SIBERIAN SNAKES FOR ELECTRON STORAGE RINGS

V.Ptitsin and Yu.Shatunov, BINP, 630090, Novosibirsk, Russia

Abstract

Applying a Siberian snake to obtain longitudinally polarized electron beam is discussed. Depolarization effects are analysed and spin matching conditions to decrease the depolarization are derived.

1 INTRODUCTION

Siberian snake was invented at mid-70th mainly as a mean against spin resonances which can destroy the beam polarization in circular accelerators[1]. Recently this concept has been tested and confirmed practically in a series of experiments carried out at the IUCF cooler ring[2].

However, there is another important area of the snake application, it is for obtaining longitudinal polarization at an interaction point. First longitudinally polarized electron beam was obtained at the energy of 27 Gev on the HERA electron ring where a pair of spin rotators was installed around the interaction point[3]. However, such the method is difficult to apply when beam energy is lower than 10 Gev because dipole magnets of the rotator will provide enormous orbit excursions inside the rotators. The Siberian snake is a special kind of a spin rotator which rotates particle spin by 180° angle around a direction lying in the horizontal plane. This direction is called by snake axis. An insertion of Siberian snake with longitudinal snake axis provides automatically the longitudinal beam polarization on a ring azimuth opposite to the snake insertion. Such a snake can be naturally performed with the use of solenoidal magnets. For a compensation of betatron modes coupling introduced by the solenoids, the snake must contain also quadrupole lenses (normal and/or rotated)[1]. The first experiments with the solenoidal Siberian snake for obtaining longitudinally polarized electrons was carried out at the AmPS storage ring at NIKHEF[4, 5]. and demonstrated the success of the given method.

The AmPS ring is operating in 300-900 Mev energy range. When using a Siberian snake at higher energies one must take into account a sharp depolarization increase with energy.

2 SPIN MATCHING CONDITIONS

In fact, the use of one Siberian snake leads to an unusual situation when the direction of the beam polarization is in horizontal plane everywhere over a ring excepting the snake itself. In this case the Sokolov-Ternov self-polarizing mechanism does not work and, on the other hand, a depolarization mechanism caused by quantum fluctuations of synchrotron radiation is considerably enhanced. Thus a special care must be taken to keep the beam polarization decay as slow as it is required by experiment needs. The depolarization rate is described by the well-known DK formula[7] as:

$$\tau^{-1} = \frac{5\sqrt{3}}{8}\frac{e^2\hbar\gamma^5}{m^2c^3}\,\alpha_+$$
$$\text{with}\qquad \alpha_+ = \Big\langle \frac{1}{|\rho|^3}\big[1-\frac{2}{9}(\hat{\mathbf{n}}\hat{\mathbf{v}})^2+\frac{11}{18}|\mathbf{d}|^2\big]\Big\rangle \quad (1)$$

where $\hat{\mathbf{n}}$ is a periodical spin solution, $\mathbf{d}=\gamma\frac{\partial\hat{\mathbf{n}}}{\partial\gamma}$ vector, ρ is bending magnet curvature, $\hat{\mathbf{v}}$ is an unit vector in the direction of particle velocity. The average is taken over the ring azimuth θ and, generally, over beam distribution.

The reduce of the depolarization time τ is completely determined by the value of the spin-orbit coupling vector $\mathbf{d}$. At the first-order approximation the $\mathbf{d}$ vector is orthogonal to $\hat{\mathbf{n}}$: $\mathbf{d}=\mathrm{Re}(iD\hat{\eta}^*)$ where $\hat{\eta}$ is an eigen solution of spin motion equation orthogonal to $\hat{\mathbf{n}}$. In general case the D function can be represented as a sum of two contributions: $D = D_\gamma + D_\beta$. Here D_γ comes from the direct dependence of the vector $\hat{\mathbf{n}}$ on the particle energy while D_β results from a jump of betatron amplitudes during an emission of a quanta.

We consider a practical case when there is no dispersion in the insertion region ($\psi_x=\psi_z=0$) and the coupling introduced by the solenoids is fully compensated by the insertion quadrupoles. For the snake located at $[\theta_1;2\pi]$ we obtain the expressions for $D_{\gamma,\beta}$ outside the snake insertion. The D_γ does not depend on internal structure of a snake:

$$D_\gamma = -\frac{\pi}{2}\sin(\pi\nu_0) + i\nu_0\left(\pi - \int_0^\theta \frac{H_z}{<H_z>}d\theta'\right)$$

where H_z and $<H_z>$ are the vertical field and its average value over ring azimuth, $\nu_0=\gamma a$.

The form of the expression for D_β depends on a number of solenoids used in the snake. For instance, for a snake containing one solenoid:

$$D_\beta = -\frac{\nu_0\pi}{2\cos(\pi\nu_x)}\Big[\cos(\pi\nu_0)\,\mathrm{Im}\big(\mathrm{e}^{i\pi\nu_x}J(\theta)f'^{*}_{Iz0}\big) - i\,\mathrm{Im}\big(\mathrm{e}^{i\pi\nu_x}f'^{*}_{Ix0}J(\theta)\big)\Big]$$

where f'_{Ix0} and f'_{Iz0} are first mode Floquet function derivatives taken at the midpoint of the solenoid and $J(\theta)=f_{Ix}\psi'_x - f'_{Ix}\psi_x$.

Two solenoids snake was applied at the AmPS storage ring This compact snake, designed and builded at BINP (Novosibirsk), uses two superconducting solenoids with up to 7.5 T field and five compensating quadrupoles:

[1] A general method of solenoid compensation is described in[6]

sq1 sq2 sol q sol -sq2 -sq1

Here (**sol**) stands for solenoid, (**sq1,sq2**) are quads rotated by 45° and (**q**) is a normal quadrupole. For two solenoids snake one finds:

$$D_\beta = -\frac{\nu_0 \pi}{4\cos(\pi\nu_x)}\Big[\cos(\pi\nu_0)\,\mathrm{Im}\big(e^{i\pi\nu_x} J(\theta) G^*_{Ix}\big) + \\ + \; i\,\mathrm{Im}\big(e^{i\pi\nu_x} G^*_{Iz} J(\theta)\big)\Big] \quad (2)$$

where $G_{Ix,z} = f'_{Ix,z(out)} - f'_{Ix,z(in)}$ is the difference of the first mode Floquet function derivatives at the entrance of the first solenoid and at the exit from second one. Also each solenoid has been treated here as having infinitely short edges. In the above expression a point just after the first edge is called by the solenoid entrance and a point just before the second edge is called by the exit.

Unlike the D_γ, the D_β contribution depends on lattice functions. In particular, this fact causes the dependence of the depolarization time on horizontal betatron tune and on any manipulation with optics of the storage ring. For example, in the Figure 1 τ is drawn versus horizontal tune for AmPS snake for two possible sets of gradients of compensating quadrupoles.

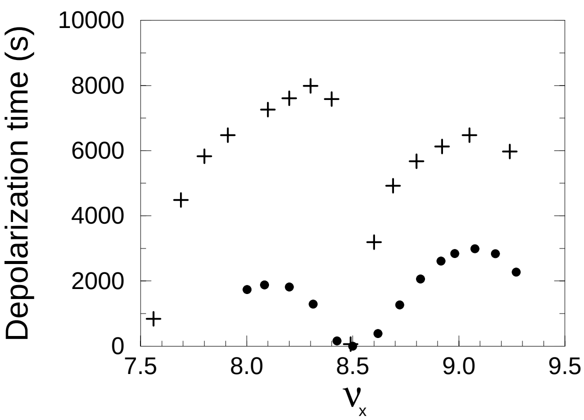

Figure 1: The dependence of depolarization time on the horizontal betatron tune, $E = 0.9$ Gev, $\nu_z = 7.25$. Circles for normal scheme, pluses for reversed gradient scheme.

The contribution of D_β term to the depolarization time can be comparable or even larger than the contribution resulting from D_γ. Figure 2 demonstrates the $|\mathbf{d}|$ variation along azimuth of AmPS ring again for the two possible sets of snake quadrupoles. At the top picture the D_β causes large oscillations of $|\mathbf{d}|$ in the magnets on background of smooth variation of D_γ. At the bottom picture D_β is small enough and this variant provides considerably larger depolarization time.

In order to increase the depolarization time and avoid its dependence on the ring optics one should apply snake scheme that cancels D_β term. For the case of two solenoids snake such the snake must provide $G_{Ix} = G_{Iz} = 0$ and,

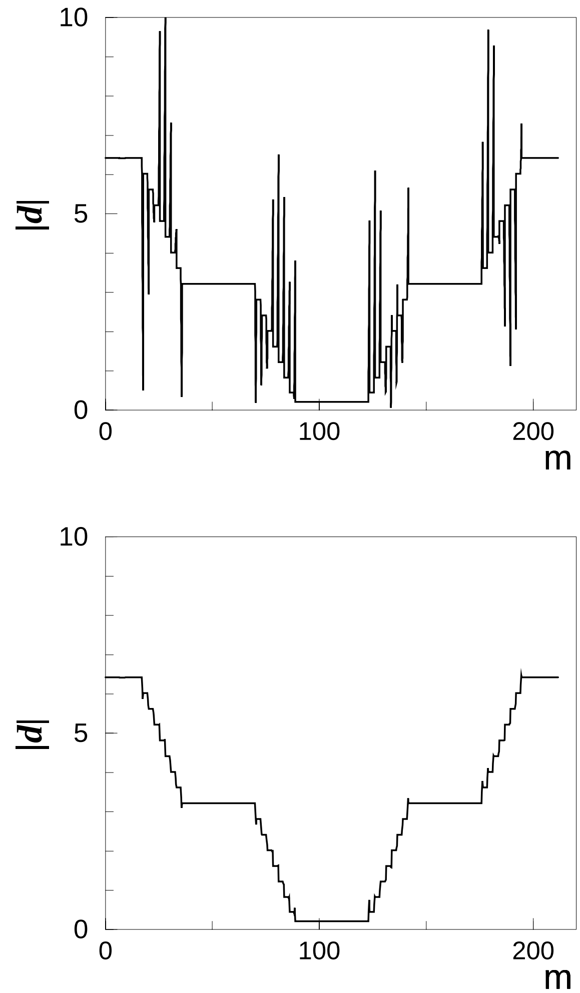

Figure 2: $|\mathbf{d}|$ along azimuth of the AmPS ring for normal (top) and reversed (bottom) gradient scheme.

therefore:

$$f'_{Ix(out)} = f'_{Ix(in)}, \quad f'_{Iz(out)} = f'_{Iz(in)} \quad (3)$$

It provides some relations on elements of transport matrix between the solenoids. Such a snake can be called a spin matched snake on the analogy with spin matching concept, introduced for spin rotators [3]. Actually in the considered case we have only partial spin matching since it is not possible to cancel the contribution of D_γ.

The simplest variant of spin matched snake is a scheme where the coupling compensation is carried out by six normal quadrupoles (**q1-6**) inserted between two solenoids:

sol q1 q2 q3 q4 q5 q6 sol

From the relations (3), taking into account the transformation of optical Floquet functions on solenoid edges, one can find that a transport matrix of the whole snake must have a form:

$$\begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

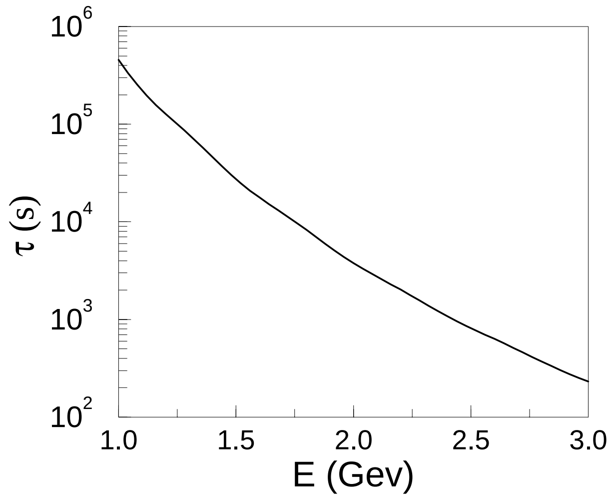

Figure 3: The depolarization time versus beam energy for the VEPP-4 with matched snake.

where I is the identity matrix. The strengths of the insertion quadrupoles are choosen to provide such the matrix.

Applying the matched snake increases the depolarization time. Nevertheless the depolarization time decreases rapidly with energy:

$$\tau \simeq \frac{54}{11}\frac{1}{\nu_0^2\pi^2}\,\tau_p \sim \frac{1}{E^7}$$

where τ_p is self-polarization time calculated when the snake is switched off.

For example, in the Figure 3 the dependence τ on beam energy is shown for the VEPP-4 storage ring with the spin matched snake. Thus for typical electron storage rings in this energy range the use of this method of obtaining longitudinally polarization is restricted to the energies below 3 Gev.

3 KINETIC POLARIZATION

As follows from DK formulas[7], though Sokolov-Ternov polarizing mechanism does not work, the equilibrium polarization should differ from zero level due to so-called kinetic polarizing mechanism caused by the dependence of synchrotron radiation probability on a spin projection on a field direction:

$$P_{eq} = -\frac{8}{5\sqrt{3}}\frac{\alpha_-}{\alpha_+} \quad \text{with} \quad \alpha_- = -\Big\langle \frac{\hat{\mathbf{b}}\cdot\mathbf{d}}{|\rho|^3}\Big\rangle$$

α_+ is given by (1) and $\hat{\mathbf{b}}$ is an unit vector in the direction of magnetic field.

This mechanism, predicted theoretically, has been never observed yet. A storage ring with a Siberian snake provides the unique possibility to confirm its existence. The kinetic polarization is driven by the vertical projection of $\mathbf{d}$ vector in bending magnets. For a snake with two solenoids this projection is:

$$d_z = -\frac{\pi\sin(\pi\nu_0)}{2} + \frac{\nu_0\pi\cos(\pi\nu_0)}{4\cos(\pi\nu_x)}\,\mathrm{Im}\big(\mathrm{e}^{i\pi\nu_x}J(\theta)G^*_{Ix}\big)$$

Thus the value of polarization and its sign depends both on the beam energy and, for an unmatched snake, on the ring optics. In the Figure 4 the equilibrium polarization dependence on energy is demonstrated for two variants of AmPS optics.

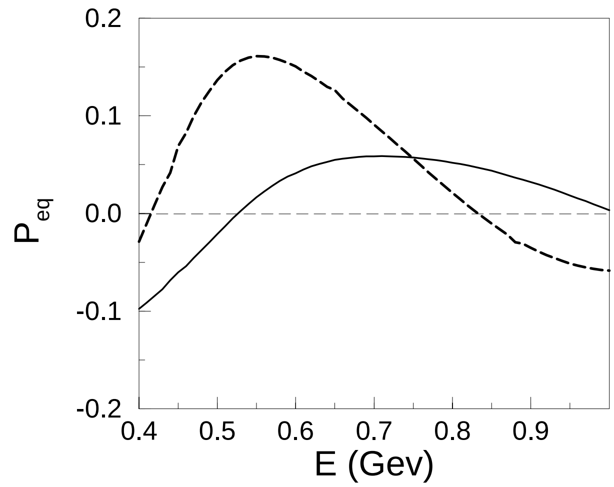

Figure 4: The equilibrium polarization versus beam energy for two variants of the AmPS ring with $\nu_x = 8.3$ (solid) and $\nu_x = 9.2$ (dashed).

Spin-matched Siberian snake provides higher level of the equilibrium polarization, but even in this case at energies higher than 1 Gev the equilibrium polarization level becomes very small:

$$P_{eq} \simeq \frac{72\sqrt{3}}{55}\,\frac{\sin\nu_0\pi}{\nu_0^2\pi} = 0.72\,\frac{\sin\nu_0\pi}{\nu_0^2}$$

4 REFERENCES

[1] Ya.S.Derbenev and A.M.Kondratenko, Part.Accel. **8**, 115 (1978).

[2] A.D.Krish et al., Phys.Rev.Letters **63**, 1137 (1989).

[3] D.P.Barber et al., Phys.Lett. **B 343**, 436 (1995).

[4] I.Passchier et al., Report presented on Worksh. on High-Energy Electron Polarimeters, Amsterdam, (1996).

[5] C.W. de Jager, V.Ptitsin and Yu.M.Shatunov, Report presented on 12th Intern. Symp. on High Energy Spin Physics, Amsterdam, (1996).

[6] A.A.Zholents, V.N.Litvinenko, Preprint INP 81-80, 1981.

[7] Ya.S.Derbenev and A.M.Kondratenko, Sov.Phys.JETP **37**, 968 (1973).