SIMULATIONS OF NON-LINEAR BEAM DYNAMICS EFFECTS DUE TO AN ELECTROMAGNETIC ELLIPTICAL WIGGLER

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Abstract

We present the results of extensive beam dynamic simulations that have been carried out in order to study the non-linear effects of an electromagnetic elliptical wiggler, presently under construction for ELETTRA, using different symplectic tracking methods implemented in the codes BETA and RACETRACK.

1 INTRODUCTION

A novel electromagnetic elliptical wiggler (EEW) has been designed and is currently under construction for the ELETTRA facility [1]. The device is designed in order to provide a source of circularly polarised light in the VUV/Soft X-ray region with a variable helicity up to 100 Hz switching rate. The device has a maximum vertical on-axis field of 0.5 T, a horizontal field of 0.1 T, and 15 periods of length 0.212 m.

In this report we present the results of extensive beam dynamic simulations that have been carried out in order to study the non-linear effects of the device and to optimise its performance in ELETTRA. It has already been noticed in previous simulations [2,3] that the introduction of a helical insertion device in the ELETTRA lattice may lead to significant reduction in the dynamic aperture due to the non-linear terms present in the extra field components compared to standard plane insertion devices. It was expected therefore that special care would have to be taken in the design of the pole shapes in order to assure the least impact on the beam dynamics within the mechanical and magnetic possibilities of the design. For this purpose, extensive simulations have been carried out for various pole configurations with the two codes BETA and RACETRACK. Since 3D magnetic field calculations showed that the transverse field distributions differed significantly from the standard model based on circular and hyperbolic functions [4], and also the existence of higher longitudinal harmonics, the two codes have been modified in order to include a more realistic transverse and longitudinal field distribution. Section 2 deals firstly with the field representation, while the results of the simulations are presented and compared in Section 3.

2 MODEL OF THE WIGGLER FIELD

In order to accurately model the magnetic field produced by the wiggler it has to be taken into account that due to the large period to gap ratio, the longitudinal distribution of the fields is non-sinusoidal, with strong odd harmonics present, and also that due to the unusual geometry of the poles, the transverse field distribution is quite complex (see fig. 1).

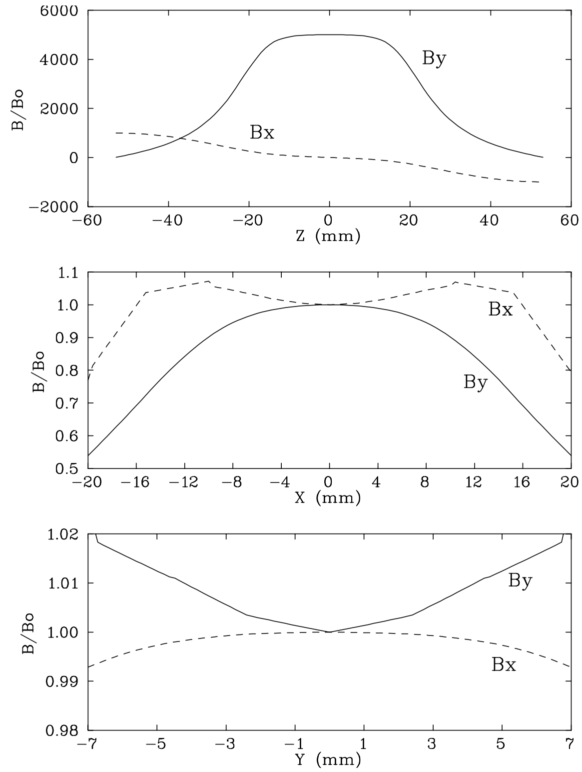

Figure 1. Field distributions along the wiggler axis (upper) and transversely in the horizontal (x) and vertical (y) directions.

A suitable description of the field can be obtained using a polynomial expansion for the off-axis field variation in the following form:

$$\phi_h = \sum_n B_{hn}\{x + a_{2hn}x^3 + b_{2hn}y^2x + a_{3hn}x^5 + b_{3hn}y^2x^3 + c_{3hn}y^4x + a_{4hn}x^7 + b_{4hn}y^2x^5 + c_{4hn}y^4x^3 + d_{4hn}y^6x\}\cos nkz$$

$$\phi_v = \sum_n B_{vn}\{y + a_{2vn}y^3 + b_{2vn}x^2y + a_{3vn}y^5 + b_{3vn}x^2y^3 + c_{3vn}x^4y + a_{4vn}y^7 + b_{4vn}x^2y^5 + c_{4vn}x^4y^3 + d_{4vn}x^6y\}\sin nkz$$

where ϕ_h and ϕ_v are the scalar potentials corresponding to the horizontal and vertical poles respectively, and $k=2\pi/\lambda_0$. The coefficients are derived by fitting the expressions above to the Fourier-analysed computed field distribution. The fit is in practice performed only in the horizontal plane, the remaining terms being derived by the following relations, imposed by Maxwell's equations :

$$6a_{2v1} = k^2 - 2b_{2v1} \qquad 6a_{2v3} = 9k^2 - 2b_{2v3}$$

$$6b_{3v1} = k^2 b_{2v1} - 12c_{3v1} \qquad 6b_{3v3} = 9k^2 b_{2v3} - 12c_{3v3}$$

$$6c_{4v1} = k^2 c_{3v1} - 30d_{4v1} \qquad 6c_{4v3} = 9k^2 c_{3v3} - 30d_{4v3}$$

$$20a_{3v1} = k^2 a_{2v1} - 2b_{3v1} \qquad 20a_{3v3} = 9k^2 a_{2v3} - 2b_{3v3}$$

$$20b_{4v1} = k^2 b_{3v1} - 12c_{4v1} \qquad 20b_{4v3} = 9k^2 b_{3v3} - 12c_{4v3}$$

$$42a_{4v1} = k^2 a_{3v1} - 2b_{4v1} \qquad 42a_{4v3} = 9k^2 a_{3v3} - 2b_{4v3}$$

By comparing the fit with the actual field distribution at arbitrary positions it was concluded that sufficient accuracy was obtained using only the 1st and 3rd harmonics. In order to evaluate the effects of different pole profiles on the beam dynamics, various shapes of both the horizontal and the vertical poles have been considered. Figure 2 shows the transverse distribution of the two field components under the respective poles for different pole profiles. Not shown are the variations in the other plane, namely By(y) and Bx(x). The fits also demonstrated the different transverse variations for the 1st and 3rd harmonics, a flat variation of the total field being due to a different curvature of the 1st and 3rd harmonics.

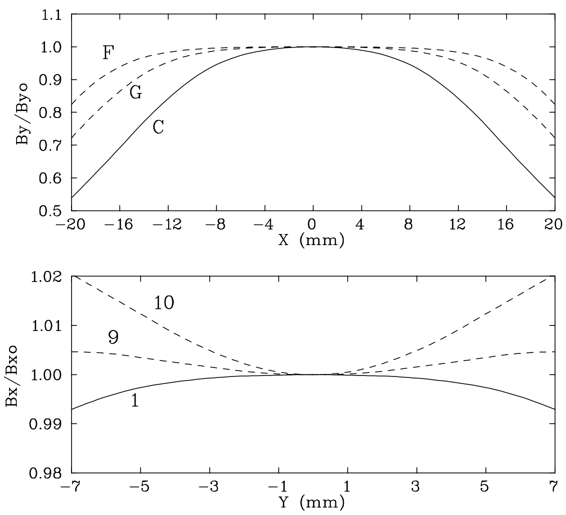

Figure 2. Field distributions due to various vertical (upper) and horizontal (lower) pole profiles.

3 BEAM DYNAMICS

New tracking routines, which take into account the polynomial expression of the field described in the previous section, have been added to the codes BETA and RACETRACK. Both integration methods implemented in the codes are symplectic up to second order [5,6]. In order to check the correct functioning of the two routines, comparisons of tracking results and phase space plots were firstly made. The results show an overall good agreement with occasional differences of 1 mm, which can be considered within the precision of the methods. During this process however the importance of setting the correct initial conditions at the entrance and at the exit of a helical insertion device emerged. In reality, insertion devices have end poles which set the particles on the

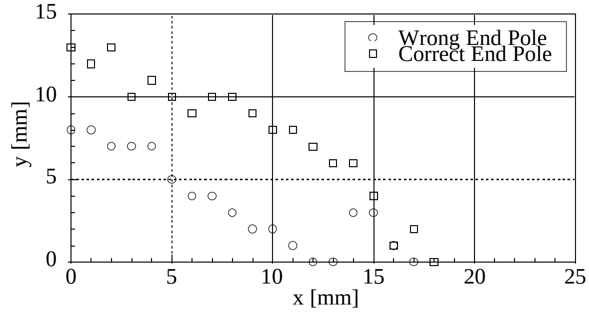

Figure 3. Comparison of tracking results with correct and incorrect treatment of the end poles.

appropriate trajectory but which however are not included in the model. In order to simulate these, both codes shifted the particle's coordinates and angles by a constant amount corresponding to the closed orbit. It was found that this procedure was not sufficient to guarantee the correct entrance and exit conditions, especially at large amplitudes due to the non-linear nature of the fields. The two codes adopted different equivalent solutions to overcome the problem. While BETA has included an end pole on either side of the device of reduced field amplitude [7], RACETRACK has adopted the solution of imposing the continuity of the conjugate momenta by adjusting properly the slope. Figure 3 shows the effect on the dynamic aperture when incorrect conditions are used. The problem of the end poles arises also in the case of a planar device, if the model takes a sine variation longitudinally.

Four particles with different initial conditions were tracked over 1000 turns and for 0.0 and ±2.5% energy deviation corresponding to the ELETTRA momentum acceptance. The linear effects of the EEW on the optics are modest, giving rise to tune shifts less than 0.01 and beta asymmetries less than 5% in both planes. In this respect, retuning of the machine to the nominal tunes of 0.3 horizontally and 0.2 vertically did not lead to any significant improvement of the dynamics.

Figure 4 illustrates the dynamic aperture investigating different pole configurations for the vertical field. The three different cases correspond to the pole shapes specified in the previous section. It can be seen that the major differences occur at small horizontal amplitudes,

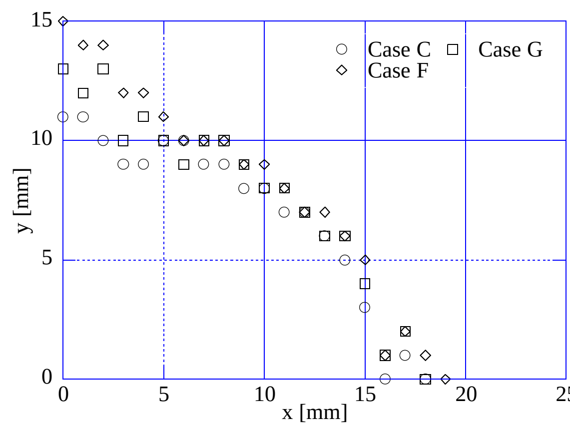

Figure 4. Comparison of dynamic apertures for various vertical pole profiles.

but with vertical amplitudes which are anyway outside the 15 mm full physical aperture of the machine. Tracking particles which were off momentum by ± 2.5% showed also no substantial differences between the three investigated cases. Thus case C, being the optimal configuration from the iron saturation point of view, was chosen as the final design for the vertical pole.

Figure 5 shows the dynamic aperture for different horizontal field configurations, using case C for the vertical poles. Here, the difference among the various cases is even less pronounced, thus Pole 1 was chosen for the final design. Figure 6 instead compares the dynamic apertures of the device with its final poles with the bare lattice when one introduces the 15 mm physical aperture of the machine at the entrance and exit of the device.

Dynamic apertures computations were also performed including all of the five plane insertion devices presently installed in ELETTRA. The result is shown in figure 7.

During the pole shape optimization, also the influence of the presence of the third harmonics was investigated by setting to zero the corresponding field amplitudes. Results showed that the dynamics are determined mainly by the first harmonic.

Another important issue which was investigated is the effect of a positioning error of the device. This was studied by creating in both planes symmetric and asymmetric bumps in the device of 1.0 mm and 0.5 mm with 0.34 mrad respectively. Even though a slight increase in the coupling was noticed, there was no substantial difference in the dynamic apertures with respect to the non-misaligned case.

4 CONCLUSION

The correct procedure for treatment of insertion device end fields in tracking routines has been determined. Simulations show that the EEW should have only a minor effect on the beam dynamics in ELETTRA.

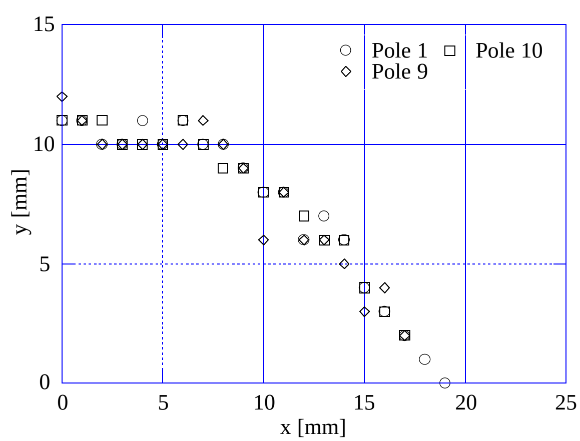

Figure 5. Comparison of dynamic apertures for various horizontal pole profiles.

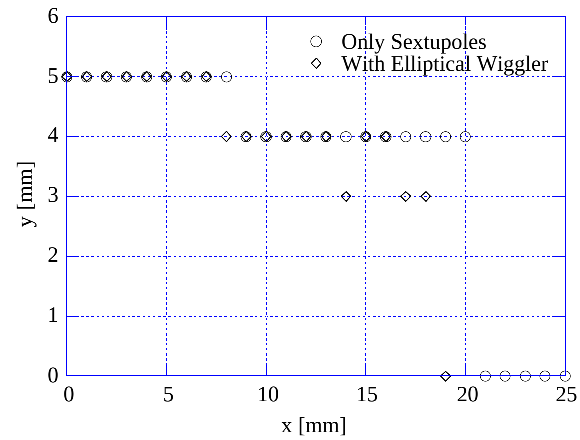

Figure 6. Dynamic aperture of the EEW including the physical aperture of the vacuum chamber.

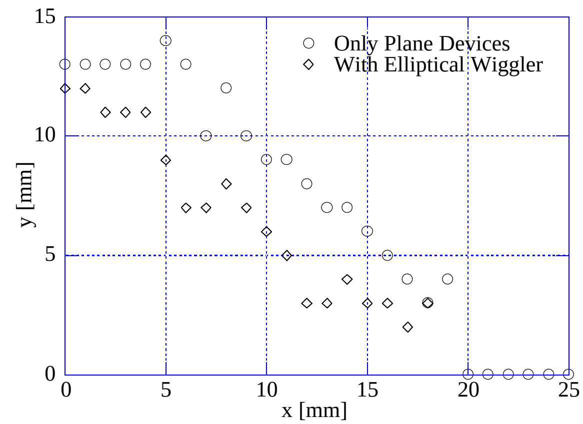

Figure 7. Comparison of dynamic aperture of the EEW with other ELETTRA insertion devices.

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