MONTE CARLO SIMULATION OF A NONLAMINAR DC BEAM INCLUDING THE INFLUENCE OF SELF-FIELDS

G. Zapalac, Imatron Inc., 389 Oyster Point Blvd.,
So. San Francisco, CA

Abstract

We present a method to simulate the transport of a nonlaminar DC beam under the influence of self-fields. The simulation randomly populates an initial phase space distribution with n particles. These particles are propagated in steps through the transport region, and the particle positions at each step are used in an unbinned maximum likelihood fit for the beam profile distribution. The self-fields at the position of each particle are calculated using the fitted beam profile distribution. The total computation required by the simulation scales linearly with n. The simulation for an electron beam with a phase space given by the Kapchinskij-Vladimirskij distribution is in good agreement with the beam profile predicted by the Kapchinskij-Vladimirskij envelope equations.

1 INTRODUCTION

The optics for a charged particle beam that does not satisfy the condition $\beta = v/c \approx 1$ is influenced by the beam self-forces. These forces include the electrostatic repulsion between the like charges in the beam and the focusing due to the self-magnetic field. The beam self-forces may become an important consideration in the transport region near the output of the particle source for the beam. In some applications, an electron beam is delivered directly from the gun to the target with no further acceleration, and the electrostatic repulsion cannot be tolerated. The beam may then be neutralized by allowing it to ionize a dilute gas[1]. We will denote the ratio of electron to ion charge densities as the beam neutralization factor f. Typically, $f = 1$ immediately after the beam encounters the ionizing gas.

The evolution of the beam envelope for a uniform elliptical beam in the absence of longitudinal magnetic fields may be calculated by integrating the Kapchinskij-Vladimirskij (K-V) equations if the phase space of the beam is given by the K-V distribution[2]:

$$f(x,y,x',y') = \delta\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{a^2x'^2}{\epsilon_x^2}+\frac{b^2y'^2}{\epsilon_y^2}-1\right). \quad (1)$$

The quantities a and b are the semi-axis of the uniform beam profile ellipse in the $x-y$ plane normal to the reference trajectory along the z direction. The slopes x' and y' yield the transverse momenta p_x and p_y in the $x-y$ plane, where p_x, $p_y << p_z$. ϵ_x and ϵ_y are the emittances (area/π) in the $x-p_x$ and $y-p_y$ planes. The K-V equations for $a(z)$ and $b(z)$ in the absence of external fields are given by[2]:

$$\frac{d^2a}{dz^2}-\frac{2K}{a+b}-\frac{\epsilon_x^2}{a^3}=0$$

$$\frac{d^2b}{dz^2}-\frac{2K}{a+b}-\frac{\epsilon_y^2}{b^3}=0$$

$$K=\frac{2Nr_e}{\beta^2\gamma}(1/\gamma^2-f), \quad (2)$$

where r_e is the classical electron radius and N is the number of electrons per unit length of the beam. These equations are easily integrated in the presence of transverse magnetic fields.

A realistic beam is unlikely to satisfy the conditions required by the K-V equations. The initial transverse momentum distribution is more likely to be Maxwellian, and longitudinal magnetic fields may be present. This note describes a Monte Carlo simulation that in principle may be adapted to simulate an arbitrary beam under the influence of self-fields. The simulation has been investigated for a DC beam satisfying the K-V distribution; this allows the results from the simulation to be compared to the beam envelope predicted by the K-V equations.

2 MONTE CARLO SIMULATION

The simulation assumes that the profile of the current density in a plane normal to the reference trajectory may be modeled by a profile function $J(x,y;\vec{\alpha})$, where the parameters $\vec{\alpha}$ are allowed to evolve as the beam propagates. The profile function should be flexible enough to represent the beam profile within the transport region of interest. Choosing an appropriate profile function with a minimum number of parameters is not a trivial task and requires some physical guidance. The profile function chosen to represent the K-V beam is discussed in the next section.

The simulation randomly generates an ensemble of n particles that populate an initial beam profile $J(x,y;\vec{\alpha_0})$ and an initial transverse momentum distribution. These initial distributions must reproduce the beam emittance.

The initial beam profile is used by the simulation to calculate the beam self-fields at the position of each particle. For a round (azimuthally symmetric) beam these fields are easily obtained from the integral

$$2\pi\int_0^R J(\rho^2=x^2+y^2;\vec{\alpha_0})\rho d\rho,$$

where R is the radial position of the particle. If the beam is not azimuthally symmetric, the fields must be obtained by numerical integration.

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Once the self-fields are calculated, the external fields are added and each particle is propagated by a short step dz. Because the self-fields scale with the dimensions of the beam, the step length is chosen to be a scale factor F_s times a parameter representing the size of the beam. If we were propagating a round beam, for example, we might choose this parameter to be the radius enclosing half of the particles. For the uniform elliptical beam used with the K-V distribution, we have chosen the minor semi-axis of the ellipse.

After the particles are propagated by dz, the new $x-y$ positions of the particles are used in an unbinned maximum likelihood fit to determine the parameters $\vec{\alpha}$ of the new beam profile $J(x,y;\vec{\alpha})$. We have used the downhill simplex method[3] to perform the fit, using the parameters from the previous step as initial values. The self-fields are recalculated at the new particle positions using the new beam profile, and the particles are each propagated through the next step. This process continues until the particles have been propagated through the desired transport region, yielding the solution $J(x,y;\vec{\alpha}(z))$ for the beam profile at each step.

The number of computations performed by the simulation scales linearly with the number of particles. For applications requiring a numerical integration of the beam profile, the computation requirements may become quite large. Fortunately, the simulation is ideally suited for distributed processing. Distributed processing has been implemented on the workstation network at Imatron. Several client processes on different CPUs each generate a fraction of the total number of particles. The clients calculate the fields and propagate the particles for each step. The positions of the particles for all of the clients are read over the network by a single server process, which performs the fit to determine the parameters $\vec{\alpha}$. Each client reads $\vec{\alpha}$ back from the server so that it can calculate the self-fields for propagating the particles through the next step.

3 SIMULATION USING THE K-V DISTRIBUTION

A beam with a K-V distribution in phase space has a uniform elliptical profile. We approximate a uniform profile by using a profile function similar to the Fermi-Dirac distribution. For a round beam we perform a one parameter fit to the radius a:

$$J(\rho^2=x^2+y^2;a)=\frac{I_0}{\aleph\left[1+\exp\left(\lambda^2(\eta^2-1)\right)\right]}. \quad (3)$$

I_0 is the total current in the beam, λ controls the cutoff of the profile near the radius a, $\aleph=\pi a^2\log(1+\exp(\lambda^2))/\lambda^2$ is a normalization constant, and $\eta^2\equiv\rho^2/a^2$. Eq. (3) may be immediately generalized to a uniform elliptical beam with semi-axes a and b by setting $\eta^2=x^2/a^2+y^2/b^2$ and $\aleph=\pi ab\log(1+\exp(\lambda^2))/\lambda^2$. The beam profile $J(x,y;a,b)$ then becomes a two parameter fit to a and b.

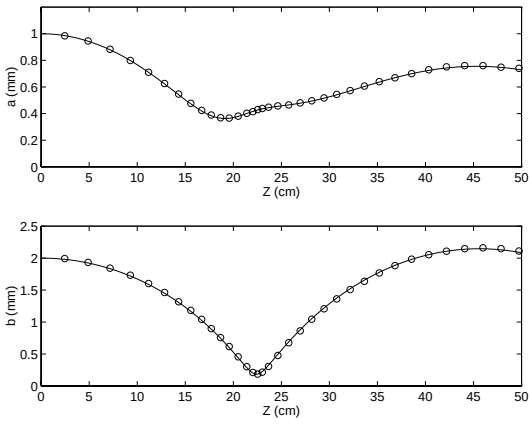

Figure 1: The beam envelope semi-axes predicted by the K-V equations (solid line) and the simulation (open circles) for a 0.635 Amp neutralized electron beam with a kinetic energy of 130 KeV and an emittance of 4.3×10^{-4} π cm-radians.

The initial phase space is populated with n electrons using Eq. (1) with $\epsilon=\epsilon_x=\epsilon_y$. For each electron we generate three random numbers between 0 and 1 which we denote as s_1, s_2, and s_3. Let a_0 and b_0 be the initial semi-axes of the beam profile ellipse. We first map s_1 into the unit area underneath the normalized beam profile distribution and solve for η_s:

$$s_1=\frac{\pi a_0 b_0}{I_0}\int_0^{\eta_s} J(x,y;a,b)\eta d\eta. \quad (4)$$

We then have:

$$\begin{aligned}\theta&=2\pi s_2\\ \phi&=2\pi s_3\\ x&=a_0\eta_s\cos\theta\\ y&=b_0\eta_s\sin\theta\\ x'&=\epsilon\sqrt{(1-\eta_s^2)}\cos\phi/a_0\\ y'&=\epsilon\sqrt{(1-\eta_s^2)}\sin\phi/b_0.\end{aligned} \quad (5)$$

The electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$ are generated at the position of each electron by numerical integration of the beam profile. Each surface element $dS'=dx'dy'$ of the beam profile is treated as a filament of current at the position $\mathbf{r}'=(x',y')$ with the current $dI=J(\mathbf{r}';a,b)dS'$. The current filament generates the fields $d\mathbf{E}(\mathbf{r})$ and $d\mathbf{B}(\mathbf{r})$ at the position $\mathbf{r}=(x,y)$ of an electron:

$$\begin{aligned}d\mathbf{E}(\mathbf{r})&=\left(\frac{ec\mu_0}{2\pi\beta}\right)\frac{(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^2}dI\\ d\mathbf{B}(\mathbf{r})&=\left(\frac{e\mu_0}{2\pi}\right)\frac{\hat{z}\times(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^2}dI.\end{aligned} \quad (6)$$

For a profile with elliptical symmetry, we make the substitutions $u=x/a$ and $v=y/b$ to map the ellipse onto a

unit disk in the $u - v$ plane. The disk is divided into two domains: a central core of radius $1 - \pi/4$ containing about 5% of the total area, and a surrounding annulus. The core is integrated by 21 point Gaussian quadrature. The annulus is further subdivided into square tiles using m radial divisions and $8m$ azimuthal divisions. Each tile is integrated by 9 point Gaussian quadrature. We have found that setting $m = 7$ provides a sufficiently accurate determination of the self-fields.

To obtain close agreement with the K-V envelope equation, we have set $F_s = 1/4$ and used several hundred electrons. Figure 1 shows the results from a simulation (plotted at 100 step intervals) of 840 electrons and the K-V envelope equation for a 0.635 Amp neutralized ($f = 1$) beam with a kinetic energy of 130 KeV and an emittance of 4.3×10^{-4} π cm-radians.

4 COMMENTS ON SIMULATING A MAXWELLIAN BEAM

Preliminary investigations are now in progress for a realistic electron beam with a Maxwellian momentum distribution that begins at a round cathode of radius a_0. The slopes x' and y' now have Gaussian distributions with $\sigma = \epsilon/(2a_0)$, where ϵ is the r.m.s. emittance[2]. The profile distribution is expected to include the profile from Eq. (3) because the beam is initially uniform.

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6 REFERENCES

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