SIMULATION OF TAIL DISTRIBUTIONS DUE TO RANDOM PROCESSES AND BEAM-BEAM INTERACTION IN KEKB

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Abstract

A simulation that uses a new technique to obtain tail distributions in electron-positron storage rings is applied to KEKB. This program makes it possible to investigate tail distribution in simple and fast simulation technique and shows good agreement with solvable cases. The simulation now includes exact solution for six dimensional beam-beam interaction and several rare random processes. An estimate of the lifetimes for KEKB is also presented.

1 INTRODUCTION

It is important to calculate beam distributions by considering several processes which affect the beam tails. However, it is not always possible to obtain beam distributions by analytical treatments.

D.N. Shatilov developed a method to reduce the CPU time in simulating the beam tail due to scattering by the residual gas[1]. The beam tail is then obtained by considering the contribution of small angle scattering which occurs with a high degree of probability. However, it is not sufficient to consider small amplitude and frequent random processes only when we estimate the beam tail. Thus, we will investigate the beam tails caused by the rare and large amplitude processes from the core.

The aim of this paper is to propose a simple and fast simulation technique for various rare random processes on the beam tails. It is shown that results of this simulation technique show good agreement with solvable cases[2]. We thus expect that this simulation technique is applicable to obtain beam tails in the cases of analytically unsolvable random processes. The beam lifetime are also obtained by counting number of the particles extending beyond energy and transverse apertures.

In Sec. 2 the simulation technique is described and results of the simulation are compared with those of analytically solvable examples. Some applications are given in Sec. 3, and Sec. 4 is devoted to a discussion and conclusions.

2 DESCRIPTION OF SIMULATION TECHNIQUE

As an example, we can consider the case in which an electron loses energy by random processes in a ring. The initial distributions of n macroparticles in the phase spaces are given randomly with specified variances. Each macroparticle i has a particle number (N_i) and p is the probability that an electron undergoes a random process in one turn. Once an electron in a maroparticle undergoes this process, we create a new macroparticle $(n+1)$-th. This new macroparticle has one particle (N_{i+1}=1) and the macroparticle which undergone a random process now has a number of particles $N_i - 1$.

We assume that the variation in the random variable due to a random process is limited to a range between a minimum and a maximum value. The variation in the random variable due to a random process can be obtained by the following way. First, calculate the probability $(P \equiv N_i p)$ that a macroparticle undergoes a random process in the ring, and generate one uniform random number (0$\leq x \leq$1). If $x < P$, a random process occurs for the macroparticle. Second, generate a uniform random number (ε_1) in the interval between the minimum value (ε_c) and the maximum value (ε_m) and one uniform random number in the interval $0 < y < (\frac{d\sigma(\varepsilon)}{d\varepsilon})_{max}$, and compare y and $(\frac{\mathrm{d}\sigma(\varepsilon)}{\mathrm{d}\varepsilon})_{\varepsilon=\varepsilon_1}$. Here ε is the energy random variable and $(\frac{\mathrm{d}\sigma(\varepsilon)}{\mathrm{d}\varepsilon})_{\varepsilon=\varepsilon'}$ is the cross section corresponding to the ε'. If $y < (\frac{\mathrm{d}\sigma(\varepsilon)}{\mathrm{d}\varepsilon})_{\varepsilon=\varepsilon_1}$, the random variation corresponding to ε_1 is given to an electron. If $y > (\frac{\mathrm{d}\sigma(\varepsilon)}{\mathrm{d}\varepsilon})_{\varepsilon=\varepsilon_1}$, discard these ε_1 and y, and generate new ε_1 and y until the relation $y < (\frac{\mathrm{d}\sigma(\varepsilon)}{\mathrm{d}\varepsilon})_{\varepsilon=\varepsilon_1}$ holds.

It is shown that equilibrium distributions are little changed by variations of ε_c of around 0.001% in beam-gas bremsstrahlung. The number of initial macroparticles that are used in simulation is 40000. We investigate the present method using design parameters of KEKB, τ_ε=2300, τ_x=τ_y=4600, ν_s=0.01, ν_x=47.52 and ν_y=43.08[3].

The motion of each macroparticle is as follows:

1. Input

We rescale the variables as follows:

$$X = \frac{\mathrm{x}}{\sigma_x^o}, \quad P = \frac{\beta^{IP} P_x}{\sigma_x^o}, \quad Y = \frac{\mathrm{y}}{\sigma_y^o},$$

$$Q = \frac{\beta^{IP} P_y}{\sigma_y^o}, \quad Z = \frac{\mathrm{z}}{\sigma_z^o}, \qquad E = \frac{\varepsilon'}{E_o\ \sigma_\varepsilon^o}$$

where $\sigma_{x,y}^o$ and σ_z^o are nominal beam sizes in transverse and longitudinal directions. β^{IP}, E_o, σ_ε^o and ε' (=$E - E_o$) are betatron function at IP, nominal energy, relative energy spread and energy deviation due to a random process, respectively.

2. Random Process

$$E' = E - \frac{\varepsilon'}{E_o \sigma_\varepsilon^o}, \tag{1}$$

where ε' is given by values between the minimum energy and the energy aperture of the beam.

$$P = P - \frac{\theta_x}{\sigma'_x}, \qquad Q = Q - \frac{\theta_y}{\sigma'_y}, \tag{2}$$

where transverse scattering angles $\theta_{x,y}$ are given by values between the minimum angle and the transverse apertures of the beam. $\sigma'_x = \frac{\sigma_x}{\beta_x}$ and $\sigma'_y = \frac{\sigma_y}{\beta_y}$. Here, $\sigma_{x,y}$ and $\beta_{x,y}$ are the transverse beam sizes and betatron functions at the position where the random process occurs, respectively. Equations (1) and (2) can be applied for longitudinal random processes and beam-gas scattering, respectively.

3. Synchrotron and Betatron Oscillations
4. Synchrotron Radiation

On the other hand, the probability that an electron loses energy ε is given by

$$f(\varepsilon) = \frac{1}{\sigma_{tot}} \frac{d\sigma}{d\varepsilon}. \tag{3}$$

When a random proces occurs, then its contribution to equilibrium distribution has the expression

$$\exp[N \int_0^\infty dt \widetilde{f}\{K e^{-dt} sin(\phi + wt)/(E_o \sigma_\epsilon^o)\} - 1], \tag{4}$$

where $\phi = tan^{-1} K_2/K_1$[4]. We assume that the number of synchrotron oscillations during one damping time is very large. We can then replace the synchrotron oscillation by an average over each synchrotron period. A solvable model to obtain the distribution functions in longitudinal and transverse random processes was shown in Refs. [2] and [4].

To show the validity of the simulation technique, we compare the results of the simulation for the cases of beam-gas bremsstrahlung and beam-gas scattering with those of the solvable model.

2.1 Beam-Gas Bremsstrahlung

An electron with energy E_o, which passes a molecule of the residual gas, loses its energy due to the radiation emitted when an electron is deflected. There is a certain probability that a photon with energy u is emitted, producing an electron with energy E', where $E' + u = E_o$. The differential cross section for an energy loss due to bremsstrahlung between E and $E + dE$ is given by

$$d\sigma = 4\alpha r_e^2 Z(Z+1) \frac{\mathrm{du}}{\mathrm{u}} \frac{E'}{E_o} [(\frac{E_o^2 + E'^2}{E_o E'} - \frac{2}{3}) \log \frac{183}{\mathrm{Z}^{1/3}} + \frac{1}{9}], \tag{5}$$

where Z, α and r_e denote the atomic number, the fine-structure constant and the classical electron radius, respectively[5].

We assume that one type of molecule uniformly exists in the ring, so that N=$Q\sigma c$, Q=2.65 $\times 10^{20} n P_a$, where σ, c, Q, n and P_a are cross section of the beam-gas bremsstrahlung, velocity of the light, number of gas molecules in a unit volume, number of atoms in each gas molecule and partial pressure of the gas in pascals. Figure 1.(a) shows energy distribution produced from the tracking of synchrotron oscillation, synchrotron radiation and beam-gas bremsstrahlung for a vacuum pressure of 10^{-9} Torr. The dotted lines and square symbols are equilibrium distributions obtained from the simulation and solvable model, respectively. Figure 1.(b) shows energy distribution produced from the tracking of the synchrotron oscillation and synchrotron radiation. It may be inferred that the beam tails in Figs. 1.(a) and (b) are caused by the influence of beam-gas bremsstrahlung.

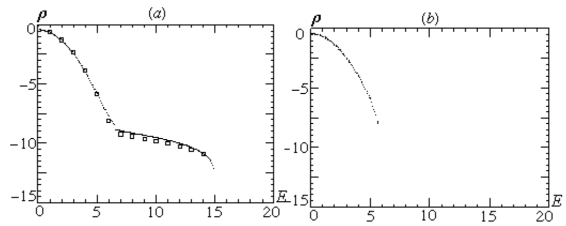

Figure 1: The horizontal axis is E, the energy deviation normalized by the energy spread. The vertical axis represents the distribution in E measured using a logarithmic scale. The number of turns is 230000.

2.2 Beam-Gas Scattering

The cross section of the elastic scattering with an atom is given by Rutherford scattering formula. Figure 2.(a) shows the vertical distribution produced from the tracking of beam-residual gas scattering, betatron oscillation, synchrotron oscillation and synchrotron radiation. Here we considered scattering angle occuring between $8\sigma'_y$ radian and $300\sigma'_y$ radian. The β_y value at the position where the scattering occurs is set to 10 meters. The dotted lines and square symbols show equilibrium distributions from obtained the simulation and solvable model, respectively.

We see from the above two examples that the simulation shows good agreements with the solvable model.

3 APPLICATION

For practical purposes, we consider beam-beam interaction and random processes with a non-uniform density distribution such as beam-beam bremsstrahlung and Bhabha scattering. Transverse beam-beam force is given by Bassetti-Erskine formula[6] and synchro-beam mapping is considered[7].

3.1 Beam-gas Scattering and Beam-Beam Interaction

Figures 2.(b) and (c) show the horizontal and vertical distributions due to beam-residual gas scattering for a pressure of 10^{-9} Torr and beam-beam interaction. It is shown that particle loss is increased by the beam-beam interaction. It is also shown that the vertical tail is caused by the beam-gas scattering rather than the beam-beam interaction.

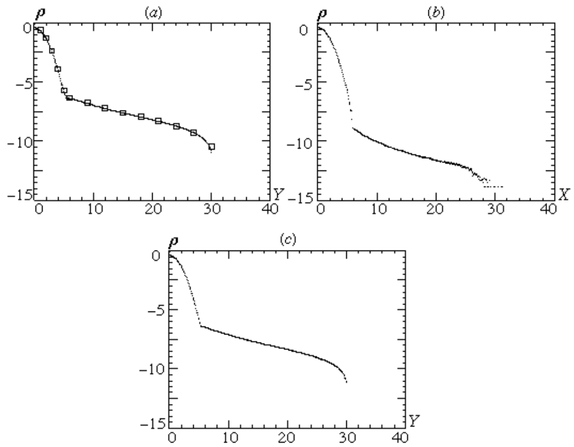

Figure 2: (a) shows vertical distribution due to beam-gas scattering. (b) and (c) show horizontal and vertical distributions due to beam-gas scattering and beam-beam interaction. The horizontal axis is Y, the distance normalized by the nominal vertical beam size. The vertical axis represents the distribution in Y measured using a logarithmic scale. The number of turns is 46000. Horizontal and vertical apertures are assumed to $34\sigma_x^{o\prime}$ and and $30\sigma_y^{o\prime}$ at IP, respectively, where $\sigma_x^{o\prime}=\sigma_x^o/\beta_x^{IP}$ and $\sigma_y^{o\prime}=\sigma_y^o/\beta_y^{IP}$.

3.2 Random Processes in Longitudinal and Transverse Motions and Beam-Beam Interaction

Figures 3.(a), (b) and (c) show the horizontal, vertical and longitudinal distributions due to the random processes of Bhabha scattering, beam-gas scattering, beam-gas bremsstrahlung, beam-beam bremsstrahlung and beam-beam interaction. The apertures are limited by $34\sigma_x^{o\prime}$, $30\sigma_y^{o\prime}$ and E=15 in the horizontal, the vertical and the energy directions, respectively. We can see that particle losses are mainly limited by the energy and the vertical apertures.

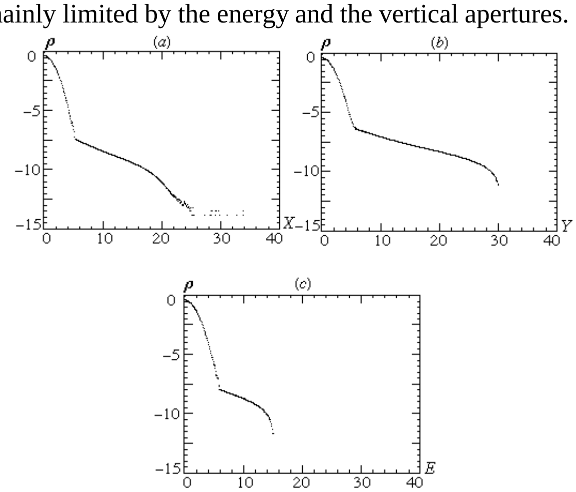

Figure 3: (a) Horizontal, (b) vertical and (c) longitudinal distributions due to longitudinal and transverse random processes and beam-beam interaction after 46000 turns. Pressure in ring is 10^{-9} Torr.

3.3 Lifetime as a Function of Apertures in KEKB

Beam lifetimes for bremsstrahlungs and scatterings as a function of energy and vertical apertures are listed in unit of hours below. Horizontal aperture is assumed to $34\sigma_x^{o\prime}$. If

Energy aperture	E=10	E=15	E=20
Beam-Beam Brems.	6.6	7.3	7.8
Beam-gas Brems.	32.4	35.3	37.7
Vertical aperture	Y=$25\sigma_y^{o\prime}$	Y=$30\sigma_y^{o\prime}$	Y=$35\sigma_y^{o\prime}$
Bhabha Scattering	13182	18595	25252
Beam-gas Scattreing	21.8	31.3	42.7

we estimate the lifetime including the beam-beam interaction to four random processes, it gives the lifetime around 5 hours in the aperture limitations of $34\sigma_x^{o\prime}$, $30\sigma_y^{o\prime}$ and E=15 for KEKB. It is shown that particle loss due to the beam-beam bremsstrahlung plays important role on electron beam lifetime for KEKB.

4 DISCUSSION AND CONCLUSION

Two simplifications were employed for our simulation to be valid: first, once a particle undergoes a random process, it does not undergo a random process again. The choice of a too small ε_c reduces the efficiency of this simulation, without giving any contribution to the tail distribution; and large ε_c can result in larger statistical fluctuations. Secondly, we assumed that new macroparticles in longitudinal and transverse directions reach an equilibrium state during two longitudinal and transverse damping times, respectively. This assumption makes it possible to simulate long-term runs.

The tail distributions are obtained by considering only the events of relative large amplitudes in several rare random processes for KEKB. It is shown that the beam-beam bremsstrahlung in KEKB is the predominate random process leading to particle losses.

A new simulation technique for beam tails is presented. This simulation method provides a simple and fast means to obtain the tail distributions due to various random processes in the storage rings. This simulation showed a good agreement with the solvable cases.

On the other hand, if random processes that produce beam tails occur with high probability, this situation requires us to generate more new macroparticles in our simulation. We would thus need more tracking times in this case.

The contribution of small amplitude scattering to the tail can be obtained by the simulation method of D.N. Shatilov. The effect of large amplitude scattering on the tail can be treated by this simulation method.

5 REFERENCES

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