beam currents is being varied. Ten different currents have been considered, ranging from $I = .02$ to $I = .112$ A, and corresponding to tune depression factors ranging from $\nu/\nu_o = .91$ to $\nu/\nu_o = .20$. In each case the scaled rms values of the horizontal size of the beam $(< x^2 >_{\delta_o} / < x^2 >_o)^{\frac{1}{2}}$ have been calculated and are shown in Fig. 4. The gray curve in the picture is described by eq. (1) with $D = \rho_o/\nu^2$, (ν replacing ν_o). Although eq. (1) was derived for a gaussian beam we can see that it gives a good approximation also for a KV distribution if the ratio ν/ν_o is sufficiently high. However, it gives increasingly and excessively larger values as ν/ν_o decreases.

5 CONCLUSIONS

As expected the presence of dispersion increases the emittance in the horizontal plane. However, for a tune depression factor of $\nu/\nu_o \sim .3$ and $\delta_o = .01$ the growth in emittance, relative to the case where no dispersion is present, remains below 5%.

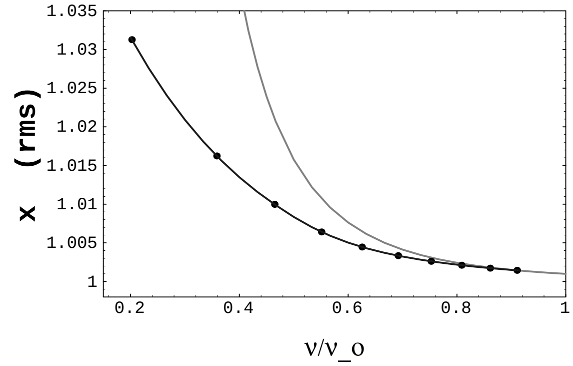

Figure 4: $(< x^2 >_{\delta_o} / < x^2 >_o)^{\frac{1}{2}}$ as a function of ν/ν_o, ($\delta_o = .01$).

6 ACKNOWLEDGMENTS

We thank N. Brown and S. Bernal for useful discussions. The work was supported by the US Department of Energy.

7 REFERENCES

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[1] The tune depression is evaluated for the KV-beam in the limit $\delta_o \rightarrow 0$. Only in this limit the tune depression is a well defined quantity.