COMPARISON OF BEAM-POSITION-TRANSFER FUNCTIONS USING CIRCULAR BEAM-POSITION MONITORS *

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Abstract
A cylindrical beam-position monitor (BPM) used in many accelerator facilities has four electrodes on which beam-image currents induce bunched-beam signals. These probe-electrode signals are geometrically configured to provide beam-position information about two orthogonal axes. An electronic processor performs a mathematical transfer function (TF) on these BPM-electrode signals to produce output signals whose time-varying amplitude is proportional to the beam's vertical and horizontal position. This paper will compare various beam-position TFs using both pencil beams and will further discuss how diffuse beams interact with some of these TFs.

1 BPM SENSITIVITY

BPMs typically have four electrodes on which beam image currents induce bunched-beam signals. These probe signals are initially processed or naturally configured to provide information about the horizontal and vertical axes that describes the beam's position. BPMs may have a variety of cross-sectional shapes, such as circular, rectangular, elliptical, etc. For the circular-cross-section BPM detecting thin beams, sufficient beam position information is contained within $R_{\bar{y}}$. Specifically,

$$R_y = |I_T| / |I_B| \tag{1}$$

where I_T and I_B are the top and bottom BPM-electrode signal amplitudes. I_T and I_B are defined as

$$I_T = \frac{\hat{i}_0}{2\pi R_p}\left[\frac{\theta_0 \, \mathrm{I}_0\left(g\frac{r_0}{R_p}\right)}{\mathrm{I}_0(g)} + \sum_{m=1}^{\infty}\frac{4\,\mathrm{I}_m\left(g\frac{r_0}{R_p}\right)}{m\,\mathrm{I}_m(g)}\sin\left(\frac{m\theta_0}{2}\right)\cos(m\phi_0)\right], \tag{2}$$

and

$$I_B = \frac{\hat{i}_0}{2\pi R_p}\left[\frac{\theta_0 \, \mathrm{I}_0\left(g\frac{r_0}{R_p}\right)}{\mathrm{I}_0(g)} + \sum_{m=1}^{\infty}\frac{4\,\mathrm{I}_m\left(g\frac{r_0}{R_p}\right)}{m\,\mathrm{I}_m(g)}\sin\left(m\left(\pi+\frac{\theta_0}{2}\right)\right)\cos(m\phi_0)\right], \tag{3}$$

where $\hat{i}_0$ is the Fourier component of the beam current, θ_0 is the electrode subtended angle, and r_0 and ϕ_0 are the polar coordinates of the beam position, and R_p is the BPM-probe-electrode radius[1]. The functions I_0 and I_m are the zeroth and mth order-modified-Bessel functions, respectively[1,2]. The term "g" includes the effects of the relative beam velocity, β, and is calculated to be

$$g = 2\pi R_p / \beta\gamma\lambda \tag{4}$$

where γ is the Lorentz factor, $\gamma = \left(1-\beta^2\right)^{-1/2}$.

References 3, 4, and 5 show that Eq. (1) may be described with the less complicated 2-D polynomial equation

$$R_{\bar{y}} = y_0 + S_y\bar{y} + S_{y^3}\bar{y}^3 + S_{yx^2}\bar{y}\bar{x}^2 \tag{5}$$

where y_0 and S_y is the manufactured probe offset and sensitivity, S_{y^3} and S_{yx^2} are third-order nonlinear coefficients, and $\bar{x}$ and $\bar{y}$ is the horizontal and vertical beam position[3,4,5]. While the original equation does not have an analytically expressible inverse function, Eq. (5) does by performing a least-squares inverse fit to the original equation or set of measured BPM data. The resulting equation from this inverse fit procedure may be written as a function of $R_{\bar{x}}$ and $R_{\bar{y}}$.

2 PROCESSOR TRANSFER FUNCTIONS

The electronic BPM processor performs a mathematical TF using the four BPM signals to produce output signals whose time-varying values are proportional to the beam's horizontal and vertical position. Mathematically, a position-processor's TF accepts $R_{\bar{y}}$ as an input and its output signal must be proportional to the beam position. However, in practice, the two output signals from the probe's opposite electrodes are cabled to two processor-input connectors.

To be an effective position measurement TF, the mathematical functions describing the combined BPM and processor TF have several characteristics. First, the effective combined TF output signal, V_{Proc}, must satisfy the odd symmetry equation of

$$V_{\mathrm{Proc}}(\bar{y}) = f(\bar{y}) = -f(-\bar{y}) \tag{6}$$

where f is a particular mathematical function. Note that if the beam is centered (i.e., $\bar{y}=0$), then the processor's output signal is zero. Eq. (5) may also be expressed as a function of $R_{\bar{y}}[dB]$ as

$$V_{\mathrm{Proc}}\left(R_y[dB]\right) = f\left(R_y[dB]\right) = -f\left(-R_y[dB]\right), \tag{7}$$

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where $R_y[dB] = 20\log(R_y)$, therefore, fulfilling the odd symmetry criterion.

Second, it is preferred, but not necessary, that the combined BPM and processor TF be linear or very nearly linear. If this combined function is highly non-linear, then the sensitivity or gain will vary with beam position and either the range or the precision of the overall position measurement will be adversely affected.

Third, it is preferred, but not necessary, that the combined BPM and processor TF have a single-variable analytically expressible inverse function. The existence of an analytic inverse function allows an accelerator control system to easily recalculate the beam position from the processors output signal amplitude. If the TF has an inverse function that is not analytically expressible with specific variables (e.g., R_x and R_y), the control system may still translate the processors output signal into a beam position by using look-up tables stored in the accelerator-control system's memory. However, to meet the required measurement resolution, these look-up tables are often very large.

There are many hardware-realizable mathematical functions for a beam position processor. Some of the more common functions are the difference over sum (Δ/Σ), arctan (AT), log ratio (LR), and normalized power difference (NPD) functions[6]. The forward and analytically expressible inverse TFs for each of these mathematical functions are shown in Table 1. $K_{\Delta/\Sigma}$, K_{AT}, K_{LR}, and K_{NPD} are the processor TF sensitivity or gain constants. Note that for centered beams, the AT function reduces to the Δ/Σ function[7].

Table 1: Processor transfer functions

	Forward TF $V_{Proc}[v] =$	Analytic Inverse TF $R_{\bar{y}}[dB] =$
Δ/Σ	$K_{\Delta/\Sigma}\dfrac{R_y - 1}{R_y + 1}$	$20\log\left(\dfrac{K_{\Delta/\Sigma} + V_{Proc}}{K_{\Delta/\Sigma} - V_{Proc}}\right)$
AT	$K_{AT}\left[\tan^{-1}(R_y) - \dfrac{\pi}{4}\right]$	$20\log\left[\tan\left(\dfrac{V_{Proc}}{K_{AT}} + \dfrac{\pi}{4}\right)\right]$
LR	$K_{LR}\log(R_y)$	$20\dfrac{V_{Proc}}{K_{LR}}$
NPD	$K_{NPD}\left(R_y - R_y^{-1}\right)$	Does Not Exist

3 FUNCTION COMPARISON

Fig. 1 displays the combined BPM and processor TFs using a linear BPM response whose sensitivity is 1.11 mm^{-1} or 0.87 dB/mm. This linear BPM position response allows for a true and direct comparison of the individual processor TFs. Under centered beam conditions, the "K" sensitivity constants were normalized, resulting in processor TFs sensitivities $K_{\Delta/\Sigma}$, K_{AT}, K_{LR}, and K_{NPD} equaling 17.4-, 17.4-, 20-, 4.3-v, respectively. The LR function has a slightly larger gain constant than the Δ/Σ and AT function and the NPD's gain constant is significantly smaller. However, having a low gain constant is not advantageous if the function is highly nonlinear.

As displayed in Fig. 2, the Δ/Σ processor TF is the only function whose sensitivity is linear with beam position. The AT function is the least non-linear function and the only nonlinear function whose sensitivity reduces with increased displacement from the BPM's center. The NPD function is highly non-linear. This non-linearity either will limit the processor's bandwidth from beam-position-dependent gain switching, will have too large of a digital word for control system digitizers, or will provide inadequate beam position resolution for centered-beam conditions.

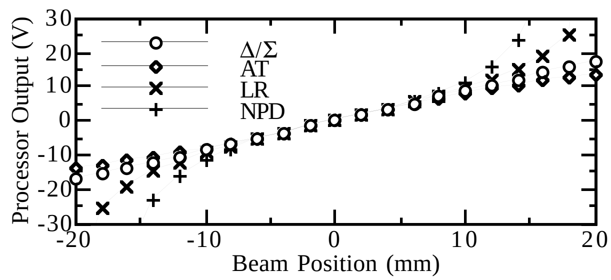

Figure 1. Processor output signal versus beam position for Δ/Σ, AT, LR, and NPD transfer functions. A linear 0.87-dB/mm BPM response was used and all processors were normalized to have the same centered beam response.

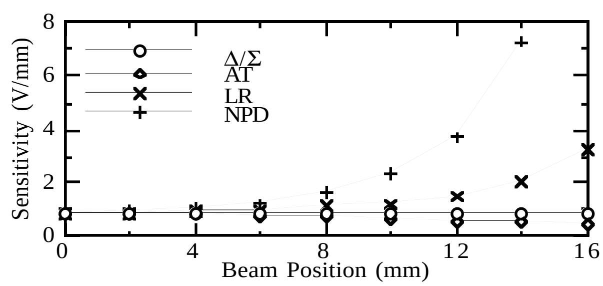

Figure 2. Combined sensitivities versus beam position for Δ/Σ, AT, LR, and NPD processor transfer functions using a linear 0.87-dB/mm-sensitivity BPM.

The cylindrical-geometry BPM changes the combined BPM and processor TF. Fig. 3 shows the processor TFs' sensitivities for a 6.7-MeV proton beam drifting through a BPM described by Eq. (1) with θ_0 and R_p equal to 45° and 25 mm, respectively. This particular BPM's sensitivity is 1.6 dB/mm. The added nonlinear position sensitivity of the BPM changes the shape of the combined position-sensitivity response. All of the TFs are nonlinear and the LR function is the least nonlinear. Both the AT and Δ/Σ position sensitivities approach zero as the beam displacement from the BPM's center is increased. Finally note that the NPD function continues to be highly nonlinear.

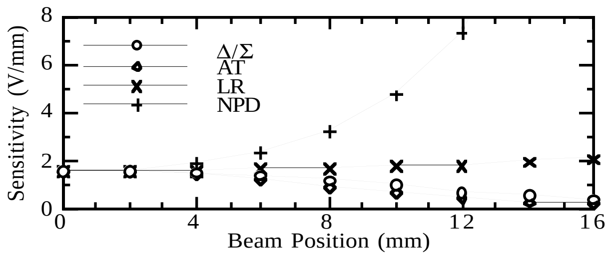

Figure 3. Combined sensitivities versus beam position for Δ/Σ, AT, LR, and NPD processor transfer functions using a cylindrical 1.6-dB/mm-sensitive BPM.

4 BPM RESPONSE TO DIFFUSE BEAMS

The calculation of a BPM's sensitivity, Eqs. (1), (2), and (3) assume the beam rms width is a significant portion of the BPM electrode radius, R_p. In most applications, this thin-beam assumption is adequate. However, in some low energy linacs, it is necessary to keep the beam pipe radius small. Because the particle beam has a finite size, the resultant beam-pipe radius to rms beam width ratio can be approximately 7:1. If the beam widths are sufficiently wide and these pipe-to-beam-width ratios are sufficient small, the BPM's position sensitivity diverges from the nominal thin-beam position sensitivity. This diffuse beam effect was experimentally observed in Fig. 6 of Reference 1.

To initially explore these diffuse-beam effects, the BPM electrode currents, as defined in Eqs. (2) and (3), were redefined as diffuse beams using a superposition of multiple thin beamlets whose beam currents were distributed in a two-dimensional gaussian distribution. Eq. (2) then becomes

$$I_T = \sum_{n=-3\sigma}^{+3\sigma}\sum_{m=-3\sigma}^{+3\sigma} \hat{I}_T a_n b_m \, e^{\frac{-\left(x_n^2-\bar{x}^2\right)}{2\sigma^2}} e^{\frac{-\left(y_m^2-\bar{y}^2\right)}{2\sigma^2}}, \qquad (8)$$

where $\hat{I}_T$ is the original top electrode current as defined in Eq. (2), σ is the round-beam rms width, and a_n and b_m are normalization constants. A similar equation was redefined for Eq. (3). The width for each of the bins in the diffuse beam distribution was 1σ. Fig. 4 displays the diffuse-beam effects to a BPM sensitivity using two rms beam widths of 2.85 mm and 0.01 mm and BPM electrode θ_0 and R_p of 45° and 7 mm, respectively. For the 2.85-mm diffuse beam, the BPM position sensitivity increases more than the thin-beam BPM sensitivity as the beam displacement from the BPM center is increased. However, beam pipe radius to rms beam-width ratios of 3:1, as shown in this example, are rare. Further calculations have shown that the sum of beam displacement and rms beam width must be greater than approximately 65 % of the BPM electrode radius for the BPM's response to significantly diverge from a BPM's thin-beam response.

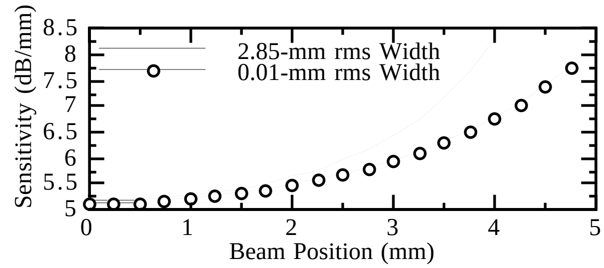

Figure 4. Diffuse- and thin-beam BPM sensitivity versus beam displacement from a BPM center. The BPM's radius and subtended angle were 7.0 mm and 45°, respectively.

5 CONCLUSION

All of the combined circular-cross-section BPM and processor TFs described in this note have odd symmetry and are nonlinear. The LR function is the least nonlinear, and therefore, the optimum choice. All of the processor functions have a single-value analytically expressible inverse function except for the normalized power-difference function. Finally, displaced diffuse-beam effects to BPM sensitivities were initially explored. It has been observed that sum of the rms beam width and displacement from BPM center must be >65% of the BPM radius for the diffuse beam effects to be significant.

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