GdfidL: A FINITE DIFFERENCE PROGRAM WITH REDUCED MEMORY AND CPU USAGE

W. Bruns*, Technische Universität Berlin, Einsteinufer 17, 10587 Berlin

Abstract

Two new field solvers based on the FDTD Algorithm are described. One solver computes resonant fields in loss-free structures, the other computes time dependent fields, scattering parameters and wake potentials in lossy or loss-free structures. The program uses linked lists instead of three dimensional arrays to model the computational volume. The main advantage of this approach is: Grid cells are only needed in regions, which are not filled with electric or magnetic conducting materials. This may result in an appreciable reduction of memory and CPU usage. The modeling via the linked lists is described in detail and some examples of calculated realistic geometries are given.

1 GENERAL METHOD

GdfidL [1] is another Finite-Difference Code that approximately solves MAXWELLs equations by discretizing the curl operators. For homogeneous materials this discretization was invented by Yee [1]. In case of inhomogeneous dielectric or permeable materials, the discretization of the curl operators is the same as by Weiland [2].

1.1 Curl Equations with linked lists

The discretized curl operators are a recipe how to compute the time derivative of a field component in the grid from the values of the four surrounding dual components. By attaching a triple index i, j, k to each field component in the computational volume, the resulting difference equations are very simple. Eg. the $\frac{d}{dt}B_z$ component in a lossfree region is approximated as

$$\frac{d}{dt}B_z^{i,j,k} \approx \frac{\Delta_x(E_x^{i+1,j,k} - E_x^{i,j,k}) + \Delta_y(E_y^{i,j+1,k} - E_y^{i,j,k})}{\Delta_x\Delta_y}$$

(1)

One of the advantages of this simple difference equation is: it is easy to understand and easy to implement in a computer program by declaring triple indexed variables. However for many geometries this approach has a disadvantage: Grids cells are also assigned to regions where the material is ideal conducting.

GdfidL uses an equation that looks slightly more complicated: Cells do not carry a triple index, but only a single index: The $\frac{d}{dt}B_z$ component now looks as

$$\frac{d}{dt}B_z^l \approx \frac{\Delta_x(E_x^{n_y^+(l)} - E_x^l) + \Delta_y(E_y^{n_x^+(l)} - E_y^l)}{\Delta_x\Delta_y}$$

(2)

---

* bruns@tetibm2.ee.TU-Berlin.DE
[1] Pronounciation is: Gee-Dee-fidl

The indices $n_x^+(l), n_y^+(l)$ are the indices of the neighbour cell in positive $x-$, and $y-$direction of the cell with index l. These indices and four other indices $n_z^+(l), n_x^-(l), n_y^-(l), n_z^-(l)$ constitute the linked lists.
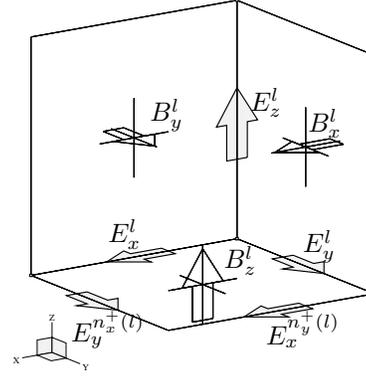
Figure 1: The l.th gridcell with its six field components. Additionally the two field components of neighbour cells are shown which are needed to compute the curl of E at the position of B_z^l.

Equations similiar to (2) for all components of B in the computational volume can be written as a single matrix equation:

$$\frac{d}{dt}\vec{b} = -\mathbf{C^e}\vec{e}$$

(3)

Here $\vec{b}$ and $\vec{e}$ are arithmetic vectors containing all the field components of E and B.

The computation of $\frac{d}{dt}E$ is slightly more complicated but can also be written as a matrix equation with a matrix $\mathbf{C^{h*}}$:

$$\frac{d}{dt}\vec{e} = [\varepsilon]^{-1}\mathbf{C^h}[\mu]^{-1}\vec{b} = \mathbf{C^{h*}}\vec{b}$$

(4)

1.2 Time domain

Given the discretized curl operators, the fields at the next timestep are computed from the previous fields by the leap-frog scheme [1].

1.3 Resonant fields

Resonant fields in principle can be found by exciting a structure with some broadband pulse and examining the ringing of the fields. If one is interested in resonances that are not well separated in frequency, such an approach needs a long simulation time T, since the frequency resolution Δf of such an approach is $\Delta f = 1/T$.

GdfidL instead solves an eigenvalue problem that is directly derived from the discretized curl equations. The time

derivatives in eqns. (3) (4) can be replaced by jω's following from the FOURIER transform. By substituting $\vec{b}$ in the second equation by the first equation, one gets:

$$-\mathbf{C^{h*}}\mathbf{C^e}\vec{e} = -\omega^2\vec{e} \qquad (5)$$

This is an algebraic eigenvalue problem with a real positive semidefinite matrix $A = \mathbf{C^{h*}}\mathbf{C^e}$. The lowest nonzero eigenvalues and their corresponding eigenvectors are found by an algorithm of Tückmantel [4].

Since the eigenvalue problem is directly derived from the discretized MAXWELLian equations, no non-MAXWELLIAN solutions are found and GdfidL does not have to check for them.

2 SCATTERING PARAMETERS

GdfidL computes S-parameters by performing a time domain simulation and subsequently FOURIER transforming the histories of the tangential fields at the ports of the structure. Absorbing boundary conditions (ABCs) have to be applied where waveguides cross the boundary of the computational volume. GdfidL implements its ABCs as an expansion of the tangential fields in a limited number of orthogonal functions, which are the tangential fields of the TE, TM or TEM-modes in the waveguides. The result of this expansion is: The parts of the field that can be described as a superposition of the expansion functions have a reflection of less than 1% independent of frequency, the remaining part is totally reflected.

3 WAKE POTENTIAL COMPUTATION

3.1 Beam traversing ABCs

For wakepotential computations, a rigid beam is traversing the computational volume. For long range wakes, ABCs have to be applied at the two beampipes where the beam enters and exits the cavities. Since the primary field of the charge cannot be expanded in the waveguide modes, a special treatment is needed: With known velocity of the charge, the primary field of the charge at the ABCs is also known and can be subtracted from the total field. The remaining field consists of the scattered waves and is expanded in orthogonal functions.

As in case of scattering parameter calculation, a reflection of less than 1% is achieved for the parts of the field that can be described by the number of expansion functions selected, the remaining part is totally reflected. Since this algorithm guarantees that no reflection larger than one occurs for any field pattern of any frequency, no late time instability occurs even for wakepotential computation.

3.2 Inhomogeneous mesh in beam direction

GdfidL exactly integrates the charge dependent factor $\vec{G}$ in the update equation (6) for the electric field independent of the selected mesh spacing and of the used time step.

$$\vec{e}^{n+1} = \vec{e}^n + \Delta t \mathbf{C^{h*}}\vec{b}^{n+1/2} - [\varepsilon]^{-1} \int\limits_{t=n\Delta t}^{(n+1)\Delta t} \vec{G}(\tau)\,d\tau \qquad (6)$$

The main advantage is: The time step Δt can be choosen as high as the stability limit allows; Therefore, a given time to simulate can be computed in less time steps.

4 EXAMPLES

4.1 Scattering parameters

Fig. 2 shows a short section of a disk loaded waveguide with attached input/ output couplers. For this case GdfidL needed 30% of CPU that would be needed if the field components were organized as triple indexed variables.
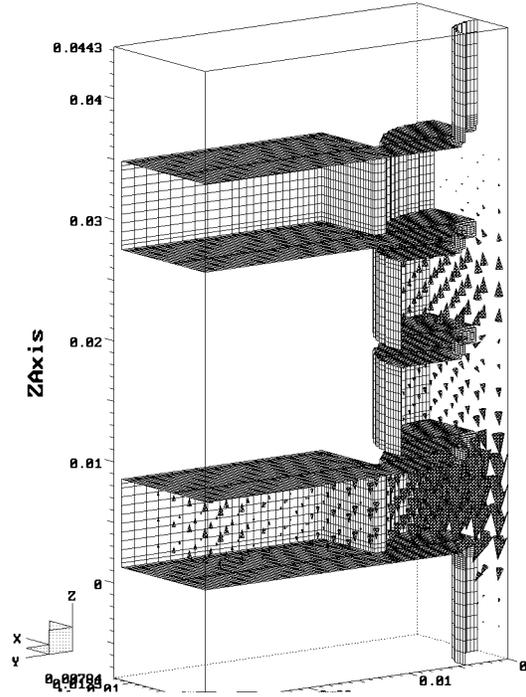
Figure 2: A quarter of a short section of a x-band linac. The shown field was excited by a GAUSSIAN pulse of the fundamental mode in the feeding waveguide left below. At the shown time, the excitation has already died, the visible field is mostly travelling upwards the cavity chain and partly back into the feed.

4.2 Long range Wakepotential

The computation of the long range wake in a simple test-geometry is shown here. It is a rectangular waveguide (w=1cm, h=0.5cm) where a rectangular cavity of depth b=0.5cm and length a=1cm is in. A relativistic GAUSSIAN line charge with $\sigma = 5$ mm is travelling in the center of the waveguide. In order to be able to compare with a code that does not allow inhomogeneous mesh in the direction of the

beam, a homogeneous grid with a spacing of Δ=0.5mm is used. In fig. 4 GdfidLs results are shown both for the short
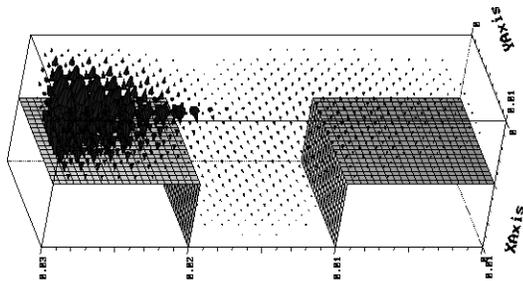
Figure 3: Primary and scattered field of a long GAUSSIAN line charge traversing the test geometry. The charge has entered the volume through an ABC at the right of the volume and is just exiting trough the left ABC. All other boundaries are electric.

and the long range wake. Fig. 5 shows the wake potential as it is computed when Mur's first ABC's [3] are applied. The short range wakes in both cases agree well, the long range wake obviously does not agree. GdfidL uses a timestep of $\Delta t = 0.928$ps and so needed 9094 timesteps for its computation of the wake in the range $0 < s < 2, 5$ m.
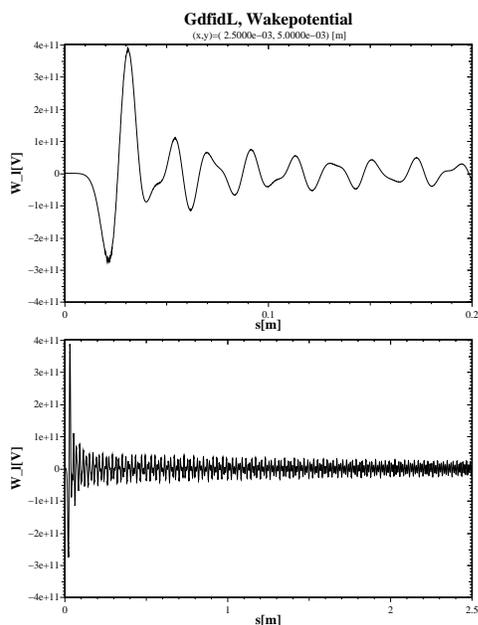
Figure 4: Above: short range wake; Below: Long range wake as computed by GdfidL.

5   AVAILABILITY

The executable programs of GdfidL for most UNIX systems are available free of charge from the author.
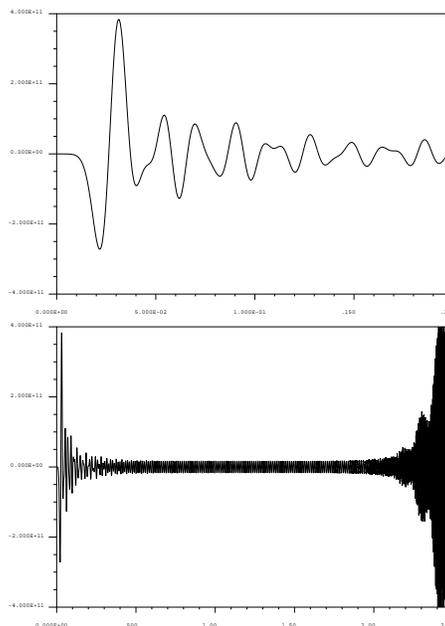
Figure 5: Above: short range wake; Below: Long range wake. Both are computed with Mur's first ABCs.

6   CONCLUSION

GdfidL implements a variant of the method of finite differences. The CPU and memory consumption is proportional to the field carrying volume. Resonant fields are computed without finding nonMAXWELLian solutions. Scattering parameters and long range wakepotentials are computed by applying broadband ABCs. For wakepotential computations, no restriction to homogeneous grids in the direction of the beam exists. No late time instability occurs.

7   ACKNOWLEDGEMENT

8   REFERENCES

[1] Kane S. Yee, "Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media", *IEEE Transactions on Antennas and Propagation*, vol. 14, May 1966

[2] Thomas Weiland, "Eine Methode zur Lösung der Maxwellschen Gleichungen für sechskomponentige Felder auf diskreter Basis", *Archiv für Elektrotechnik und Übertragungstechnik, (Electronics and Communication)*, Band 31 (1977), Seiten 116-120

[3] Gerrit Mur, "Absorbing Boundary Conditions for the Finite-Difference Approximation of the Time-Domain Electromagnetic-Field Equations", *IEEE Transactions on Electromagnetic Compatibility*, vol. EMC-23, No. 4, November 1981

[4] J. Tückmantel, "An improved version of the eigenvector processor SAP applied in URMEL", CERN/EF/RF 85-4, 4 July 1985